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Drawdown Distribution During Recovery around a Large Diameter Well

Zekâi Şen

Istanbul Technical University,
Ayazağa, Istanbul, Turkey

An analytical solution is presented for the drawdown distribution in a large diameter well during recovery. It takes into consideration the well storage. A set of type curves are given to determine the transmissivity and storage coefficient of the aquifer. The basis of the methodology is to solve simultaneous differential equations of the water balance and Darcy laws. The application of developed methodology has been given for recovery data from India and Saudi Arabia.

Introduction

Large diameter wells are usually hand dug and mostly exist in the Middle Eastern countries, Asia and Africa. Their diameters may vary between 1 and 3 metres and in the Kingdom of Saudi Arabia the diameters may reach up to 10 metres.

In the past processing of groundwater data has been achieved most often by the modified non-equilibrium method presented by Cooper and Jacob (1946). Since, the basis of this method is the well known Theis (1935) equation, all of the assumptions involved in its derivation reflect themselves.

One of the main assumptions is that the main well is substituted by a line sink which implies an infinitesimally small well diameter as a result of which no well storage is considered. Therefore, this method, due to its very basis, practically yields satisfactory results either for large time t or small radial distance r so that the dimensionless time factor $u = r^2S/4tT$, becomes sufficiently small. Herein, S and T

are the storage coefficient and transmissivity, respectively. The above referred restriction leads to the analysis of recovery data on the basis of late time solution. However, for the early stages of the recovery, the straight line assumption cannot be valid and therefore, the recovery problem deserves a special treatment. For small u values the plot of residual drawdown versus $\log(t'/t)$ gives a straight line, where t' is the pumping stop time. Hence, it is not possible to incorporate the early recovery data in such an approach. Case *et al.* (1974) have presented another method again based on the Theis equation whereby they have approximated the exponential integration by convenient series. However, their analysis suffer from the drawback of not being able to account for the well storage and valid for small diameters only. Fenske (1977) proposed a technique of time/residual drawdown analysis as an extension of Theis equation where the well storage is taken into consideration. Calculation of the type curves requires the use of exponential integral which can be taken from available tables.

Large diameter well hydraulics leading to pumping test data assessment have been tackled first by Hantush (1964) with the assumption that the rate at which water is pumped from the well is equal to that entering the well so that the well storage is ignored. Another major contribution to large diameter wells is due to Papadopulos and Cooper (1967) who developed a set of type curves for aquifer parameter determination, taking into consideration the well storage as a boundary condition.

Apart from the aforementioned analytical solutions, numerical methods of pumping test analysis for large diameter wells have been used successfully in the time/drawdown record interpretation (Rushton and Sarah 1981). The basis of these numerical methods is the differential equation of groundwater movement which is solved by using discrete space and time approximations. On the other hand, Basak (1983) found an approximate solution for the unsteady groundwater flow equation for the region around a large diameter well. Later, Şen (1983) obtained approximate type curves based on Basak's work.

This paper gives the exact analytical solution of the recovery problem in confined aquifer and large diameter wells which is hitherto unreported in published literature.

Problem Statement and Solution

Let us consider a non-leaky artesian aquifer of infinite extent, homogeneous, isotropic, uniform in thickness, b, and tapped by a large diameter well which penetrates completely the aquifer, (see Fig. 1a). The radius of well is r_w. The hydraulic parameters are regarded as spatially and temporally constant. A prerequisite in any recovery test is that the pumping will continue for a certain time t',

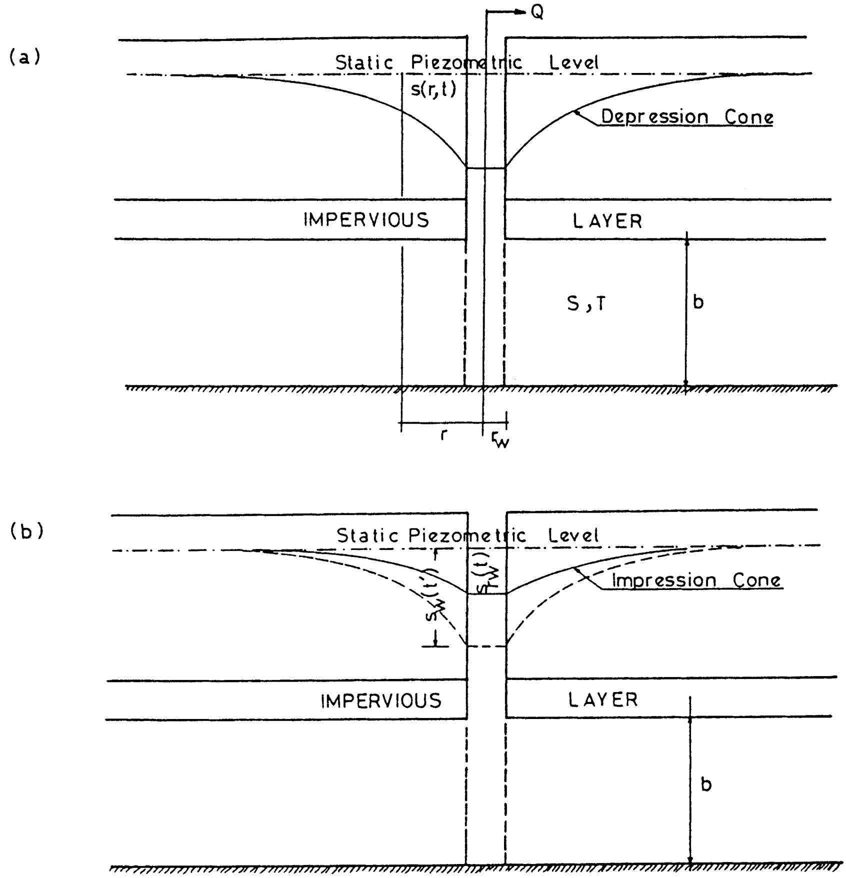

Fig. 1. (a) Description of discharging well; (b) Description of recovery well.

with a constant discharge Q, and then it is stopped. At this instance, the drawdown in the main well has its maximum value $s_w(t')$, which then starts to retreat itself to its original level with the passage of time. A record of time/drawdown is kept during this retraction, and subsequently matched with a theoretically derived type curve so as to determine the aquifer constants which are, the transmissivity T, and the storage coefficient S.

At the time of pump stop, the flow in the aquifer may or may not reach to a steady state. However, it should be preferred always to attain more or less a steady state condition. During the recovery, water is being taken into the storage of the well and the aquifer part which is affected by the depression cone. After a certain time t, from pump-stop the water level in the well retracts itself to $s_w(t)$ position corresponding to impression cone shown in Fig. 1b.

Application of the water balance equation during time interval dt in the recovery period, between two concentric cylinders of radii r and $r+dr$ centered at the well axis leads to

$$\frac{\delta Q(r,t)}{\delta r} = 2\pi r S \frac{\delta\phi(r,t)}{\delta t} \tag{1}$$

where $Q(r, t)$ and $\phi(r, t)$ are discharge and hydraulic head at concentric cylinder of radius r. The discharge can be written in terms of filter velocity as

$$Q(r\ \ t) = 2\pi r b v(r,t) \tag{2}$$

by taking derivatives of both sides with respect to r

$$\frac{\delta Q(r,t)}{\delta r} = 2\pi b v(r,t) + 2\pi r b \frac{\delta v(r,t)}{\delta t} \tag{3}$$

The substitution of Eq. (3) into Eq. (1) gives

$$\frac{\delta v(r,t)}{\delta r} + \frac{1}{r} v(r,t) = \frac{S}{b}\frac{\delta\phi(r,t)}{\delta t} \tag{4}$$

In addition, Darcy law says that

$$v(r,t) \equiv k \frac{\delta\phi(r,t)}{\delta r} \tag{5}$$

where k is the hydraulic conductivity. The last two equations completely describe the groundwater flow to a well. Substitution of Eq. (5) into Eq. (4) results in general groundwater equation

$$\frac{\delta^2\phi(r,t)}{\delta r^2} + \frac{1}{r}\frac{\delta\phi(r,t)}{\delta r} = \frac{S}{T}\frac{\delta\phi(r,t)}{\delta t}$$

This equation has been solved by Laplace transformation and Bessel functions for various groundwater problems with relevant initial and boundary conditions. However, herein the solution of the problem is achieved through simultaneous treatment of Eqs. (4) and (5) as follows. Ordinary differential forms of these equations can be solved by using Boltzman's transformation which is

$$\eta = \frac{r}{2t^{\frac{1}{2}}} \tag{6}$$

Application of chain rule for differentiation gives that

$$\frac{\delta v(r,t)}{\delta r} = \frac{1}{2t^{\frac{1}{2}}}\frac{dv(\eta)}{d\eta}$$

$$\frac{\delta\phi(r,t)}{\delta r} = \frac{1}{2t^{\frac{1}{2}}}\frac{d\phi(\eta)}{d\eta}$$

and

$$\frac{\delta\phi(r,t)}{\delta t} = \frac{\eta}{2t}\frac{d\phi(\eta)}{d\eta} \tag{7}$$

The substitution of these derivations into Eqs. (4) and (5) yields

$$\frac{1}{\eta}\frac{\delta v(\eta)}{\delta\eta}+\frac{1}{\eta^2}v(\eta)\equiv\frac{S}{b}t^{-\frac{1}{2}}\frac{d\phi(\eta)}{\delta\eta} \tag{8}$$

and

$$\frac{d\phi(\eta)}{d\eta}=\frac{v(\eta)}{k}\frac{1}{t^{\frac{1}{2}}} \tag{9}$$

Combination of these two equations is

$$\frac{1}{\eta}\frac{dv(\eta)}{d\eta}+\frac{1}{\eta^2}v(\eta)+\frac{S}{T}v(\eta)\equiv 0 \tag{10}$$

The solution of this linear differential equation is

$$v(\eta)=C\frac{\exp(-\frac{S}{T}\eta^2)}{\eta}$$

or in terms of *r* and *t* it becomes

$$v(r,t)=2Ct^{\frac{1}{2}}\frac{\exp(-\frac{r^2S}{4tT})}{r} \tag{11}$$

So far neither the initial nor the boundary conditions are used. For the problem at hand, they are

$$\phi(r_w,t)=\phi_w(t) \tag{12}$$

$$\phi(\infty,t)\equiv\phi_0 \qquad \text{(static piezometric level)} \tag{13}$$

$$\phi(r,0)=\phi(r,t') \tag{14}$$

$$\phi(r_w,0)=\phi(r_w,t') \tag{15}$$

and

$$\lim_{r\to r_w}[2\pi rbv(r,t)]=\pi r_w^2\frac{ds_w(t)}{dt} \tag{16}$$

where $ds_w(t)$ is the drawdown increment in the well after pump stop. Eq. (12) states that the drawdowns at the well face in the aquifer and in the well are equal. Conditions in Eqs. (14) and (15) state that the initial drawdown for the recovery are equal to the final (or maximum) drawdowns during the pumping period. Boundary condition in Eq. (16) expresses that the rate of discharge from the aquifer to the well is equal to the rate of increase in volume of water within the well.

Substitution of Eq. (11) into Eq. (16) yields after some mathematical manipulations

$$C \equiv + \frac{r_w^2 \frac{ds_w(t)}{dt}}{4bt^{\frac{1}{2}}} \exp\left(\frac{r_w^2 S}{4tT} \right)$$

Hence, Eq. (11) can be rewritten as

$$v(r,t) \equiv \frac{r_w^2}{2b} \frac{ds_w(t)}{dt} \exp\left(\frac{r_w^2 S}{4tT} \right) \frac{\exp\left(-\frac{r^2 S}{4tT}\right)}{r} \tag{17}$$

Eqs. (5) and (17) can be exploited to find the hydraulic head distribution around the well by taking integration from r to ∞. Hence,

$$s(r,t) = \frac{r_w^2}{4bk} \frac{ds_w(t)}{dt} \exp\left(\frac{r_w^2 S}{4tT} \right) \int_r^\infty \frac{\exp\left(-\frac{r^2 S}{4tT}\right)}{r} dr \tag{18}$$

where $s(r, t) = \phi(\infty, t) - \phi(r, t)$ is the drawdown at radial distance. In order to simplify Eq. (18) the following dimensionless time factor definitions can be considered

$$u = \frac{r^2 S}{4tT} \tag{19}$$

and

$$u_w = \frac{r_w^2 S}{4tT} \tag{20}$$

From Eq. (20) one can find that

$$dt \equiv - \frac{r_w^2 S}{4Tu_w^2} du_w \tag{21}$$

Substitution of Eqs. (19), (20) and (21) into Eq. (18) yields

$$s(r,t) = - \frac{u_w^2 \exp(u_w)}{S} \frac{ds_w(t)}{du_w} \int_u^\infty \frac{e^{-u}}{u} du \tag{22}$$

It gives the relationship between the drawdown in the main well and any observation well at distance r from the well center. In order to find drawdown variation within the main well only, it suffices to replace u and u_w in Eq. (22) which becomes

$$s_w(t) = - \frac{u_w^2 \exp(u_w)}{S} \frac{ds_w(t)}{du_w} \int_{u_w}^\infty \frac{\exp(-u_w)}{u_w} du_w \tag{23}$$

It can be rearranged in a differential equation form as

$$\frac{ds_w(t)}{s_w(t)} = -S \frac{du_w}{u_w^2 \exp(u_w) \int_{u_w}^{\infty} \frac{\exp(-u_w)}{u_w} du_w}$$

Hence, integration from 0 to $s_w(t)$ gives

$$\ln\left[\frac{s_w(t)}{s_w(t')}\right] = -S \int_{u_w}^{\infty} \frac{du_w}{u_w^2 \exp(u_w) \int_{u_w}^{\infty} \frac{\exp(-u_w)}{u_w} du_w}$$

By considering that the residual drawdown is $s_{rw}(t) = s_w(t') - s_w(t)$, it can be written as

$$\ln\left[1 - \frac{s_{rw}(t)}{s_w(t')}\right] = -S \int_{u_w}^{\infty} \frac{du_w}{u_w^2 \exp(u_w) \int_{u_w}^{\infty} \frac{\exp(-u_w)}{u_w} du_w}$$

This indicates that a logarithmic recovery response is valid. The ratio $s_{rw}(t)/s_w(t')$ is referred to as the relative residual drawdown. Finally, this relative residual drawdown which is in fact a percentage, can be found as

$$\frac{s_{rw}(t)}{s_w(t')} = 1 - e^{\int_{u_w}^{\infty} -S \frac{\exp(-u_w)}{u_w^2 \exp(u_w) \int_{u_w}^{\infty} \frac{\exp(-u_w)}{u_w} du_w} du_w} \qquad (24)$$

For very small times u_w is very large, and therefore, the right hand side of Eq. (24) is equal to zero which implies that $s_w(0) = 0$. On the other hand, for large times, the exponent term becomes $-\infty$ and hence, $s_{rw}(t') = s_w(t')$. This proves that, theoretically the recovery reaches the initial static piezometric level after infinitely large times from the pump stop.

Type Curves and Discussion

The type curves for the recovery well can be defined as variations of $s_{rw}(t)/s_w(t')$ with u_w for a given S value. They are obtained by solving Eq. (24) numerically and plotted in Fig. 2. The general characteristic common to all of these type curves is

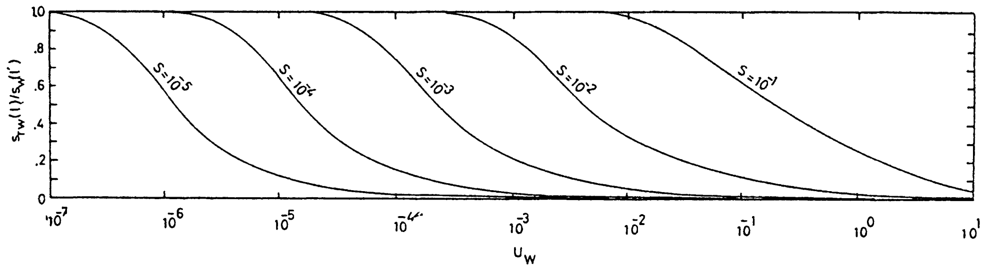

Fig. 2. Recovery type curves.

that for small times (u_w large), the recovery is quite rapid compared to large times where the relative residual drawdown attains its asymptotic value of zero in an extremely slow manner.

As the storage coefficient becomes smaller the type curve forms become indistinguishable from each other. Consequently, determination of S value by this method becomes questionable similar to the Papadopulos and Cooper (1967) method. If S is determined by some other method, then determination of T is insensitive to the choice of type curve to be matched. However, for relatively large S values, the type curves are rather distinct from each other and gives rise to the opportunity of determining S value from the type curve matching.

The method presented herein will be valid approximately for unconfined aquifer case provided that the drawdowns are comparatively smaller than the aquifer thickness.

Application

Determination of the aquifer constants is achieved through matching the best suitable type curve to the field data. The necessary steps are as follows:

(i) from the field data, the corresponding relative drawdown, $s_{rw}(t)/s_w(t')$, versus $\log(t)$ are plotted on a semi-logarithmic paper with the same scale per log cycle as the type curve set. This plot is drawn preferably on a transparent paper.

(ii) the field data scatter is overlain on the type curve sheet and moved until a good match with one of the curves is obtained in such a way that the corresponding logarithmic axis on both sheets overlaps at all times.

(iii) the storage coefficient is simply read from the best fitting type curve and the transmissivity is calculated from Eq. (20) as

$$T = \frac{r_w^2 S}{4 u_{wM} t_M} \tag{25}$$

where u_{wM} and t_M are the dimensionless time factor and time values corresponding to any arbitrarily chosen matching point on the common areas of type curves and field data sheets.

First of all recovery data observed by Basak (1982) in India will be used to demonstrate the procedure that is to be applied for interpreting field data. The first two columns in Table 1 shows observations for recovery times, and corresponding water levels, h_w, in the well with reference to the confining layer. The diameter of the well is 2.14 m. The static piezometric level and initial water level at the pump stop are 1.3 m and 0.18 m, respectively. Therefore, the maximum recovery will be 1.30-0.18 = 1.12 m.

Table 1 – Recovery data from India

Time (hours) (1)	Water Level (m) (2)	Recovery Drawdown (m) (3)	Relative Drawdown (4)
0	0.18	0	0
1	0.31	0.13	0.12
2	0.43	0.25	0.22
3	0.54	0.36	0.32
4	0.63	0.45	0.39
5	0.72	0.54	0.48
	1.30	1.12	1.00

The following procedure should be used in the interpretation of the data in Table 1.

(i) the recovery and relative drawdowns are calculated from the field data and presented in columns (3) and (4), respectively.

(ii) plot the data in Table 1 for relative drawdowns versus time on a semi-logarithmic paper with the same scale as for the type curves, (see Fig. 3).

(iii) the best fit to the field data is obtained with a type curve labelled $S = 10^{-4}$. The matching point readings are $1/t_M = 4.6 \times 10^{-1}$ 1/hours and $u_{wM} = 4.0 \times 10^{-5}$. Hence, from Eq. (25) it is possible to find $T = 1.55$ m^2/hours or about 37 m^2/day which shows that the aquifer is of moderate potential (Gheorghe 1978).

A second set of data is from the western part of Saudi Arabia. This area is composed of precambrian rocks and the water is available mostly in the quaternary alluvial deposits of wadis. A recovery test was conducted right after the pumping test in one of the large diameter wells, 2.5 m in diameter. The recovery period was rather long more than 40 hours. The relevant data for the application of the methodology developed in this paper are presented in Table 2.

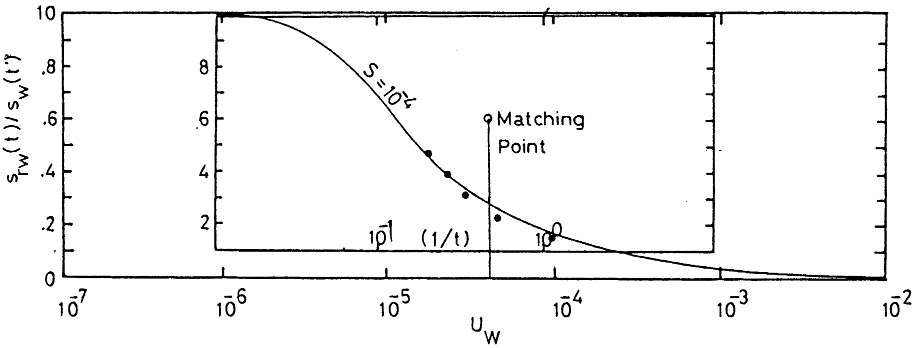

Fig. 3. Type curve matching to field data from India.

The maximum recovery is about 2.06 m. Again the field data are matched to type curve with label $S = 10^{-4}$, (see Fig. 4). The match point reading on the horizontal axis of both field and type curve sheets are $1/t_M = 7.5 \times 10^{-3}$ (1/min) and $u_{wM} = 1 \times 10^{-3}$. Substitution of these values into Eq. (25) yields $T = 1.17 \times 10^{-3}$ m^2/min which is equivalent to about 168 m^2/day, hence the aquifer is of very low potential.

Table 2 – Recovery data from Saudi Arabia

Time (min) (1)	Water Level (m) (2)	Recovery Drawdown (m) (3)	Relative Drawdown (4)
0.0	3.00	0.00	0.0
0.5	3.02	0.02	0.009
1.0	3.03	0.03	0.015
3.0	3.04	0.04	0.020
7.0	3.06	0.06	0.029
10.0	3.09	0.09	0.044
13.0	3.11	0.11	0.054
16.0	3.14	0.14	0.068
22.0	3.19	0.19	0.092
31.0	3.21	0.21	0.10
50.0	3.31	0.31	0.15
80.0	3.46	0.46	0.22
140.0	3.67	0.67	0.32
190.0	3.74	0.74	0.36
245.0	4.03	1.03	0.50
345.0	4.13	1.13	0.55
435.0	4.18	1.18	0.57
590.0	4.57	1.57	0.76
2,564.0	6.06	2.06	0.98

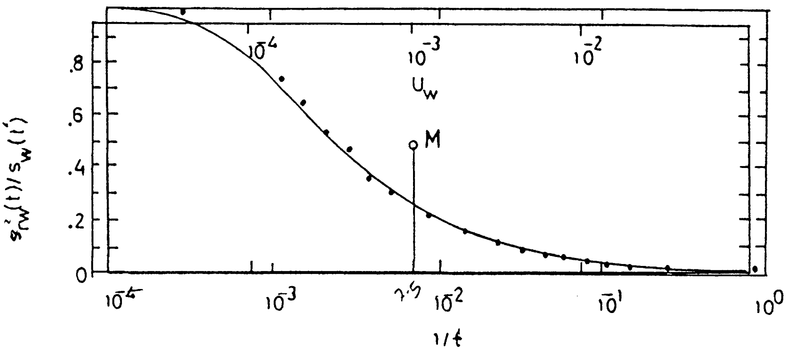

Fig. 4. Type curve matching to field data from Saudi Arabia.

Conclusions

A simple procedure has been presented for determining the formation constants, namely, the storage coefficient and the transmissivity from the recovery test. The well storage has been taken into consideration in the derivation of the basic expressions which are applicable to large diameter wells only. It has been observed that the type curves depict the change of relative drawdowns after the pump stop versus dimensionless time factor for any given storage coefficient. The expressions derived are rather simple to work with a simple hand calculator, and therefore, there is no need for a detailed table.

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Permanent Address:
Civil Engineering Faculty,
Department of Hydraulic and Water Power,
Istanbul Technical University,
Ayazağa,
Istanbul, turkey.

Present Address:
King Abdulaziz University,
Faculty of Earth Sciences,
Hydrogeology Department,
P. O. Box 1744,
Jeddah 21441,
Kingdom of Saudi Arabia.