*Nordic Hydrology* 11, 1980, 255-272



# Development and Applications of a Runoff Model for Snowcovered and Glacierized Basins

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A conceptual model has been developed for the prediction of runoff on a daily basis from snowcovered and glacierized basins. The basin is divided into an icefree area and an icecovered area, which are treated separately by the model in an attempt to account for the highly different physical conditions prevailing within these two types of areas. The heat exchange between the atmosphere and the snow- or icecover, which essentially controls the melt production, is calculated either by means of a modified degree-day approach or – if the necessary data are available – by means of the complete energy balance equation. The performance of the model has been tested on a glacierized basin in Western Canada – The Peyto Glacier Basin – from which eight years af basic meteorological observations exist for the melt period in addition to continuous discharge measurements.

Further, the model has been applied to a glacierized basin in Johan Dahl Land, Southern Greenland, which is of interest in relation to hydro power development. The purpose of this study has been to extend the existing 4 years series of discharge measurements on basis of long term meteorological observations from a station located appr. 17 km from the outlet of the basin.

The results of the two case studies are presented and discussed.

## Introduction

The utilization and control of the water resources within arctic areas and high mountain regions have become of increasing importance during the recent decades. For many years runoff models, i.e. mathematical models which simulate the runoff from a basin on basis of climatological input, have been used successfully in connection with the evaluation, planning and operation of water resources. The need for runoff models stems from the fact that long-term meteorological observation series generally exsist to a much larger extent than discharge series do, and this is even more so in arctic and alpine regions.

In arctic and alpine basins the runoff regime is often strongly influenced by the presence of permanent glaciers. Thus, to be adopted for use in these areas, runoff models should in general not only be able to predict runoff originating from rainfall and melting of seasonal snowcovers but also to simulate the meltproduction and runoff from glaciers. Only few such models exist today. One example is the adjustment of the Canadian UBC-model presented by Power and Young (1979).

The development and testing of a model for calculating runoff on a daily basis from snowcovered and glacierized basins are presented herein. The icefree and icecovered areas within the basin are treated separately by the model in an attempt to pay due respect to differences both in heat balance and in runoff from these two types of areas. The model is able to take into account the areal variability in meteorological conditions, snow coverage and other characteristics, as facilities are included for a further division of the basin into an arbitrary number of subbasins.

The heat exchange between the atmosphere and the snow- or icecover, which controls the thermal condition and thereby the melt production from the snow or the ice, is usually calculated by means of a modified degree-day approach. Alternatively the complete energy balance equation may be used, if the necessary data are available, as this equation is also included in the model. Within the icecovered areas the heat exchange between the surface layer and the underlying ice is determined by solving the equation of heat conduction.

In the icefree areas the daily amount of water that reaches the ground either as snow melt or as rain on bare ground is routed to the stream by means of a conceptual lumped-parameter model – the NAM-model – developed and presented by Nielsen and Hansen (1973). In the present model-version the melt production from the icecovered areas is routed to the stream by modelling the glacier storage as a simple linear reservoir.

To test its performance the model has been applied to a small (22.8 km), highly glacierized basin in Western Canada – the Peyto Glacier Basin – from which eight years of discharge measurements have been obtained during the International Hydrological Decade (IHD), together with recordings of basic meteorological data within the basin (Young and Stanley 1976). The resulting predictions of daily runoff have been compared with the observed hydrographs.

Further, the model has been applied to a 125 km$^2$ basin in Southern Greenland – the Johan Dahl Land Basin. In this case 4 years of discharge measurements from the outlet of the basin were available together with 19 years of meteorological observation from a station located appr. 17 km south of the outlet. The purpose of this study has been to investigate the possibility of extending the existing discharge series by means of the model in order to obtain an improved basis of decision regarding hydro power development in the basin.

## General Structure of the Model

Fig. 1 is a diagram of the general structure of the model. As it is seen, the model operates with two main types of areas – icefree areas and areas covered by permanent glaciers. Each of these areas can be divided further into an arbitrary number of subareas of assumed homogeneity with respect to surface properties (ex. slope and aspect, vegetation cover) and meteorological conditions.

### Icefree Areas

In the icefree areas the runoff originates partly from rain on bare ground and partly from the melting of seasonal snowcover.

Within each icefree subarea based on local conditions the model calculates the daily amount of water reaching the ground in form of rain and snowmelt. These contributions are summed up for all subareas and routed to the stream by means of a conceptual, lumped rainfall-runoff model – the NAM-model.

NAM is based on the conception that the transposition to streamflow of water reaching the ground takes place through a sequence of storages. Thus, NAM operates with a surface storage, a lower-zone storage and a baseflow storage. The streamflow is considered to consist of an overland component, an interflow component and a baseflow component, and the ratio magnitude of these components are controlled by the amount of water currently available in the different storages. The actual evaporation from the snowfree parts of the area is controlled by the amount of water in the overland storage and the lower-zone storage. A presentation and a detailed description of the NAM-model is given in Nielsen and Hansen (1973).

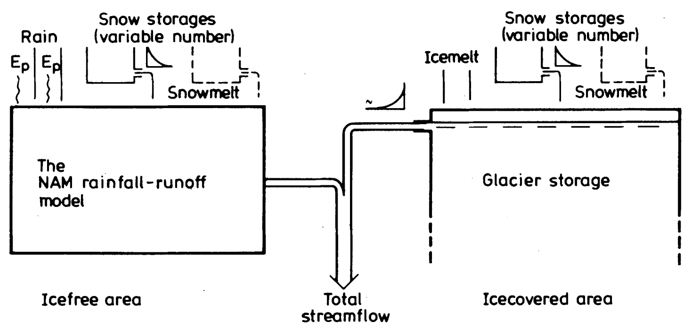


Fig. 1. General model structure. Principal sketch.

### Icecovered Areas

In the icecovered areas the runoff originates from the melting of seasonal snowcover as well as melt production from the permanent ice masses.

It is assumed that ice melt does not take place before the ice masses become snowfree.

Within each subarea the model calculates the daily amount of snow melt reaching the surface of the glaciers and the daily amount of ice melt from the snowfree parts of the glaciers. The total amount of daily melt production within the icecovered areas is determined by the summation of the contributions from all subareas.

As the runoff response of a glacier is usually markedly different from that of icefree areas, a separate procedure has been adopted for the routing of melt water through the icecovered areas. In the present version of the model the routing procedure consists of a simple linear reservoir. In general, glaciers respond rapidly due to channelization within the ice, and thus the time constant of the glacier storage is usually set to an order of magnitude of a few days.

## Snow Accumulation and Snow Melt

### Accumulation

When the local mean daily temperature is below a specified critical level (close to 0°C) the precipitation is assumed to fall as snow and to accumulate in the snow storage until melting conditions occur.

### Heat Exchange

The thermal condition of a snowpack is controlled by the exchange of heat between the pack and the surroundings, i.e. the atmosphere and – to a much lesser extent – the underlying ground. The dominating processes of the heat exchange are

- net short wave radiation
- net long wave radiation
- sensible heat transfer
- latent heat transfer

The model yields the possibility of calculating the daily heat exchange based on a mathematical formulation of the above mentioned processes. The adopted energy balance equation is described in Gottlieb (1978). However, the use of this approach requires knowledge of daily values of cloud cover, wind force, temperature and relative humidity, and as this information is seldomly available, an alternative approach has been developed which only requires temperature as input.

This approach assumes that the heat exchange between the atmosphere and the snowpack is proportional to the temperature difference between the air and the surface of the snowpack, i.e.

$$\Delta H = CS\,(T_a - T_{so}) \tag{1}$$

where

$\Delta H$ – daily heat exchange
$T_a$ – daily mean air temperature
$T_{SO}$ – daily mean temperature of snow surface
$CS$ – degree-day factor for heat exchange

Initially $CS$ was taken as a constant. However, several measurements (Linsley, Kohler and Paulhus 1958, Kuusisto 1980) indicate that the degree-day factor varies considerably throughout the season. This variation is often ascribed to the seasonal variation of short wave radiation and of the albedo of the snow surface. In the present version of the model the degree-day factor is calculated as the sum of a constant term and a term which varies throughout the season with the radiation reaching the outer atmosphere and with the albedo

$$CS = C1\,\frac{S_{RO}}{S_{RO\max}}\,(1-\alpha) + C2 \tag{2}$$

where

$S_{RO}$ – incoming radiation reaching the outer atmosphere
$S_{ROmax}$ – max $\{S_{RO}\}$
$\alpha$ – albedo of snow surface
$C1$ and $C2$ – constants to be determined by calibration

The albedo is determined by the approach given in Anderson (1968).

Eq. (2) is somewhat similar to the one proposed in Laramie and Schaake (1972) where the first term on the right hand side of Eq. (2) is used to calculate $CS$. However, in the present approach an attempt has been made to give a crude representation of the complete energy balance equation. Thus, the first term on the right hand side of Eq. (2) is considered to represent the radiation budget, while the second term to some extent represents the contributions from the sensible and the latent heat transfer.

In Fig. 2 a comparison is given of predictions based on a constant CS and on the seasonally varying $CS$ (Eq. (2)) respectively. Fig. 2 is a representative example of the predictions undertaken so far and demonstrates the improvements that may be obtained by the adoption of a seasonally varying $CS$, especially – it is believed – when the melting season is relatively long.

It should be noted that the present approach (Eqs. (1) and (2)) does not require more information and does not involve more calibration constants than the most commonly used degree-day approach in which the daily *melt* is set in linear proportion to the air temperature

$$M \equiv \begin{cases} a(T_a - b) & T_a > b \\ 0 & T_a \le 0 \end{cases} \tag{3}$$

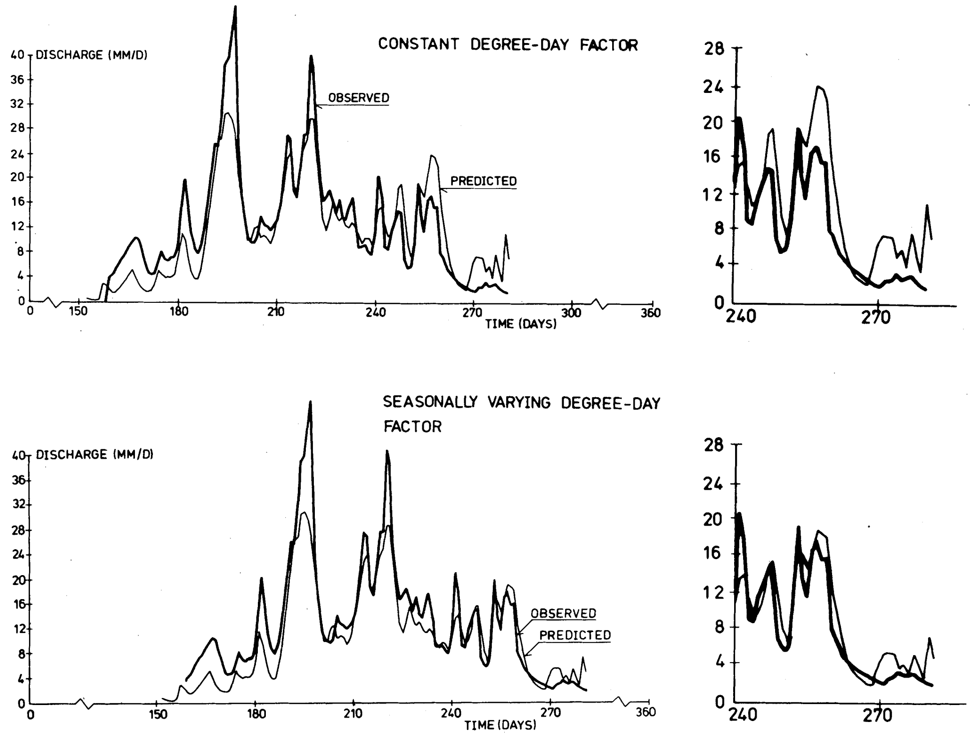


Fig. 2. Effect of using a seasonal varying degree-day factor. Peyto Glacier Basin 1968.

where

| | | |
|---|---|---|
| $M$ | – | daily melt |
| *a and b* | – | calibration constants. |

**Thermal Condition of the Snowpack**

The modelling of the heat flow within the snowpack is illustrated in Fig. 3. If the average temperature of the snow is below the melting point, any heat supply from the atmosphere ($T_a > T_{so}$) is first used to raise the snow temperature. Further heat supply is used for melt production.

Some of the melt water is held as free water within the snowpack. Thus, the water-holding capacity of the snowpack has to be fulfilled before any runoff can occur. The model operates with a constant water-holding capacity of 4% (by weight).

Heat loss from the snowpack ($T_{so} > T_a$) will first result in refreezing any free water within the pack. Further heat loss will result in a lowering of the average snow temperature.

The temperature, $T_{so}$, of the snow surface is determined as follows

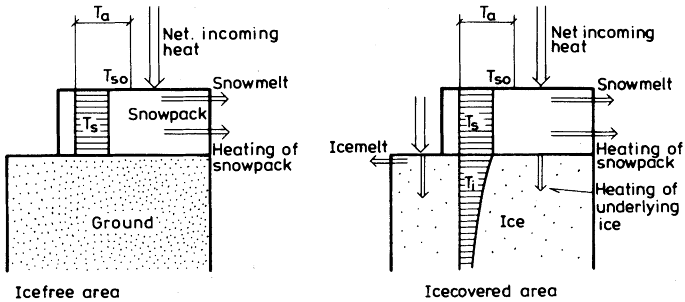


Fig. 3. Modelling of heat flow. Principal sketch.

$$T_{so} = \begin{cases} \dfrac{T_a+T_s}{2} , & \dfrac{T_a+T_s}{2} < 0 \\[2ex] 0 , & \dfrac{T_a+T_s}{2} \geq 0 \end{cases} \tag{4}$$

where

$T_s$ average temperature of the snowpack.

## Areal Extent of the Snowcover

It is well known that aside from large scale variations in the water-equivalent of accumulated snow due to areal variability of precipitation, smaller scale variations exist, mainly due to local variability with respect to wind exposure.

The large scale variations can more or less be accounted for by the subdivision of the basin. However, variability in snow water-equivalent within the individual subbasins will lead to a gradual decrease in snowcoverage of the subbasin during the melting. This effect has been accounted for by means of the method suggested in Anderson (1968). The method is based on the assumption that a relation exists between the snow water-equivalent and the areal extent of the snowcover as illustrated in Fig. 4. The method also implies the prediction of a return to complete snowcoverage under snowfall conditions during the melting period, cf. Fig. 4.

## Ice Melt

The modelling of the heat flow within the icecovered areas is illustrated in Fig. 3. When the glacier is covered by snow, the assumption is made that no ice melt takes place. The surface temperature of the glacier is set equal to the temperature of the overlying snowpack, and the daily heat transport from the snow to the glacier is determined by numerically solving the equation of heat conduction for the glacier.

As areas of the glacier become exposed, the heat exchange between the at-

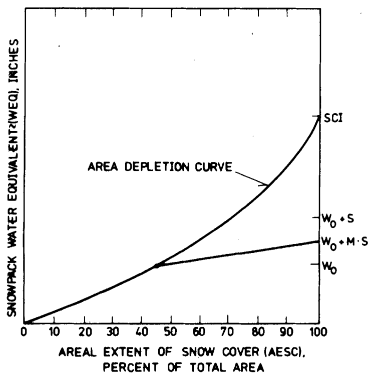


Fig. 4. Modelling of areal extent of snowcover.

mosphere and the ice is calculated in the same way as for the snow. Some of the incoming heat is lost due to heating of the underlying ice. Again this portion is determined by means of the equation of heat conduction for the ice. Any surplus of heat is considered to be used for ice melt production.

## Performance and Applicability of the Model

### General

The main objective for the developing of the present model has been to enable the utilization of meteorological data for the generation of synthetic runoff series from glacierized areas with the purpose of improving the basis for evaluation, planning and operation of water utilization schemes (hydro power, water consumption, etc.).

Obviously the model should be able to adequately simulate the different physical processes involved in glacierized areas in the transformation of precipitation into runoff. However, in practice at least two factors set an upper limit to the attainable degree of correspondence between simulated and observed runoff

- inaccuracy in the measurements of runoff as well as in meteorological point data
- Uncertainty in establishing areal values of meteorological data based on point measurements either within the basin – or very often – in considerable distance from the basin.

In order to test specifically the performance of the model, representative areas should be selected with the best possible data base. However, to obtain an indication of the practical applicability of the model, test cases should be selected which involve extrapolation of meteorological point measurements over a considerable distance.

### Selected Test Cases

The performance of the model has been tested on the Peyto Glacier Basin, Western Canada. Further, a case study has been undertaken, aiming at extending the existing discharge series from Johan Dahl Land, Southern Greenland, on the basis of extrapolated meteorological data.

The two test cases will be discussed in the following.

## Test Case – The Peyto Glacier Basin

### Basin Characteristics

The Peyto Glacier Basin was one of five glacier basins in Canada selected for detailed hydrological studies during the IHD. It is located longitude 116°33'W, latitude 51°40'N on the eastern flank of the Rockey Mountain, Alberta. The total basin area is 22.8 km$^2$ of which approximately 61% is covered by the Peyto Glacier. Fig. 5 illustrates the basin topography. A detailed description of the basin characteristics is given in Young and Stanley (1976).

In Young (1977) a review is given of the runoff regime. The runoff varies considerably during the year as well as from year to year. Approximately 90% of the total annual runoff occurs in the months of June to September.

### The Data Base

During the IHD a field measuring program was set up consisting of

- recording of the runoff at the basin outlet.
- recording of basic meteorological variables near the snout of the glacier (cf. Fig. 5) including temperature and precipitation.
- recording of the accumulation of the winter snowpack and the summer melting of the snowpack and the glacier ice.

The field measurements commenced in 1965 and continued during each summer from May to September until 1974. However, the runoff recordings did not start until the summer of 1967. The measuring program is described in details in Young and Stanley (1976).

### Calculation Procedure

The model calculations were performed by means of the mentioned degree-day approach. Daily mean values of the recorded temperature and precipitation were used as the basic meteorological indata. The basin was divided into 200 m elevation zones, and local values of temperature and precipitation were estimated from the recorded data by simple linear elevation corrections. The environmental temperature lapse rate was set to

- 1.3°C/200 m at days of precipitation
- 1.9°C/200 m at days of no precipitation.

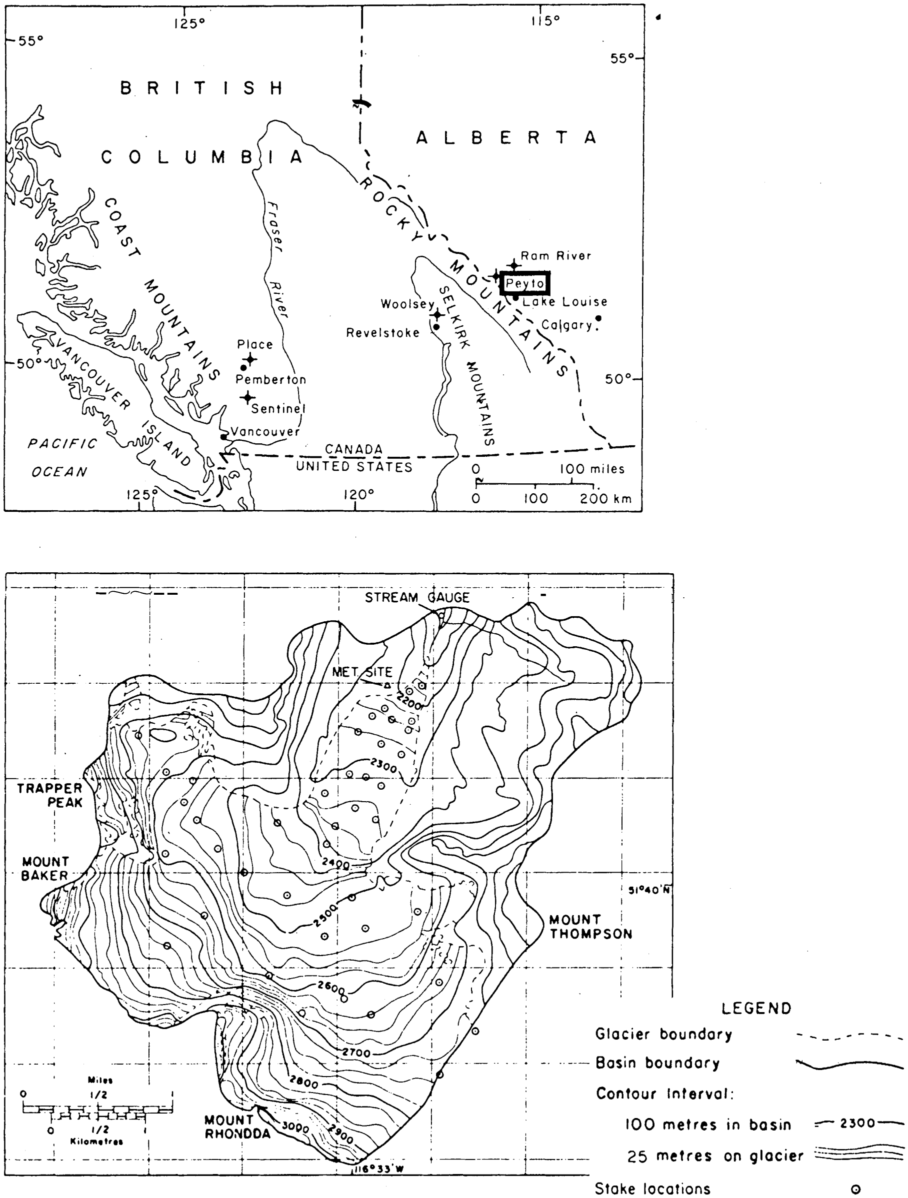


Fig. 5. Peyto Glacier basin.

The elevation increase of the precipitation was estimated to 40%/200 m while the deficit in gage catch were set to

- 20% for snow
- 10% for rain.

In each elevation zone the local temperature was used to determine whether the precipitation fell as snow or rain.

Initial values of the snowdepth in the individual ice-covered elevation zones were determined on the basis of reported values of winter snow accumulation (Young and Stanley 1976). In (Power and Young 1979) the winter snow accumulation in the ice-free areas was estimated to be half the accumulation on the glacier. The same procedure was followed herein.

Predictions of the daily runoff were performed and compared with recordings for all summer periods form 1967 to 1974.

**Results of Predictions**

The computed hydrographs are plotted in Figs. 6-7 together with the recorded ones. As it is seen, the overall agreement is quite good considering, among other things, the uncertainty in establishing areal values of temperature and precipitation based on the point measurements near the snout of the glacier. A noteworthy shortcoming, however, seems to be the general inability of the model to correctly predict the major peak flows. This might be attributed either to deficiencies of the degree-day approach in accounting for combinations of weather situations giving rise to particularly large heat inflows or to oversimplifications in the description of the runoff routing from the glacier.

As a quantitative measure of the model performance the 'explained variance', $R^2$, was calculated for each simulation period

$$R^2 = 1 - \frac{\sum(Q_i^o - Q_i^c)^2}{\sum(Q_i^o - \bar{Q}^o)^2} \tag{5}$$

where

$Q_i^o$ – observed runoff on the $i$'th day
$Q_i^c$ – computed runoff on the $i$'th day
$\bar{Q}^o$ – mean observed runoff

Table 1 gives the calculated $R^2$-values.

Table 1 – Peyto Glacier Basin. $R^2$-values.

| Period | $R^2$ |
|---|---|
| July – August 1967 | 0.72 |
| June – August 1968 | 0.80 |
| June – August 1969 | 0.75 |
| June – August 1970 | 0.71 |
| June – August 1971 | 0.79 |
| July – August 1972 | 0.56 |
| June – August 1973 | 0.69 |
| June – August 1974 | 0.83 |

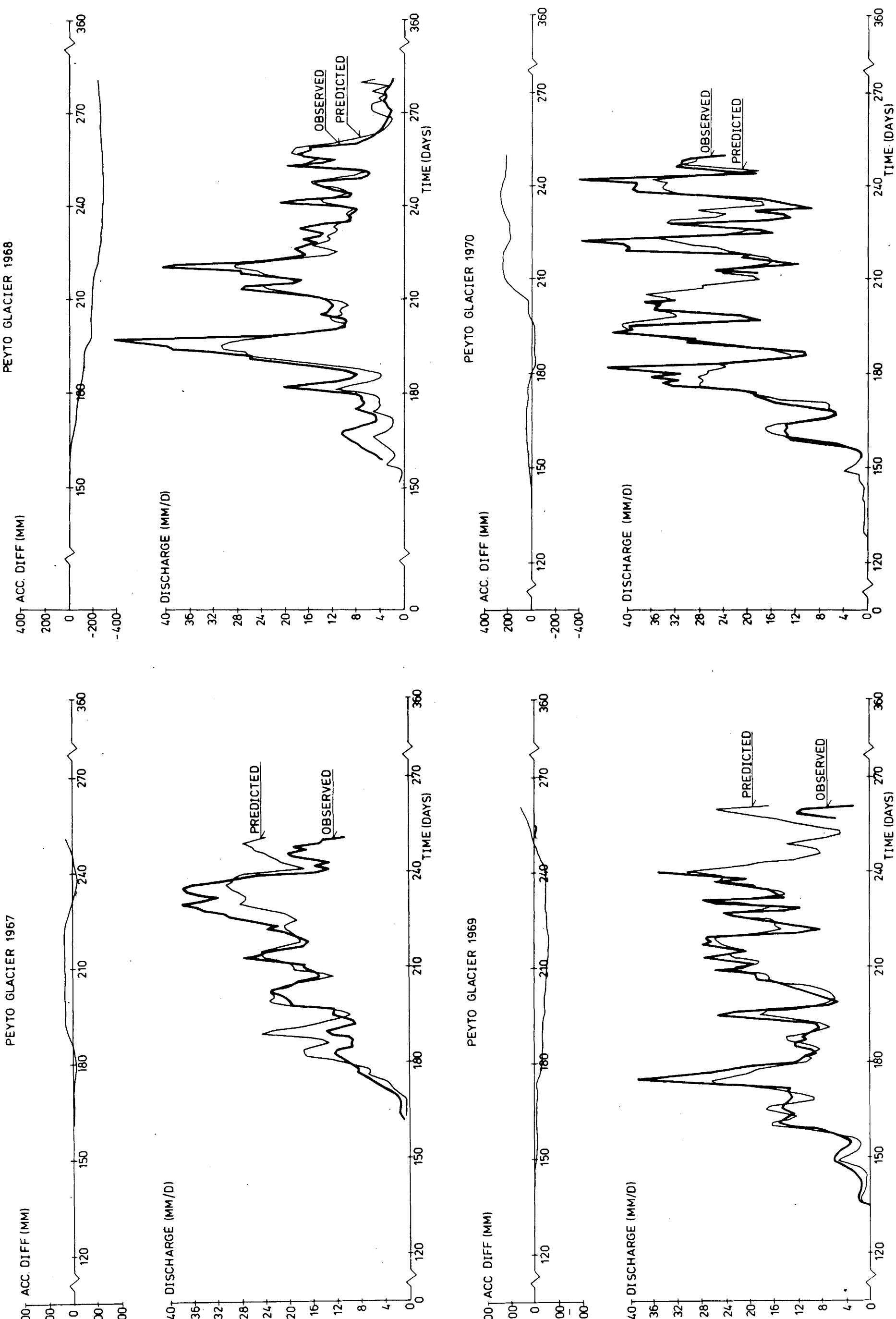


Fig. 6. Peyto Glacier basin. Discharge simulations 1967-70.

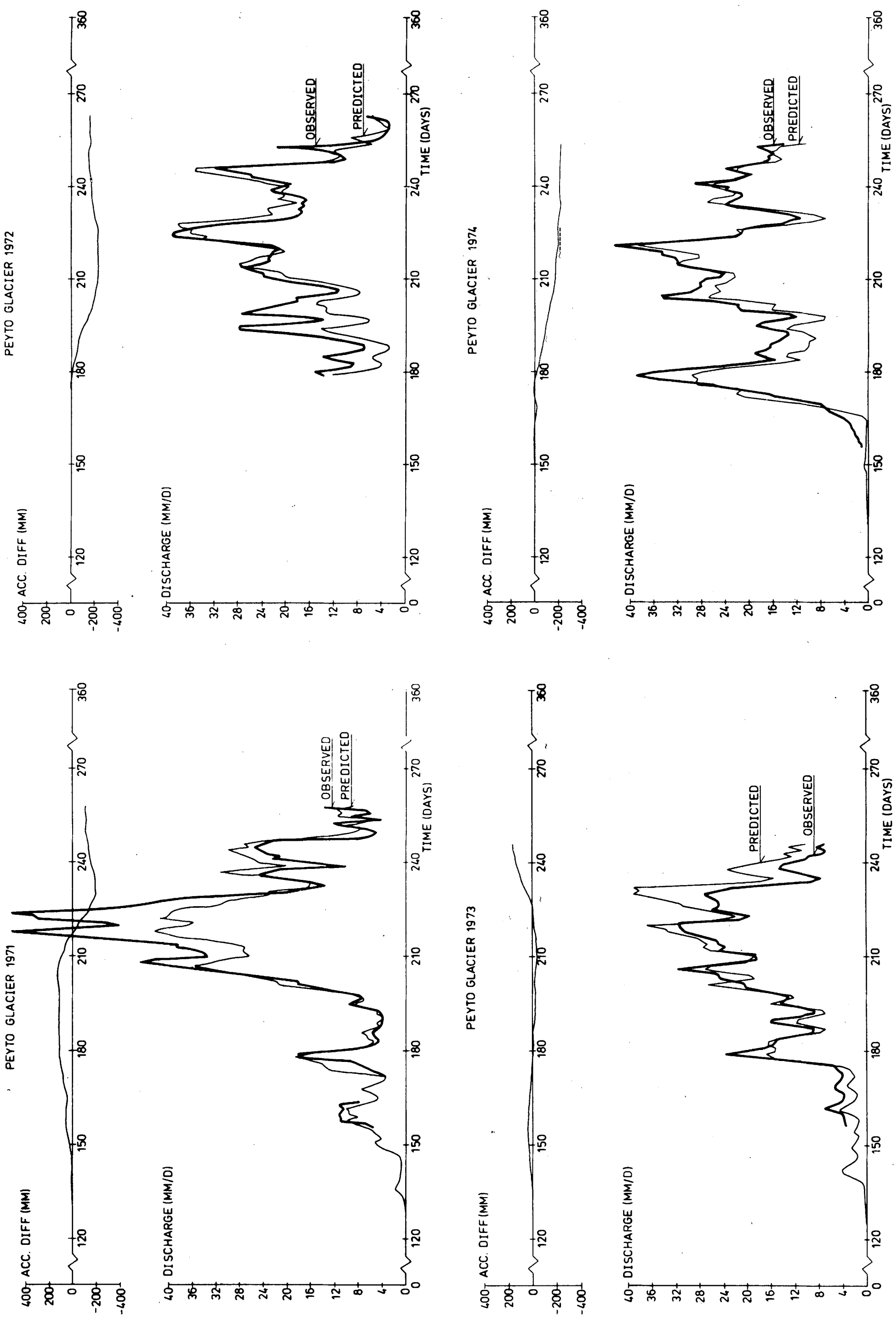


Fig. 7. Peyto Glacier basin. Discharge simulations 1971-74.

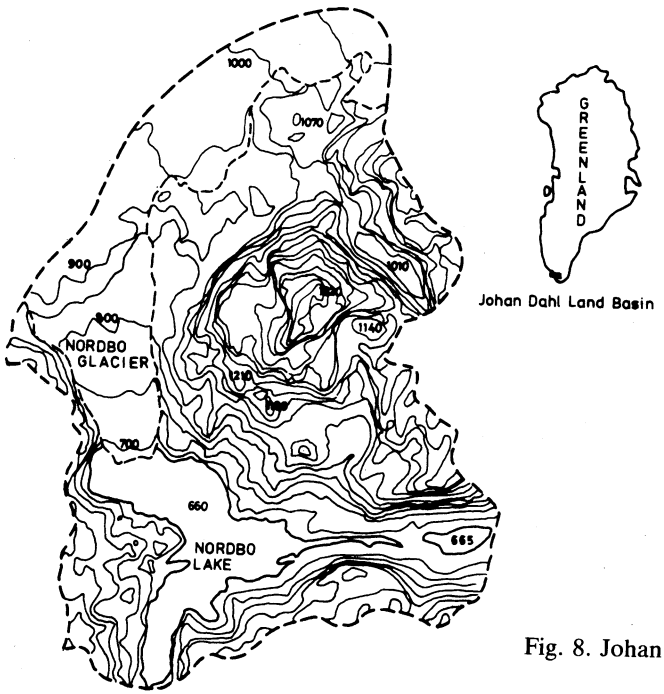


Fig. 8. Johan Dahl Land basin.

## Test Case – Johan Dahl Land Basin

### Basin Characteristics

The Johan Dahl Land Basin constitutes the catchment of the Nordbo Sø which is located longitude 45°22'W, latitude 61°21'N in Southern Greenland. The basin abuts on the Inland Ice, and the extent of the area is approximately 125 km$^2$ of which approximately 25 km$^2$ is covered by the drainage area of the Nordbo Glacier. Fig. 8 is an illustration of the basin characteristics including topographical features. The elevation ranges from approximately 600 m.s.l. to approximately 1,200 m.s.l.

Johan Dahl Land is of interest in relation to hydro power development, and consequently it is of importance to establish the best possible knowledge of yield and necessary reservoir capacity.

### Data Base

Four years of continuous discharge measurements (1976-79) were available from the outlet of Nordbo Sø together with sporadic observations of basic meteorological variables including temperature and precipitation at different locations within the basin. (The Greenland Technical Organization 1980 personal communication, The Greenland Geological Survey 1980).

Further 19 years of meteorological observations (1961-79) were available from the Narssarssuaq Airport located near sea level approximately 17 km south of the outlet of Nordbo Sø.

### Calculation Procedure

The model was calibrated on the 4 years of discharge measurements using daily values of temperature and precipitation from Narssarssuaq as input data.

The basin was divided into 200 m elevation zones, and elevation corrections of temperature and precipitation were established by means of the sporadic meteorological observations within the basin. The temperature lapse rate used was 1.3°C/200 m while the elevation increase of the precipitation was set to 4%/200 m.

### Results of Predictions

In Fig. 9 the computed hydrographs are compared to the runoff recording for the 4 year period of 1976-79 while Table 2 gives the calculated $R^2$ values. Again the overall agreement in computed and recorded runoff regime is fair, although not quite as good as in the Peyto case. This was to be expected, as the extrapolation of the meteorological input data is encumbered with greater uncertainty in the present case. Furthermore, in the present case the location of the boundaries of the basin is not determined very accurately.

The large peak recorded in 1978 which has not been picked up by the model is known to originate mainly from the outburst of an ice-dammed lake on Nordbo Glacier.

Table 2 – Johan Dahl Land. $R^2$-values.

| Period | $R^2$ |
|---|---|
| 1976 | 0.72 |
| 1977 | 0.80 |
| 1978 | 0.79 |
| 1979 | 0.88 |

### Model – Generated Runoff Series

After the calibration of the model an additional 15 year series of runoff was generated based on the meteorological data from Narssarssuaq for the period 1961-1975.

The generated series has been used for improved estimates of the average runoff as well as the necessary reservoir capacity of a possible hydro power scheme (Ammentorp and Korsgaard 1980).

Regarding the average runoff the following results have been obtained

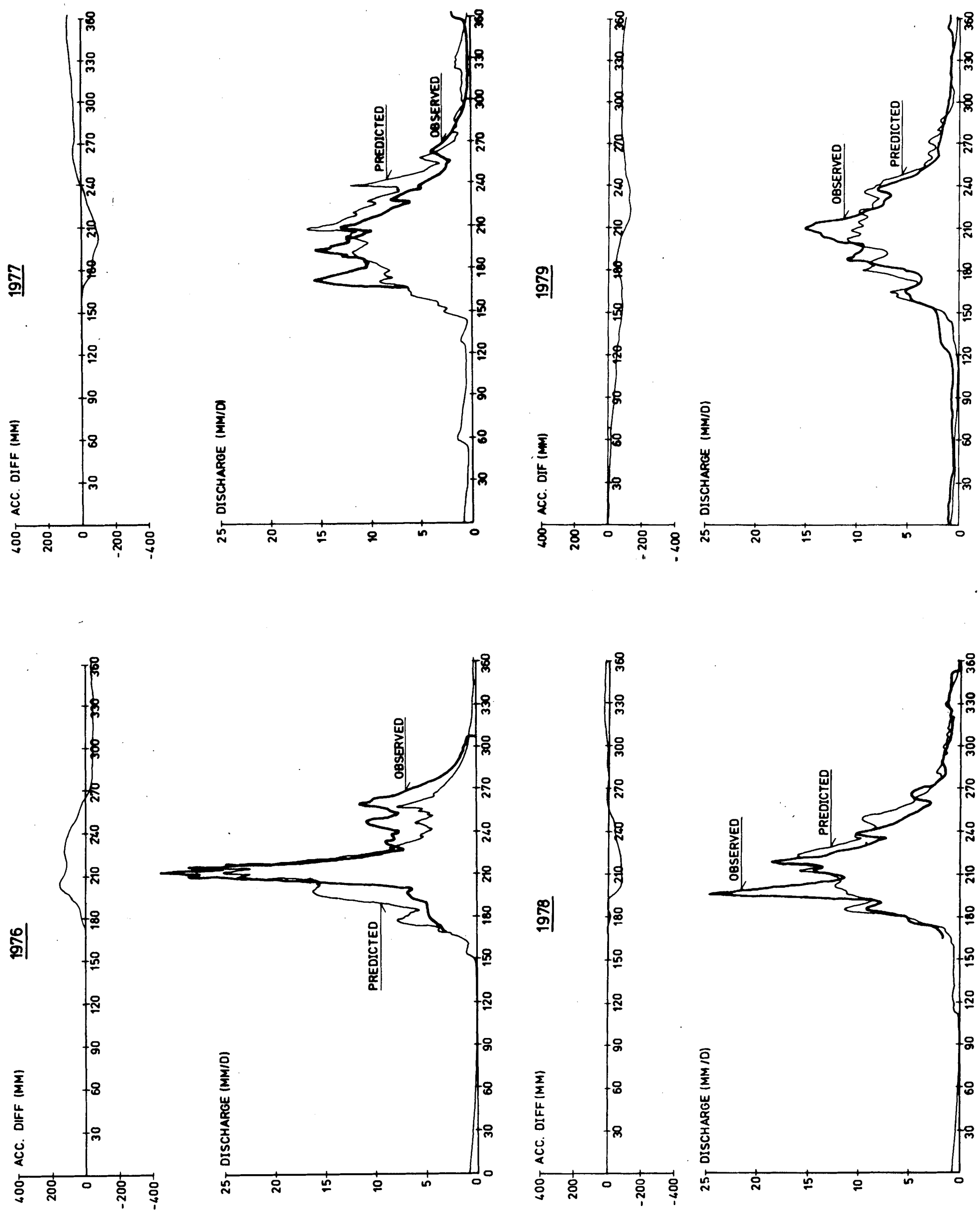


Fig. 9. Johan Dahl Land basin. Discharge simulations 1976-1979.

Estimates of average runoff

| Based on 4 years runoff observations | | Based on 15 years generated runoff series in addition to 4 years runoff observations | |
|---|---|---|---|
| Average runoff | 2 × standard deviation | Average runoff | 2 × standard deviation |
| $140 \times 10^6 m^3/yr$ | $19.2 \times 10^6 m^3/yr$ | $130 \times 10^6 m^3/yr$ | $10.8 \times 10^6 m^3/yr$ |

The standard deviation of the estimates is determined according to the method proposed in Gottlieb and Rosbjerg (1980). As it is seen, the application of the model has lead to a considerably improved estimate of the average runoff, as the standard deviation has decreased to about one half of the standard deviation obtained, if the estimate is based on only the runoff measurements.

In order to obtain the equivalent reduction in standard deviation by means of measurements, appr. 8.5 years of runoff measurements should be performed in addition to the existing 4 years. In other words, the information contained in the 15 year generated series – with regard to average runoff – is worth appr. 8.5 years of measurements.

## Conclusion

A runoff model to be applied to snow-covered and glacierized basins has been developed and the model performance tested on a highly glacierized IHD-basin in Western Canada. The results of the predictions are quite satisfactory.

Further, the model has been applied to a basin in Southern Greenland, which is of interest in relation to hydro power development. As these predictions involved the extrapolation of meteorological point data over a considerable distance and elevation range, the quality of the predictions decreased in comparison to the results obtained in the first test case. The model was used to extend the existing 4 year series of runoff measurements by generating an additional 15 year synthetic runoff series. A statistical analysis showed that the application of the model led to a considerable improvement of the estimate of the average runoff as the standard deviation of the estimate reduced at abt. 50%.

## Acknowledgement

The data from the Peyto Glacier Basin were put at the author's disposal by Dr. Gordon Young and Dr. John Power, National Hydrology Research Institute, Ottawa, Ontario.

The Greenland Technical Organization and the Danish Meteorological Institute supplied the discharge data from Johan Dahl Land and the basic meteorological data from Narssarssuaq respectively.

The author whiches to express his gratitude to the above mentioned persons and institutions for their efforts in making these data available for the present study.

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Received: 1 October, 1980

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