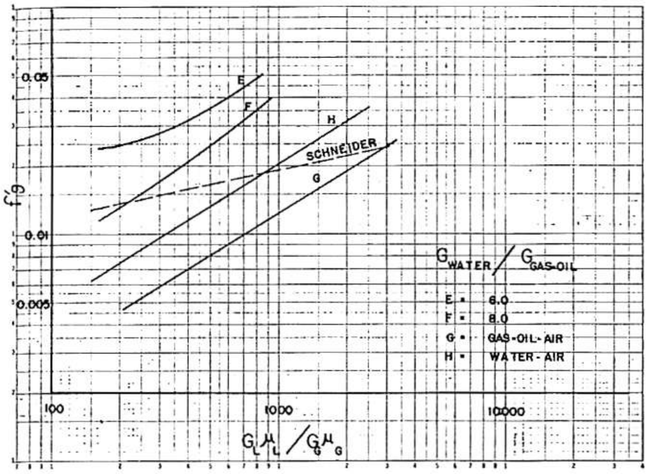

FIG. 10 CORRELATION OF THREE-PHASE AND TWO-PHASE FLOW WITH SCHNEIDER FRICTION FACTOR

of 12.2 per cent is found over the range, G_L = 15,000 to 68,000 lb/hr(sq ft) and Gg = 3000 to 17,500 lb/hr(sq ft)

SAMPLE CALCULATION

Run 99; water, gas-oil, and air in 3-in. line, temp = 75 F, D = 0.261 ft.

μ_{air} = 0.0435 lb/ft-hr
μ_{water} = 2.22 lb/ft-hr
$\mu_{\text{gas-oil}}$ = 8.3 lb/ft-hr
G_{water} = 30,200 lb/hr(sq ft)
$G_{\text{gas-oil}}$ = 20,000 lb/hr(sq ft)
$G_L = G_{\text{water}} + G_{\text{gas-oil}}$ = 50,200 lb/hr(sq ft)
G_{air} = 10,150 lb/hr(sq ft)

$$\text{Air density (neglect compressibility factor)} = \frac{PM}{RT}$$

$$= \frac{(14.6)(28.97)}{(10.72)(535)} = 0.0736 \text{ lb/ cu ft}$$

$$f'g = \frac{(\Delta P/\Delta L)g_c D\rho_g}{2G^2_g} \quad \ldots\ldots\ldots\ldots\ldots\ldots [1]$$

$$\left(\frac{\Delta P}{\Delta L}\right) = \frac{f^1 g 2 Gg^2}{g_c D\rho_g} \qquad f'g = f\left(\frac{G_L\mu L}{G_g\mu g}\right) \quad \ldots\ldots\ldots [2]$$

$$\mu L = \frac{30{,}200}{50{,}200}(2.22) + \frac{20{,}000}{50{,}200}(8.3) = 4.64 \text{ lb/ft(hr)}$$

$$\mu L = \text{mass fraction water} \times \mu_{\text{water}} + \text{mass fraction gas-oil} \times \mu_{\text{gas-oil}}$$

$$\frac{G_L\mu L}{G_g\mu g} = \frac{(50{,}200)(4.64)}{(10{,}150)(0.0435)} = 527; \quad R = \frac{G_{\text{water}}}{G_{\text{gas-oil}}}$$

$$= \frac{30{,}200}{20{,}000} = 1.51$$

Interpolating from Fig. 9: f^1g = 0.021
Substituting into Equation [2]

$$\left(\frac{\Delta P}{\Delta L}\right)_{\text{calc.}} = \frac{(0.021)(2)(10{,}150)^2}{(32.2)(3600)^2(0.261)(0.736)} = 0.54 \text{ lb/sq ft-ft}$$

$$\left(\frac{\Delta P}{\Delta L}\right)_{\text{expt'l}} = 0.56 \text{ lb/sq ft-ft}$$

G_L = liquid-mass velocity based on full pipe diameter, lb/hr(sq ft)
G_g = gas-mass velocity based on full pipe diameter, lb/hr(sq ft)
g_c = conversion factor, lb_M ft/lb_F (sq hr)
D = pipe diameter, ft
ρg = gas density, lb/cu ft
f^1g = pseudo-friction factor, based on gas flow, dimensionless
$(\Delta P/\Delta L)$ = static pressure drop per unit length, lb/sq ft-ft
μL = liquid viscosity, lb/ft-hr
μg = gas viscosity, lb/ft-hr
R = water-oil flowing ratio, dimensionless

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Discussion

OVID BAKER.[11] The authors have presented us with a method of estimating pressure drop in flow lines from oil wells to the gas-oil separators. Through these lines flows a mixture of gas, oil, and water. Oil-field flow lines are usually two, three, or four inches in diameter and may vary from 150 to 3500 ft in length. Inside diameters are often reduced by paraffin accumulations after the line has been in service for some time. Probably there are applications of three-phase flow in many industries.

The usual methods of correlating pressure drop for simultaneous flow of gas and liquid involve the calculation of pressure drop for the liquid phase. So far as the writer knows, no data are available for calculating pressure drop of oil-water mixtures in turbulent flow through a horizontal pipe. The Schneider friction-factor correlation presented in Fig. 9 avoids the necessity for calculating the pressure drop of an oil-water mixture.

The authors have not described what liquid viscosity was used in making the three-phase correlation in Fig. 9. It would be helpful if we could have some additional details on the use of this correlation and an idea of the magnitude of the wide deviations mentioned.

As implied by the authors, the Schneider-type correlation should be used with caution. The original Schneider friction-factor correlation failed to account for pressure drops in pipelines where a slug-flow pattern prevailed.[12] Although slug flow was not included in these experiments it does occur in many oil-well flow lines.

The authors also have included new data on two-phase flow of air and water and oil and water in three-inch pipelines. These data will be very welcome to those working with two-phase pressure drop in large-diameter pipes. Most of the work done on two-phase flow has been in pipes of two-inch and smaller diameters. This will be the third paper to present data for three-inch diameter pipe. For other large sizes there are four papers covering four-inch pipe, four papers for six-inch pipe, two papers on eight-inch, and one which covers two-phase flow in ten-inch pipe. There are additional unpublished data on six, twelve, and sixteen-inch pipe which probably will be made available in the future. It seems probable that a generalized correlation for two-phase

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[11] Magnolia Petroleum Company, Dallas, Texas.

[12] "Designing for Simultaneous Flow of Oil and Gas," by Ovid Baker, *Oil and Gas Journal*, vol. 53, July 26, 1954, pp. 185–190, 192, 195.

flow in all sizes of horizontal pipes will become possible before very long.

The authors are to be commended for their many contributions on this subject.

M. R. Tek.[13] The writer would like briefly to preface his remarks concerning a short technical discussion of the paper by complimenting the authors on an interesting study, a well-organized paper, and a fine presentation.

This work, to the writer's knowledge, represents the first attempt in analyzing the multiphase pipe-flow problem with more than two phases flowing simultaneously. The mechanics and the characteristics of the multiphase-flow systems have been of everyday interest to many industries for some time. In numerous engineering installations such as pipelines, heat exchangers, chemical or nuclear reactors multiphase-flow conditions are of common occurrence. As was pointed out by the authors, quite often in oil-production operations, centrally located stock tank batteries lead to long gathering lines through which oil, water, and natural gas flow simultaneously. The prediction of pressure drops in such systems from the physical properties of the flowing fluids and the geometric properties of vehicle conduit constitutes a very complicated problem.

One of the main difficulties is due to the fact that several flow patterns of widely different geometry and characteristics may co-exist. These flow patterns usually referred to as bubble, plug, stratified, wave, and so on, not only impose unstable undefinable geometry on the system, but also critically affect the relative magnitudes of several force fields each active to different extents.

The classical approach of attempting to solve the Navier-Stokes' equations becomes hopelessly devoid of any promise of success not only due to analytical difficulties in setting up the boundary conditions but also because of their inherent non-linearity. In a general formulation, the necessity to include interfacial and gravitational forces, along with viscous inertia and pressure forces further complicates the theoretical approach.

These major difficulties often lead the researchers to semi-empirical approaches where ideas from the far better established theory of single-phase flow are combined with experimental observations on multiphase-flow systems.

In general, in three-phase flow systems it may be expected that the flowing mixture density can be different from *in situ* mixture density due to slippage and hold-up phenomena. This is shown very significantly by the data presented in this paper. The curves indicating the variation of *in situ* water-oil ratios versus the air-mass velocity in Fig. 4 are interesting and informative. The physical significance of Fig. 4 may be analyzed by considering the action of the gas phase, respectively, on oil and water phases.

At low air-mass velocities the free surface of the oil and oil-water interface are usually quiescent and undisturbed. As the air-mass velocity is increased the free surface of the oil flowing on top of the water becomes disturbed by small capillary waves. As these small ripples become unstable and grow in magnitude under the effect of increasing wind, the drag forces effected on the oil phase by the flow of gas become large due to viscous shear and the ruffled condition of the free surface. This in turn, would cause an increase on the net transport of the oil resulting in an increased water-oil ratio. As the air velocity reaches a critical value, presumably around 4000 lb/hr × sq ft, the growth of waves at the free surface of the oil apparently increases all the way across the depth of the oil film and disturbs the oil-water interface as well. Similarly as before, this would then also increase the drag on the water and the net transport of water phase. Although this viewpoint seems restricted to a rather narrow range of stratified-flow regime it appears quite compatible with the observed data.

Again in reference to Fig. 4 it also would seem worth while to know the variation of total liquid in place with increasing air rates.

In comparing the pressure-drop quantities given in Figs. 5, 6, 7, and 8, respectively, for single, two, and three-phase flows, it is interesting to note that at a given air-mass velocity, the presence of a second liquid phase increases the pressure drop approximately five-fold. These data also appear significant, interesting, and important. The specific points should be spotted on Fig. 8. A tabulation of basic raw data also should prove very useful in further studies.

In connection with Fig. 9, it would seem desirable to plot data points on or about the correlating curves.

Finally, in addition, the writer believes the method used in computing μ_L oil-water-mixture viscosity should be included in the paper.

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