The available information thus indicates that the authors' results may well prove useful in predicting real flows as have the results of others.

Minor points are worth mention. (1) F should coincide with E in the lower part of Fig. 5(b). (2) The authors should be consistent in setting $V_j = 1$ or not. Equations (1) and (2) differ in this respect; and, if $V_j = 1$, then $\eta = V_1$. (3) They also use radians and degrees interchangeably; if $K = 4/3$, then π/K is $3\pi/4$ not 135 deg (caption to Fig. 6). (4) The authors' Table 1 contains several inconsistencies and errors in the light of the foregoing discussion. Configurations 3, 4, and 5 are not alike, nor can all small angles be solved in closed form. (5) Reference [7] should be to Michell, not Mitchell.

Additional References

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23 McNown, J. S., and McCaig, I. W., "Complexities of Manifold Flow," *Proc. II Hydraulics Conf.*, Wash. State Univ., 1960, pp. 131–159.

Authors' Closure

For lateral flow past a conduit fitted with a barrier (Fig. A), Prof. McNown has stated that the heavy broken lines indicate two possible streamline positions characterizing the asymmetric case. However, in an earlier publication [23] it is stated that either case "has no counterpart in real flow, and sizeable departures from ideal (or locally symmetric) divisions would doubtless lead to pronounced separation." In the discussion of the present paper too, this fact is implied since he states that "in real flows, the jet would surely separate from the boundary as the authors indicate in Fig. 8" ($\alpha > \pi/K$). The present results for cases (Fig. 8) in which the barrier does not guide the flow, agree with existing solutions for the free jet [7] in the entire range of variables presented ($\alpha > \pi/K$, Fig. 8). It is interesting to note that specific cases of configurations presented (Fig. 1) can be viewed as a form of Von Mises flow [21] and that the present value of C_{co} agrees with earlier results when proper scaling factors are used. In view of the explanations provided, it can be stated that configurations 3, 4 and 5(a) provide results which agree with the corresponding free jet model for which closed form solutions have been obtained.

In the present analysis, D is forced to be the stagnation point and the stagnation streamline reaches D in all cases. In the more general case, stagnation may occur on DE or on DC with a lip cavity downstream of D. The hodograph drawn was not intended to include the formation of a lip cavity especially since this study was undertaken essentially to supplement existing simple cavity free models for lateral flow past barriers [1, 22]. The formation of a lip cavity can be accommodated in a more complex model. The analysis suggested by McNown which includes the artificial barrier does not account for the lip cavity.

The main thrust of th argument in the discussion appears to be that the flow is always guided by the barriers "no matter what." This thesis is flawed as one can easily conceive a barrier which is redundant for the cases where the free jet angle exceeds the barrier angle (Fig. 8). In Figs. 5(a) and 5(b), we are dealing with finite barriers and hence E need not coincide with F.

The authors wish to thank Prof. McNown for providing additional valuable information related to this theoretical analysis. An experimental program is underway to verify some of the assumptions made in the analysis of lateral flow past a two dimensional conduit fitted with a barrier.

Predicted and Measured Pressure Drop in Parallel Plate Rotary Regenerators[1]

A. L. LONDON.[2] This paper describes an interesting rotary regenerator incorporating a unique rectangular passage geometry heat and mass transfer surface. The following listing summarizes the parameters relating to this discussion.

Mean aspect ratio of the rectangular passages (Fig. 2) = 1/121
Hydraulic diameter, $D_h = 1.764 \times 10^{-3}$ m (0.0694 inches)
through flow porosity, $\sigma = 0.796$
area density, $\beta = 4\sigma/D_h = 1805$ m^2/m^3 (550 ft^2/ft^3)
passage length to hydraulic diameter ratio, $L/D_h = 57.6$
Reynolds No. test range 160 to 370

The area density of this surface is about twice that of the air-side of a modern automobile radiator (12 fins/in.).

This small aspect ratio geometry, approximating infinite parallel plates, has excellent characteristics with respect to both small fontal area and small volume requirements when operating with fully developed laminar flow [a]. The inherent advantages, however, are not realized unless manufacturing tolerances are closely controlled [b]. The effect of passage-to-passage nonuniformities is to produce a relatively small reduction of pressure drop and, unfortunately, a relatively large reduction of heat transfer. Are the authors planning to present the predicted and measured heat transfer performance for the regenerator?

There is virtually a one-to-one correspondence between the author's treatment of overall exchanger pressure drop and the methodology proposed in [5], as noted in Section 2.3. One point of difference is the use of a factor K of equations (1, 6, 7), termed a "pressure drop coefficient" in the paper and the "incremental pressure drop number" in [a], in place of K_c of [5], termed the "contraction loss coefficient." It is noted in Section 2.3 that $K_c = 0.55$ while $K = 0.686$. Both of these factors include the pressure drop effect required to produce a flow velocity profile typical of fully developed laminar flow in the small passage. Additionally, K_c includes flow irreversibility effects associated with the flow contraction, $C_c = 0.73$ for $\sigma = 0.796$, Fig. 3 of [c], followed by the reexpansion of the flow downstream from the vena contracta (Fig. 2 of [c]). No such effect is included in K. The authors' flow model, Fig. 3 of the paper, specifies loss free flow between Sections 1 and 2; that is, $C_c = 1.00$. However, K does include the effect of higher wall shear associated with the higher wall velocity gradient downstream from Section 2. There is no allowance for incremental wall friction in K_c in the entrance length region. This is consistent with the flow model Fig. 2 of [c], as the wall velocity gradients are small in the entrance length region when C_c is less than unity. As noted by the authors, the test data experimental uncertainty is of the order of ± 2.35 percent and the influence of a 0.55 (K_c) slope on the equation (1) line of Fig. 6, instead of 0.686 (K), shows as less than -1 percent at the highest test Reynolds number. Thus, the best flow model, Fig. 3 of this paper or Fig. 2 of Kays work [c], remains in doubt. The discrepancy in questions, however, varies inversely with Reynolds number and thus would be important at Re = 1000.

As a concluding comment, in view of the fact that Kay's pioneer paper [c] is 30 years old and is based on contraction

[1] By I. L. Maclain-Cross and C. W. Ambrose, published in the March, 1980, issue of the JOURNAL OF FLUIDS ENGINEERING, Vol. 102, No. 1, pp. 59–63.

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coefficient data that is almost 100 years old, it would be worthwhile to update the data base, the flow modeling, and the analysis for the entrance effects for abrupt changes in flow cross-section with low Reynolds number flow.

Additional References

a Shah, R. K., and London, A. L., *Laminar Flow Forced Convection in Ducts,* Academic Press, New York, 1978, pp. 383-397, 42.

b Shah, R. K., and London, A. L., "Effects of Nonuniform Passages on Compact Heat Exchanger Performance," ASME Paper 79-WA/GT-9 (to be published in the ASME *J. of Eng. for Power).*

c Kays, W. M., "Loss Coefficients for Abrupt Changes in Flow Cross Section with Low Reynolds Number Flow in Single and Multiple-Tube Systems," Trans. ASME, 1950, pp. 1067-1074.

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Professor London's warning on manufacturing tolerances [b] is appropriate. Uniform plate spacing with the wound parallel plate regenerator requires a correct choice of the winding tension and careful manufacturing control of both winding tension and spacer uniformity.

The predicted fully developed heat transfer coefficient [a] for the sensible heat regenerator tested is $123 W/m^2 K$ for dry air at 300 K away from the spokes. All heat transfer coefficient values calculated from the authors measurements [1 Chap. 4, 3] for practical operating conditions are within $\pm$ 20 percent of this. The authors are planning to report on predicted and measured heat transfer performance after revising their predictions using [a] and making more careful measurements.

The differences between Kays [c] and the authors entrance loss predictions are important in other applications [3, d].

Finite difference solutions of the complete Navier-Stokes equations with the Boussinesq approximation for buoyancy forces have recently been obtained for constant temperature horizontal parallel plates including the region far upstream of the entrance as well as the flow development length [9, e, f]. For $20 \leqslant Re \leqslant 1000$, $0.7 \leqslant \sigma \leqslant 1$, $Pr = 0.7$ and Rayleigh number $Ra < 10 Re^2$ no vena contracta or separation bubble occurs at the entrance as proposed by Kays [c] but incremental total pressure loss does decrease as σ increases. This incremental loss is above either Kays or the authors but decreases as Re increases. Incremental pressure drop number K, local Nusselt number and incremental heat transfer number [a] for constant heat flux calculated using this or similar solutions to four significant figures with $Ra = 0$, $Pr = 0.7$, $20 < Re < 2000$ and $0 < \sigma < 1$ would be useful and interesting but costly.

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e Nguyen, T. V., Maclain-cross, I. L., and deVahl Davis, G., "Combined Forced and Free Convecton between Parallel Plates," International Conference on Numerical Methods in Thermal Problems, Swansea, July 2nd-6th, 1979.

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A Note on the Phase Relationships Involved in the Whirling Instability in Tube Arrays[1]

R. D. GIBSON,[2] N. TONKS[2] and T. S. WILKINSON.[3]

The author has examined and extended the earlier attempts by Connors [1] and Blevins [2] to model the self-induced whirling vibrations of tube arrays, such as are found in heat exchangers. All such attempts have been based on the premise that the vibrations are due to the mechanical cross-coupling of quasi-static forces, the force on a tube being dependent upon the relative displacements of the nearest neighboring tubes. Connors and Blevins considered the coupling between longitudinal and transverse bending modes for a single row of tubes in cross-flow assuming the phase of the nearest neighbors to be 180 degrees. The author considers the coupling between torsional and bending modes of neighboring tubes allowing for arbitrary phase relationships between nearest neighbors. The results obtained by the author can be applied to the motion considered by Connors and Blevins by a simple notational reinterpretation.

The author draws the conclusion that the Connors/Blevins quasi-static modelling does not lead to a vibratory instability but rather to a zero frequency divergent instability. We point out an error in the author's argument and show that the instability is vibratory, the frequency being the natural frequency of the system.

The author derives a fourth order homogeneous differential equation (5) for the transient displacement response of tube i. In order to examine the stability he defines the displacement in exponential vibration form

$$z_i = z e^{(\lambda + j\omega)t}, \qquad j = \sqrt{-1},$$

unstable vibrations corresponding to $\lambda > 0$ and the neutral stability curve corresponding to $\lambda = 0$. Substituting into the differential equation and equating real and imaginary parts he obtains (8a) and (8b). He then investigates the neutral stability curve by putting $\lambda = 0$ in (8b) and solving the resulting cubic in ω (8c) for the case when the phase of the nearest neighbors is 180 degrees as in the Blevins model. He shows that for the symmetric case of equal damping and stiffness in both motions there are two roots one of which is zero and the other the undamped natural frequency ω_0. He then attempts to examine the stability of the system by applying the Hurwitz criterion to (8a) obtaining an expression for the critical minimum velocity for the onset of instability. He observes that in order to reproduce the corresponding expression given by Blevins it is necessary to put $\omega = 0$ and he thus deduces that the neutral stability curve for the Connors/Blevins case corresponds to a zero frequency or divergent instability. The error arises from the fact that the Hurwitz criterion determines whether *all* the roots of a polynomial equation have negative real parts whereas the author has attempted to use it to investigate the sign of the real roots in isolation.

[1] By S. D. Savkar, published in the December, 1977, issue of the ASME Journal of Fluids Engineering, Vol. 99, No. 4, pp. 727-731.

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