satisfactory explanation in the case of Shaw's data, Fig. 6, since this machining was done at the very low speed of 1 ipm. In this regard, let us consider again the effect of rubbing on the tool flank. The equations for T and Z were based on the assumption of no flank rubbing, i.e., $b = 0$. If b is not zero, the maximum of these curves is shifted in the same way as the minimum-energy point previously discussed. As explained previously, this normally would result in shifting the maximum to the right, meaning that our computed value of ϕ_0 was too low. If a larger value of ϕ_0 were used, the value of $1 - \phi/\phi_0$ would be larger, and the difficulty would be resolved. We feel that this is the proper explanation for the intercept with the slow-speed data.

Conclusion

We have examined the case of orthogonal machining of a strain-hardening material. We conclude that the strain imparted to the chip is a measure of the deviation from Merchant's original equation, $2\phi + \tau - \alpha = 90$ deg, provided the temperature increases are not so great as to alter the stress-strain curve drastically. We conclude further that an accurate evaluation of τ in the foregoing equation (or ϕ_0 in terms of our variables) requires an assessment of the forces of rubbing on the tool flank.

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DISCUSSION

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The authors have presented an interesting analysis of the metal cutting process. A few of the assumptions used, however, should be considered in greater length.

The first of these is the assumption of strain hardening in the material being cut. As can be seen from the data of Kececioglu[3] shown in Fig. 11 this can, under the unique conditions of metal cutting, be strain softening. Also applicable here is Fig. 12 which shows an increase in shear stress due to increasing strain rate. A statistical analysis[4] of this data reveals the equations stated on each curve. Also included are the correlation coefficient r, standard deviation σ, and the confidence level. The confidence level is the probability that the observed correlation could have been achieved by a sample from a population of random data. The confidence levels of neither Figs. 11 or 12 show the data trends to be highly significant, but neither can they be disregarded. This would imply that the effect of strain rate should also be included in the analysis. In a recent paper by S. Kobayashi and others[5] the opinion was forwarded that neither γ nor $\dot{\gamma} = d\gamma/dt$ had any effect upon shear stress. Unfortunately, no statistical tests were applied to these data.

The assumption of isothermal shear must be carefully considered as the majority of the metal cutting applications approach very closely to adiabatic shear. In view of the data presented in Fig. 13[6] the assumption of isothermal shear may not impose as stringent a limitation as might be expected, since at these

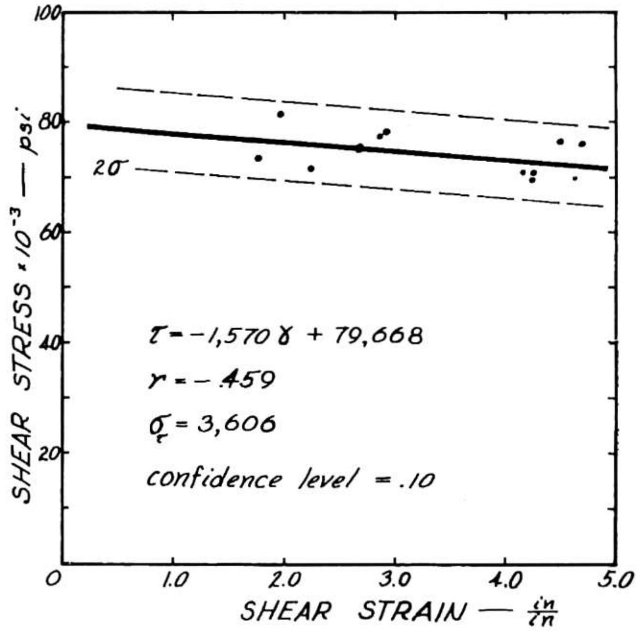

Fig. 11 Variation of shear stress with strain (from Kececioglu)

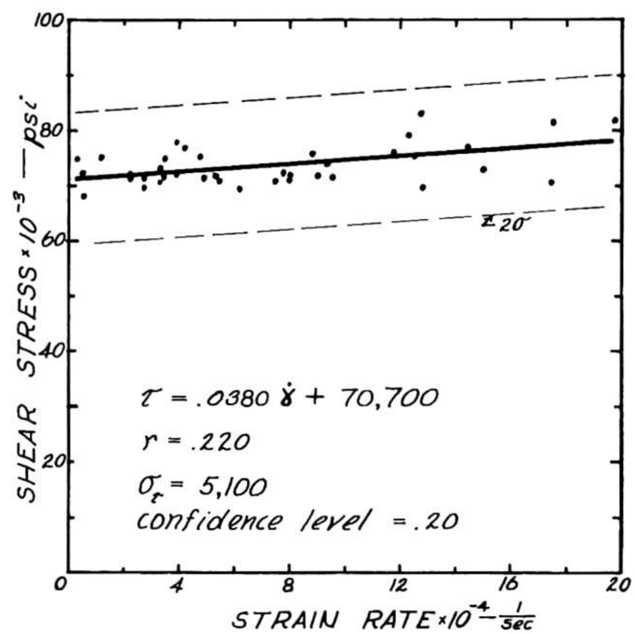

Fig. 12 Variation of shear stress with strain rate (from Kececioglu)

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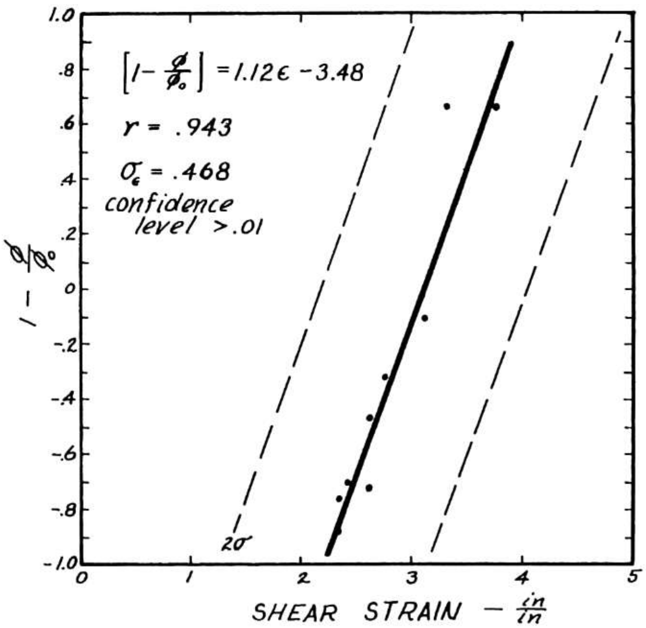

Fig. 13 Data from Bagley and Mennell; C-6 carbide cutting 235 Bhn, 4340 steel; tool signature of 0, 0, 5, 5, 5, 0, 0.020; 0.200 depth, 0.005 feed; dry; V = 50 to 700 sfpm

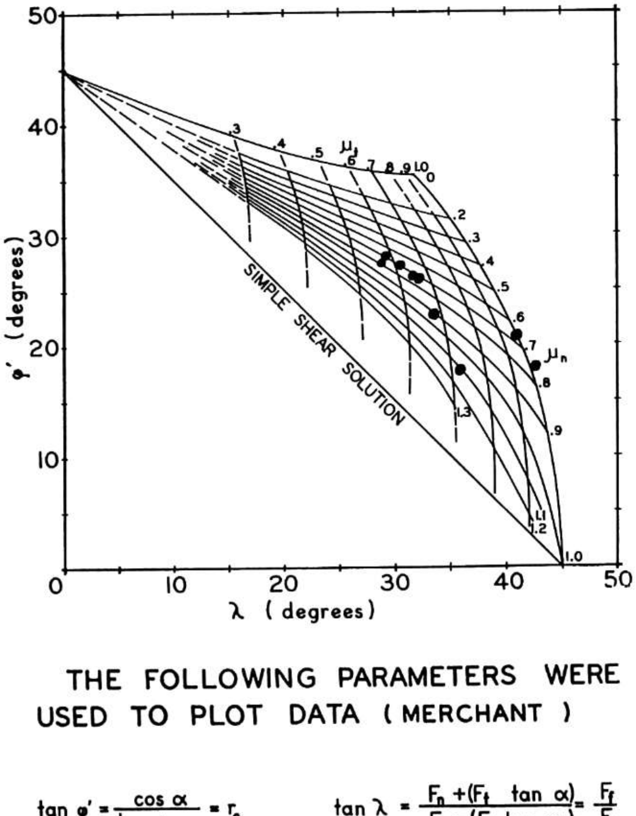

Fig. 14 Lee and Shaffer plasticity solution for α = 0 deg

high cutting speeds the correlation is very good. Regardless of this, the temperature effect on the shear zone is present and eventually must be included into the mathematical model.

The positive intercept of some of the data plotted in the paper, and also Fig. 13, was thought by the authors to be due to the assumption of no flank rubbing ($b = 0$). To obtain further insight into the cause of this positive intercept the Lee and Shaffer plasticity solution was used to approximate the coefficient of friction (μ_n) along the clearance face. To obtain the values of ϕ_0 necessary to plot the experimental data on Fig. 13 the data were first plotted on Fig. 14. From Fig. 14 the approximate values of λ and μ_n for each data point were obtained. With this information and the experimental shear angle the author's equation for the shear angle may be solved for b.

$$\cot 2\phi = \frac{\cot \gamma - b \cot \theta}{1 - b}$$

therefore:

$$b = \frac{\cot \gamma - \cot 2\phi}{\cot \theta - \cot 2\phi}$$

Selecting a data point at random and substituting in the values from the plasticity solution where:

$$\lambda = \gamma$$

$$\lambda_n = \tan^{-1} \mu_n = \theta$$

A value of 3.57 was obtained for b. This is a completely absurd value as these were new, sharp tools. This discrepancy could be due to either the ideal plastic model or the minimum energy approximation as they are both known to be in error.

The value of b was then assumed to be zero and the data plotted in Fig. 13. The correlation is seen to be excellent and the trend shown to have very high significance.

The practical application of Fig. 13 is very difficult as γ is not a constant with respect to cutting speed. Also the introduction of the ideal plastic model is not consistent with the possibility of a shear stress that is a function of shear strain. This is not serious though, since it was necessary later in the analysis to remove the possible variations of the shear stress.

Authors' Closure

We thank Mr. Bagley for his interesting discussion of our paper.

Concerning first the attempt to calculate the quantity b from the equation:

$$\cot 2\phi = \frac{\cot \gamma - b \cot \theta}{1 - b}$$

We do not understand the discusser's use of the plasticity condition to calculate γ, since γ is defined as the angle formed by the resultant of the exterior forces applied to the tool and should simply be calculated from the dynamometer readings. However, we certainly agree that this equation should not be used to calculate b. The equation was in no way intended to predict ϕ in general, but merely points out the effect that flank dragging would have on the original Merchant minimum energy solution for a material whose shear strength was a true constant.

We agree with the discusser concerning the possibility of "strain softening" occurring; in fact, we feel that the entire shape of the stress-strain curve at high strain rates is of paramount importance. Reference [4], by Zener and Holloman, points to this conclusion. We believe that, for heat sensitive materials, or for high speeds, a complex stress-strain relationship could result in rounding of the chip workpiece intersection and consequent values of ϕ greater than ϕ_0.