Honey-bee mating optimization (HBMO) algorithm in deriving optimal operation rules for reservoirs

O. Bozorg Haddad, A. Afshar and M. A. Mariño

ABSTRACT

The honey-bee mating process is considered as a typical swarm-based approach to optimization, in which the search algorithm is inspired by the process of real honey-bee mating. In this paper, the honey-bee mating optimization (HBMO) algorithm is applied to extract the linear monthly operation rules of reservoirs for both irrigation and hydropower purposes. The release rules for each month are considered as a linear function of the reservoir past-month-end storage as well as current monthly inflow to the reservoir. In such a case, the decision variables are 36 for each problem and are set so that water supply deficits are minimized. In both irrigation and hydropower purposes, 60–480 months are considered and results are compared to those from the nonlinear programming solver of the LINGO 8.0 software. The approach and the rules tend to be very promising and denote the capability of the proposed HBMO algorithm in solving reservoir operation problems. Furthermore, the results indicated that, by using the near-optimal solution from the HBMO as a starting point for the NLP solver, the obtained objective function value was enhanced significantly and a better local optimum was found.

Key words | honey-bee mating optimization, hydropower, irrigation, operating rule, optimum reservoir operation

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INTRODUCTION

In reservoir operations for water supply, water can be either released for beneficial uses or retained in the reservoir for possible future use. This simple choice becomes extremely complex in the presence of uncertain future inflows and nonlinear economic benefits for released water (Shih & ReVelle 1994, 1995). The literature concerning development of operating rules for water resource systems is extensive, particularly for water supplies (Bower *et al.* 1962; Loucks & Sigvaldason 1982; Lund & Guzman 1996, 1999). In general, reservoir operating rules guide release decisions. Good reservoir management therefore requires creating "a set of operation procedures, rules, schedules, or plans that best meet a set of objectives" (USACE 1991). For water supply systems, the so-called standard operating policy (SOP) is perhaps the simplest reservoir operating rule. The SOP (Maass *et al.* 1962; Loucks *et al.* 1981) is specified as a function of the total value of currently available water (i.e. current storage, plus projected inflows, minus evaporation during the present period).

The coordinated operation of a reservoir system for efficient management of available water, to maximize the net benefit or minimize the total deficits of the system, is a complex decision-making process. The decision policies involve many variables, objectives, and considerable risk and uncertainty. They must satisfy various constraints on system operation while maximizing releases for various purposes such as irrigation, energy production, or minimizing spills and losses. Ideally, reservoirs in a system should be designed and operated together to maximize net social benefits. This aim can be reached using optimization approaches.

Over the last decade, evolutionary and meta-heuristic algorithms (EAs) have been extensively used as search and

doi: 10.2166/hydro.2008.018

optimization tools in various problem domains. Because EAs are heuristic search techniques (a result of their stochastic nature), global optimal solutions are not guaranteed to be found using EAs. Nevertheless, EAs give alternative solutions close to the optimum after a reasonable number of evolutions that can be accepted for most real-life problems. When compared with traditional optimization techniques, EAs have performed well, generally resulting in near-optimal solutions. In some cases, EAs have performed better than traditional techniques. The advantage of EAs over conventional optimization is their handling of complex, highly nonlinear problems that are more realistic.

Genetic algorithms (GAs) are search algorithms based on the mechanics of natural selection and natural genetics (Goldberg 1989). GAs have been extensively employed as search and optimization methods in various problem domains, including science, commerce and engineering (Esat & Hall 1994; Gen & Cheng 1997; Wardlaw & Sharif 1999). In the water resources field, GAs with their modifications and extensions have been applied to reservoir operation (Oliveira & Loucks 1997; Wardlaw & Sharif 1999; Sharif & Wardlaw 2000). In addition, GAs have been linked with stochastic dynamic programming (SDP) (Huang *et al.* 2002) for the operation of a multiple reservoir system.

Modeling the behavior of social insects, such as ants and bees, for the purpose of search and problem solving are the context of the emerging area of swarm intelligence. An ant colony is a typical successful swarm-based optimization approach, where the search algorithm is inspired by the behavior of real ants (Dorigo 1992; Dorigo *et al.* 1996; Dorigo & Gambardella 1997; Abbaspour *et al.* 2001; Simpson *et al.* 2001).

Honey-bee mating may also be considered as a typical swarm-based approach to optimization in which the search algorithm is inspired by the process of mating in real honey-bees. Bozorg Haddad *et al.* (2006) demonstrated the efficiency and applicability of the HBMO algorithm by applying it to well-known mathematical optimization problems and compared the final solutions with those from analytical methods and genetic algorithms (GAs). They concluded that the results obtained were perfectly comparable with those obtained by well-developed GAs.

In this paper, the HBMO algorithm is used to derive optimum operating rules of reservoirs. The reservoirs are considered to be for the purposes of both irrigation and hydropower, to evaluate the efficiency of the algorithm in both convex and non-convex water resource problems. Also, optimized periods are considered to be in the range of 60–480 months to evaluate the capability of the HBMO algorithm in large-scale optimization problems. Evaporation losses are also taken into account as an efficient parameter to test the sufficiency of the proposed algorithm in handling real reservoir operation problems.

HBMO ALGORITHM

The optimization algorithm is developed by simulating the natural mating process and biological statements of honey-bees and translating them into algorithmic statements. The honey-bee mating optimization (HBMO) algorithm has been fully described in Bozorg Haddad *et al.* (2006) and Afshar *et al.* (2007). A brief discussion of the algorithm is presented in this paper. HBMO is a hybrid algorithm which consists of a genetic algorithm (GA), simulated annealing (SA) and local search, and combines these three approaches to make an enhanced algorithm with a high ability of finding an optimum solution for complex problems.

The HBMO model input parameters are as follows: (a) structure parameters that are mostly problem-dependent such as number of decision variables (e.g. amount of releases in each period), upper and lower bounds on decision (limits of A_i, B_i and C_i) and state (limits of storages in each period) variables and target values, penalty coefficients (for preventing reservoir storages violations, exceeding allowable limits in each period), etc., and (b) algorithm parameters used as tuning parameters, such as number of generations (mating flights), number of new solutions produced in each generation (size of hive), number of accepted solutions (spermatheca size), number of trial solutions produced in each iteration of the simulated annealing (SA) process, some constant parameters (queen's initial speed and energy), in addition to type and number of heuristic functions defined by different workers.

The algorithm begins with the random generation of a set of initial solutions. The generated solutions may or may not belong to the feasible region. In fact, most of the generated solutions may be non-feasible. Randomly generated solutions are then ranked using a penalized objective function. The fittest solution is named the queen, while the remaining solutions are categorized as drones (i.e. trial solutions). By defining the queen, drones, broods and workers (predefined functions), the hive is completely formed and mating may now be started.

The simulated annealing (SA) process is employed to map the real mating flight into a mathematical representation in the algorithm development. Using SA, a set of solutions from the search space is selected to form a mating pool for possible information exchange between the best preset solution and the selected trial solutions.

To generate a new set of solutions, different predefined cross-over operators and heuristic functions between the best current solution and the trial solutions have been used. It has been found that the type and number of cross-over operators has a significant effect on the quality of the generated new solution. Moreover, in the present algorithm, four arbitrary different cross-over operators are employed. For instance, in this study, four operators used in the breeding process (new solution generation) are: (I) one-point cross-over in which the queen's genotype has been put in the left side of the generated brood's genotype; (II) one-point cross-over in which the queen's genotype has been put in the right side of the generated brood's genotype; (III) two-point cross-over in which the queen's genotype has been put in the middle of the generated brood's genotype; and (IV) two-point cross-over in which the queen's genotype has been put in both ends of the generated brood's genotype. In general, for further studies, more than four cross-over operators can be considered. The reason for selecting various types of cross-over operators is to have different tools for new generation production in each problem. By employing these operators, the need of using sensitivity analysis in each problem will be eliminated. It will not cause any increase in the computational effort, because even in the case that there are so many operators contributing in the breeding and new solution generation, the better functions will almost always find a chance to come to the next generation, though the chances of the others will not be eliminated, even without making any improvement. The contribution rate of cross-over operators and heuristic functions to the information exchange between the solutions is made proportional to their fitness value at the previous cycle. A fitness value is assigned to each operator which is updated by considering its contribution to solution improvement at each computational step. For example, the fitness value (effectiveness weight) assigned to each cross-over operator either increases or decreases at the next generation and eventually the operator's contribution will decrease only if it is not successful.

By employing different heuristic functions and mutation operators, the best solution is improved. Again, the contribution rate of the operators in solution improvement is made proportional to their fitness value in the previous cycle. The ranking process and selection of the best heuristic functions for the next generation is the same as that described for cross-over operators. However, in its present form, the algorithm benefits from a combination of four different cross-over operators which act as breeding processes as well as two mutation operators (heuristic functions) as different feeding performance.

The HBMO algorithm continues until the termination criteria (meeting the predefined number of iteration) are satisfied, and the best solution from the set of current best solutions and improved solutions are selected. If the termination criteria are not satisfied, all trial solutions are discarded and a new set of trial solutions is generated to make the search process more extensive. To generate a new set of trial solutions, remaining broods with desirable fitness are partially used along with the random generation of new (trial) solutions.

It should be noted that one of the main characteristics or advantages of the proposed algorithm is its independence from its related parameters. This is due to the fact that the algorithm tunes its parameters during the process and so it is not reliant on the initial value of such parameters. Hence, this highlights the performance of the algorithm which decreases the need for the algorithm parameters' sensitivity analysis required in the case of other evolutionary algorithms.

APPLICATION

To illustrate the model application and performance of a nonlinear, non-convex, real-world water resource problem, the operation of the Dez reservoir in southern Iran is selected as a case study. Monthly inflows to the reservoir along with monthly demands are presented in Figure 1. The average annual inflow to the reservoir and annual demand are estimated as $5{,}900 \times 10^6\,m^3$ and $5{,}303 \times 10^6\,m^3$, respectively. The effective storage volume of the reservoir is $2{,}510 \times 10^6\,m^3$. In the following problems, monthly release from the reservoirs in month t (R_t) of a 12 month period is considered as a linear function of end storage in the past month (S_{t-1}) and the monthly inflow to the reservoir in the current period t ($Q(t)$):

$$R_{(t)} = A_i + B_i \times S_{(t-1)} + C_i \times Q_{(t)} \tag{1}$$

where A_i, B_i, and C_i are decision variables of the problems.

Irrigation purpose

The objective of the study is to minimize the total squared deviation (TSD) of releases ($R_{(t)}$) from the target demands ($D_{(t)}$):

$$\text{Min TSD} = \sum_{t=1}^{\text{nt}} ((R_{(t)} - D_{(t)})/D_{\max})^2 \tag{2}$$

s.t.:

$$S(t) = S(t+1) - Q(t) + R(t) + \text{Loss}(t); \quad \forall t \tag{3}$$

$$R_{\min}(t) \le R(t) \le R_{\max}(t); \quad \forall t \tag{4}$$

$$S_{\min}(t) \le S(t) \le \text{Cap}; \quad \forall t \tag{5}$$

$$S(1) = S_{\min} \tag{6}$$

in which $D_{\max}$ = maximum demands; $S(t)$ = storage at the start of period t; $Q(t)$ = inflow to the reservoir during period t; $R_{\min}(t)$ and $R_{\max}(t)$ = minimum and maximum capacity of the outlet; $S_{\min}(t)$ = minimum storage during period t and Cap(n) = design capacity of the reservoir (MCM).

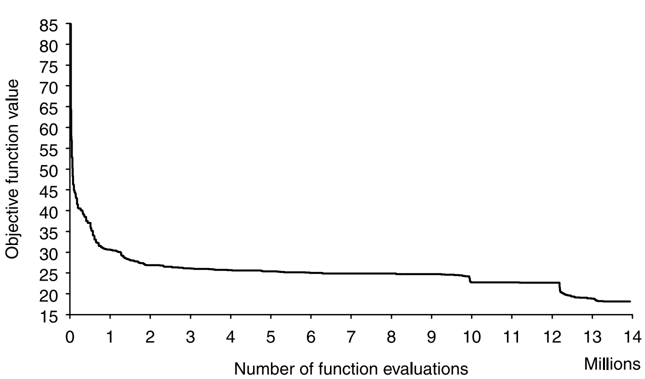

Figure 1 | Rate of convergence of the model (considering evaporation).

Considering the optimum long-term operation without any operational rule, 60 operational periods, and using the LINGO 8.0 NLP solver, this problem has a fitness value of 0.796 115. The result of 10 different runs with the proposed HBMO algorithm has an average fitness value of 0.824 and the best fitness value of 0.803. The results of this case show a very slight difference between the results of the LINGO 8.0 solver as the global optimum of the problem and those obtained by the HBMO algorithm. Increasing the operational periods from 60 to 480 makes the problem more complex for using the proposed algorithm in terms of dimensionality. In this case, the LINGO 8.0 software reports a value of 10.617, whereas the HBMO algorithm converges to 24.205. Using the linear operational rule and 480 operational periods for this problem, 26.084 and 18.147 are the results for the LINGO 8.0 and HBMO algorithm, respectively. It seems that, in the case of extracting the operational rules, the NLP solvers remain trapped in a local optimum due to the multi-modality of the problem. In contrast the HBMO algorithm overcomes this complexity and converges to a better local optima compared with NLP solvers. Decision variables obtained by those two NLP and HBMO algorithms are presented in Table 1. The convergence rate of the model using the HBMO algorithm toward a global optimum is illustrated in Figure 1. It shows a very rapid convergence of the objective function value obtained using the proposed algorithm had a very rapid convergence considering the number of function evaluations. By having a rather rapid convergence and final TSDs that are very near to those of the global optimum, it can be suggested that the HBMO algorithm is quite promising for further development and application in the field of water resource planning and management. Figure 2 shows releases obtained by using the resulting rules for each month of the last five years of operation. Results of the model in terms of storage volume at the end of each period are shown in Figure 3. For the same problem, along with the global optimum, obtained from the LINGO 8.0 NLP solver, the monthly releases and storages resulting from the HBMO model are presented in Figures 2 and 5, respectively. Monthly demands as well as the monthly inflow to

Table 1 | Decision variables obtained by HBMO algorithm and LINGO 8.0

		Month											
		1	2	3	4	5	6	7	8	9	10	11	12
HBMO	A	1940	3000	1837	2082	1372	684	616	19752	21791	522	405	2547
	B	1.03	1.58	1.01	0.99	1.03	0.71	0.65	2.00	2.00	0.30	0.53	1.51
	C	0.47	0.31	0.63	4.00	1.00	0.64	1.53	1.57	1.77	1.05	0.85	0.80
LINGO	A	0	0	0	0	0	0	0	0	0	0	0	0
	B	0.00	0.00	0.05	0.09	0.09	0.03	0.00	0.00	0.00	0.00	0.00	0.00
	C	0.50	0.64	0.87	1.33	2.14	3.15	2.57	1.03	0.88	0.77	0.62	0.63

the reservoir are also shown in Figure 2. A systematic underestimation observed in this period shows that the results of the HBMO algorithm have more adaptability with demand patterns. As is clear, the storages of both methods are among the permissible limit that denotes the feasibility of the results. In other words, the HBMO algorithm keeps the storage levels lower than those in the NLP solver, which means that the regulated water in each period by the proposed algorithm has a higher amount than those by the NLP solver.

Hydropower purpose

The objective of the study is to make the power generation as close to the installed capacity as possible. Mathematically, the objective may be defined as

$$\text{Min} \sum_{t=1}^{T} \left(1 - \frac{P(t)}{\text{PPC}}\right) \tag{7}$$

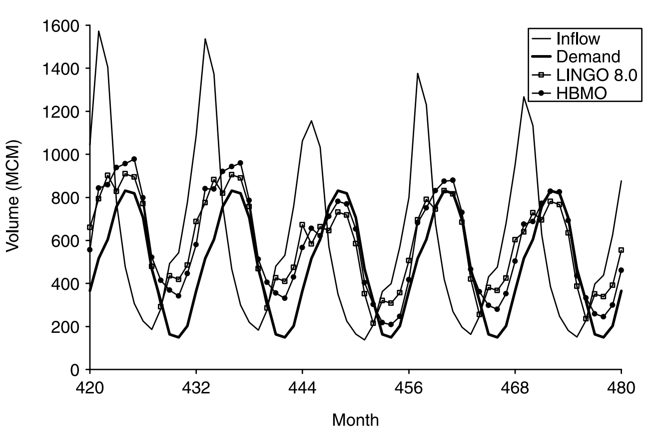

Figure 2 | Monthly optimum releases resulting from HBMO algorithm and LINGO 8.0.

where $P(t)$ = power generated during month t in MW (decision variables); PPC = installed capacity in MW and T = total number of periods (months) considered (180). The objective function is subject to the following constraints:

$$P(t) = g \times e(h) \times \text{RP}(t)/\text{PF}/\text{Mul}(t) \times (\bar{H}(t) - \text{TW})/1000 \tag{8}$$

$$\bar{H}(t) = (H(t) + H(t+1))/2 \tag{9}$$

$$H(t) = h_0 + h_1 S(t) + h_2 S(t)^2 + h_3 S(t)^3 \tag{10}$$

$$S(t) = S(t+1) - Q(t) + R(t) + \text{Loss}(t) \tag{11}$$

$$\text{Spill}(t) = R(t) - RP(t) \tag{12}$$

$$\text{Loss}(t) = Ev(t) \times A(t)/1000 \tag{13}$$

$$A(t) = a_0 + a_1 S(t) + a_2 S(t)^2 + a_3 S(t)^3 \tag{14}$$

$$R_{\min}(t) \le RP(t) \le R_{\max}(t) \tag{15}$$

$$S_{\min}(t) \le S(t) \le S_{\max}(t) \tag{16}$$

$$P_{\min}(t) \le P(t) \le \text{PPC} \tag{17}$$

in which g = gravity; $e(h)$ = system efficiency (= 0.9 in this study); RP(t) = inflow to the power plant; PF = plant factor, Mul(t) = conversion factor (MCM to CMS); TW = tail water elevation; $\bar{H}(t)$ = average reservoir head; $H(t)$ = reservoir surface elevation; $S(t)$ = storage at the start of period t; $Q(t)$equals;inflow to the reservoir during period t; $R(t)$ = release during period t (decision variable); Loss(t) = losses during period t (MCM); Spill(t) = volume of spill from the reservoir during period t; $Ev(t)$equals;evaporation height during period t (mm); a_0, a_1, a_2, a_3, h_0, h_1, h_2 and h_3 are constants; $S_{\max}(t)$ = maximum storage during period t and $P_{\min}(t)$ = minimum generated power during month t.

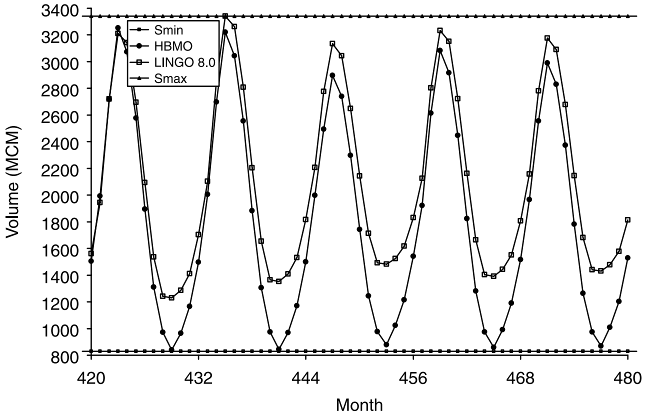

Figure 3 | Monthly optimum storages resulting from HBMO algorithm and LINGO 8.0.

The objective function defined by Equation (7) subject to constraints (8)–(17) represents the design and operation optimization problem for a hydropower system for a given number of periods. Equation (8) defines the power generated during period t as a function of average reservoir head $\bar{H}(t)$. Reservoir surface elevation, $H(t)$, is approximated through Equation (9). Constraints (15), (16) and (17) define the minimum and the maximum range for releases, storages and power generation, respectively.

The defined problem is highly nonlinear and non-convex, hence gradient-based NLP solvers may not be a good choice for its solution. As a matter of fact, they may either produce local suboptimal solutions and/or may even fail to end up with feasible solutions (Cai *et al.* 2001).

As a test, the model for 180 operational periods without considering any rules was solved using a very efficient NLP

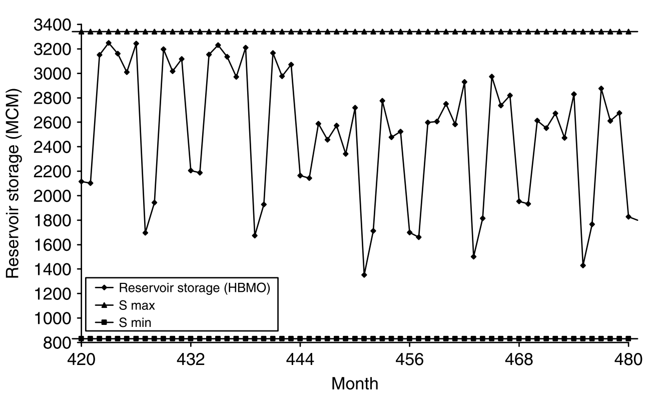

Figure 4 | Monthly optimum storages resulting from HBMO algorithm (without considering evaporation).

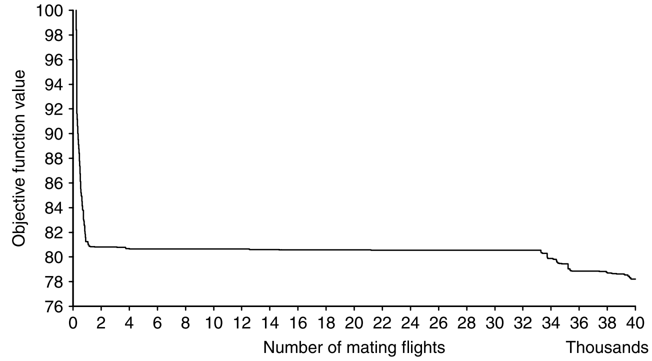

Figure 5 | Convergence rate of the model (considering evaporation).

solver from LINGO 8.0. The objective value of 20.415 was obtained for the default starting point of the algorithm, as a feasible/non-optimal solution which, in fact, was only a feasible, but not local, optimum solution. The same problem was solved using the proposed HBMO algorithm. For 65,000 mating flights and 14,223,621 function evaluations, an objective value of 3.3957 was obtained.

To further examine the performance of the HBMO algorithm for longer operation periods in highly nonlinear and non-convex problems, the same operation problem with 480 periods was considered. Disregarding evaporation, the NLP solver resulted in an objective function value of 45.80 whereas, after 40,000 mating flights, the HBMO algorithm resulted in a value of 54.7. Inclusion of evaporation further increases the nonlinearity of the model. In fact, when evaporation from the reservoir was added to the model, the LINGO 8.0 LNP solver failed to find a feasible solution, whereas the HBMO algorithm resulted in a feasible solution with an objective function value of 57.4 after 40,000 mating flights. It is interesting to note that, by using the near-optimal solution resulting from the HBMO in the LINGO 8.0 NLP solver as the starting solution, a minor improvement resulted in which the objective function value changed from 57.4 to 45.6 units.

In another attempt, using linear operation rules and eliminating the evaporation losses, the NLP solver of the LINGO 8.0 software gave a value of 78.188 for the objective function whereas the results of the HBMO (83.256) in this case has a small discrepancy (6.5%) from those obtained by LINGO 8.0. End-storage results of the HBMO algorithm are presented in Figure 4 as a proof of feasibility of the final result. In other words, the results of the HBMO algorithm are inputted into the LINGO 8.0 solver as the initial point and the NLP solver improves the objective function to 76.345. Results of this study indicate that, although the proposed HBMO algorithm does not give results that are better than those from the NLP solver, substituting the results as the initial solution of the gradient base solver increased the quality of the objective function of the problem. In such a case, using and even combining both the HBMO algorithm and the NLP solver can lead to a better solution for this type of optimization problem.

Adding the evaporation losses to this problem put the NLP solvers in trouble. For instance, LINGO 8.0 failed to find any feasible solution in this problem, whereas the HBMO algorithm converged to 78.204 as the final solution. The changing rate of the objective function with the number of mating flights is presented in Figure 5. This figure presents the convergence of the results to a near-optimal solution with an objective function value of 78.204 units after 65,000 mating flights. Clearly, the HBMO model has resulted in a much better performance than the efficient gradient-based NLP solver. Figure 6 shows storage volume at different periods for the five ending years. This research shows a strong potential for the proposed HBMO algorithm and other evolutionary algorithms of these types for solving the non-convex problems. In these cases, unlike the HBMO algorithm, the gradient-based NLP solvers fail to find any feasible solution or get trapped in local optima. Thus, as stated before, the results of the proposed algorithm can be a potential initial point of the NLP solvers to find a better solution, i.e. coupling of the HBMO algorithm with an NLP solver can be a powerful tool for solving non-convex problems.

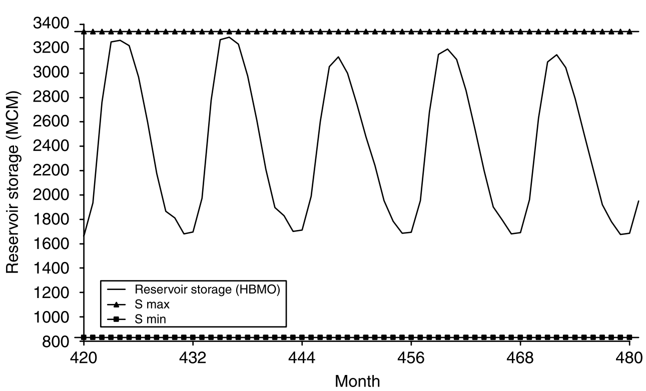

Figure 6 | Monthly optimum storages resulting from HBMO algorithm (considering evaporation).

CONCLUDING REMARKS

Application of the HBMO algorithm to a nonlinear, non-convex, recent water resources problem which is deriving the optimum operation rule for the reservoirs, resulted in a near-optimal solution with performance measures much better than the gradient-based LINGO 8.0 NLP solver. By using the near-optimal solution from the HBMO as a starting point for the NLP solver, a better local optimum resulted with appreciable improvement in the value of the objective function. Furthermore, comparing the results of both the NLP solver and the HBMO algorithms in deriving the monthly operation rules of the reservoirs indicates that the HBMO algorithm has the capability in handling such multi-modal problems. This is accomplished by adapting the heuristic functions (workers) during the process of optimization, though in some cases the results of NLP solvers show better performance. Even in those cases, applying the final results of the HBMO algorithm as the initial solution of NLP solvers improves the quality of the final results by the LINGO 8.0 software.

Model applicability and performance in a highly nonlinear, non-convex, real-world water resource problem was illustrated and tested using the Dez reservoir in southern Iran. As the nonlinearity of the problem increased by adding evaporation and/or extending the operation period, the HBMO algorithm performed better than the NLP solver. In fact, for 180 operation periods, both the NLP solver and the HBMO found feasible solutions where the HBMO algorithm performed significantly better. When evaporation was added to the model and operation periods were extended to 480 months, the NLP solver from LINGO 8.0 failed to find a feasible solution after two hours. For the same problem structure, the proposed HBMO algorithm found the first feasible solution in early mating flights. Improvement in the objective function after 40,000 mating flights was quite promising. Using the near-optimal solution from the HBMO in the LINGO 8.0 NLP solver as an initial feasible solution resulted in a slightly improved final solution.

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First received 3 January 2007; accepted in revised form 1 March 2008