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Discussion

Discussion of ''On the Evaluation of Dynamic Stresses in Pipelines Using Limited Vibration Measurements and FEA in the Frequency Domain'' (Moussa, W. A., and Abdelhamid, A. N., 1999, ASME J. Pressure Vessel Technol., 121, Aug., pp. 241–245)

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A practical technique in determining the dynamic stresses in pipelines using finite element method is investigated by Moussa et al.. The authors propose to create an FEA model to excite the piping structure by a single concentrated harmonic force at its midspan with frequencies ranging from 0 to 160 Hz. The dynamic vibrating displacements are then determined as well as the von Mises stress at all elements. The displacement responses are assumed as if they were measured and are then used as input to excite the model. The von Mises stress in both cases (force input and motion input) are compared together and plotted for different tested sections. For the case of a single displacement used as the loading source, the maximum differences are 0.0 and 9.4 percent for Sections 1 and 12, respectively (Fig. 2 of the paper).

Here, the same pipeline structure is restudied using an analytical approach as well as a user-oriented piping stress analysis software, CAESAR II. The results show that both approaches are nearly identical, but differ seriously from the aforementioned work. The piping model is described as follows:

Total length:	240 in.	Modulus of elasticity:	30E6 psi
Outside diameter:	4.5 in.	Mass density:	0.2825 lb/in^3
Wall thickness:	0.74 in.	No. of elements:	22

Boundary conditions: Clamped at point 1 and hinged at point 23 (Fig. 1)
Natural frequencies (first 3 modes): 11.67, 37.87, and 79.02 Hz

Basic Equations

Consider an undamped single degree of freedom SDOF system that is subjected to a harmonic force $P(t)$ with amplitude P_0 and a circular frequency $\overline{\omega}$ (Fig. 2). The equation of motion is given by

$$M\ddot{y}+ky=P_0\sin(\overline{\omega}t) \quad (1)$$

The solution of Eq. (1) is

$$y(t)=A\cos\omega t+B\sin\omega t+\frac{P_0}{k}\frac{1}{1-r^2}\sin\overline{\omega}t \quad (2)$$

where r is defined as the ratio of the circular frequency of the externally applied load to the natural circular frequency of the system; that is,

$$r=\frac{\overline{\omega}}{\omega}=\frac{\overline{f}}{f} \quad (3)$$

The solution given by Eq. (2) is the superposition of the free vibration problem and the effect of the exciting force exposed by the last term of Eq. (2), which involves only the frequency of the harmonic load. For frequency response or harmonic analysis, only the steady-state response is considered, and Eq. (2) becomes

$$y(t)=\frac{P_0}{k}\frac{1}{1-r^2}\sin\overline{\omega}t \quad (4)$$

The solution for maximum displacement from an unphased harmonic analysis is then

$$\delta_{\text{dyn}}=\frac{P_0}{k}\frac{1}{(1-r^2)} \quad (5)$$

Let

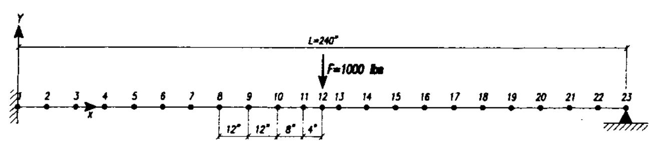

Fig. 1 Clamped-hinged piping model

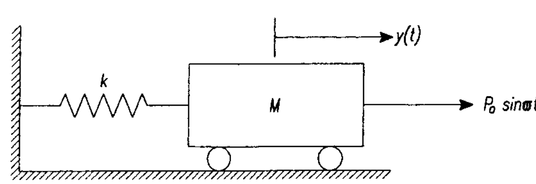

Fig. 2 Undamped SDOF system subjected to a harmonic load

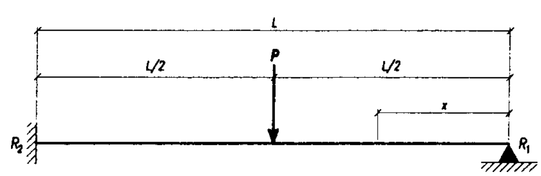

Fig. 3 Concentrated load at midspan

Table 1 Displacement results due to force load case at mid-span (F=1000 lb)

Node	Distance from clamped end	Displacement (in.) (Analytical) δanal.	Displacement (in.) (Caesar) δcaesar	$\varepsilon=\left(\frac{\delta_{anal}-\delta_{caesar}}{\delta_{anal}}\right)x100$
1	0	0.0000	0.0000	0.00
4	36	0.0606	0.0610	0.66
8	84	0.2311	0.2308	0.13
12	120	0.3205	0.3200	0.16
16	156	0.3061	0.3083	0.72
20	204	0.1587	0.1617	1.89
23	240	0.000	0.000	0.00

$$\frac{P_0}{k}=\delta_{\text{static}}\Rightarrow\delta_{\text{dyn}}=\frac{\delta_{\text{static}}}{(1-r^2)} \tag{6}$$

where k is the structural stiffness of the SDOF system.

Force Excitation at Midspan

Analytical Calculation. The beam deflection equations for the case, as shown in Fig. 3, are well known and written as follows:

$$\text{at load,}\quad \Delta_x=\frac{7PL^3}{768EI} \tag{7}$$

$$\text{when}\quad x<L/2,\quad \Delta_x=\frac{Px}{96EI}(3L^2-5x^2) \tag{8}$$

$$\text{when}\quad x>L/2,\quad \Delta_x=\frac{P}{96EI}(x-L)^2(11x-2L) \tag{9}$$

For the force load case of 1000 lb and the excitation frequency of 5 Hz, at midspan (Node 12), the static deflection is calculated as per Eq. (7) and is equal to 0.267 in. The dynamic deflection is

$$\delta_{\text{dyn}}=\frac{0.267}{1-\left(\frac{5}{11.675}\right)^2}$$
$$=0.32\ \text{in.}$$

At Node 8, 84 in. from the clamped end ($x=156>L/2$), the same procedure could be applied to calculate the dynamic deflection using Eq. (9) and is equal to 0.231 in. The foregoing procedure is then used to obtain the dynamic deflection profile, as shown in Table 1. In order to determine dynamic stresses, one need only apply the following relationships for bending moment, shear, and bending stress:

$$M=-EI\frac{d^2y}{dx^2};\quad V=\frac{dM}{dx};\quad \sigma=\frac{M}{Z}$$

Computer Simulation. A harmonic dynamic analysis of the foregoing system has been performed using CAESAR II. The harmonic load of 1000 lb at a frequency equal to 5 Hz was applied at Node 12 (midspan). The computer output is tabulated in Table 1 for comparison purpose. As one can see, the displacement results are nearly identical in both approaches.

Table 2 Displacement results due to imposed displacement at Node 8

Node	Distance from clamped end	Displacement (in.) (Analytical) δanal.	Displacement (in.) (Caesar) δcaesar	$\varepsilon=\left(\frac{\delta_{anal}-\delta_{caesar}}{\delta_{anal}}\right)x100$
1	0	0.0000	0.0000	0.00
4	36	0.0704	0.0681	3.27
8	84	0.2364	0.2308	2.37
12	120	0.2819	0.2813	0.21
16	156	0.2491	0.2542	2.05
20	204	0.1242	0.1292	4.03
23	240	0.000	0.000	0.00

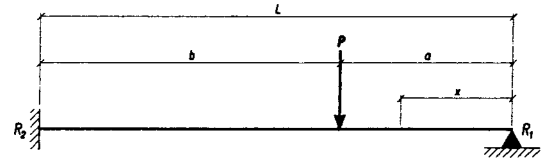

Fig. 4 Concentrated load at any point

The next step is to take displacement response at a certain location to excite the system in order to simulate the displacement excitation case and evaluate the dynamic stresses.

Displacement Excitation

Computer Simulation. In order to compare with the results of the paper, the displacement at Node 8 (equal to 0.2308 in. in CAESAR II from the force load case) was used to excite the system with the harmonic frequency of 5 Hz. A computer run was performed and the output results are shown in Table 2.

Analytical Calculation. Regarding the analytical approach for this loading case, one needs to evaluate the equivalent dynamic force applied at Node 8 to create a displacement of 0.2308 in. at this location. With trial and error processing using CAESAR II, a final force equal to 1220 lb applied at Node 8 will generate a identical result as the displacement load case. The problem now becomes how to evaluate the dynamic displacement with a concentrated load at any point. The beam deflection equations for Fig. 4 are written as follows:

$$\text{at load,}\quad \Delta_x=\frac{Pa^2b^3}{12EIL^3}(3L+a) \tag{10}$$

$$\text{when}\quad x<a\quad \Delta_x=\frac{Pb^2x}{12EIL^3}(3aL^2-2Lx^2-ax^2) \tag{11}$$

$$\text{when}\quad x>a\quad \Delta_x=\frac{Pa}{12EIL^3}(L-x)^2(3L^2x-a^2x-2a^2L) \tag{12}$$

At load point, i.e., Node 8, the static deflection is calculated using Eq. (10)

$$\Delta_x=\frac{1220(156)^2(84)^3(3\times240+156)}{12(30E6)(16.047)(240)^3}$$
$$=0.193\ \text{in.}$$

and the dynamic deflection is

$$\delta_{\text{dyn}}=\frac{0.193}{1-(5/11.675)^2}$$
$$=0.236\ \text{in.}$$

The deflection at other nodes are also calculated using Eqs. (11) and (12), and the results are also shown in Table 2.

Results

As we can see, both approaches (analytical and computer calculations), offer very similar results with the maximum error not

Table 3 Comparison of γ values at frequency of 5 Hz

Section	Stress (psi)		γ at 5 Hz	
	Force Load	Displacement load	This work	Previous work [1]
1	7733	8983	16.2	0
12	6098	4071	33.2	9.5
16	4611	3131	32.1	6.5

Table 4 Spectrum of γ using displacement at Node 8 as displacement excitation

Frequency (Hz)	γ		
	Section 1	Section 12	Section 16
5	16.16	33.24	32.10
15	19.26	56.10	36.34
45	11.61	23.71	107.71
65	66.38	29.11	62.94
85	71.99	86.81	90.72
105	65.34	98.07	85.36
125	16.18	89.12	74.22

Table 5 Spectrum of γ using displacement at Nodes 8 and 16 simultaneously as displacement excitation

Frequency (Hz)	γ		
	Section 1	Section 12	Section 16
5	5.33	29.09	14.79
25	37.33	50.20	35.86
45	20.37	34.33	128.69
65	68.78	19.95	41.62
85	22.17	66.16	40.18

exceeding 4.5 percent. We are going to evaluate the stresses induced by the force load and displacement load at 5 Hz. We use the same equation proposed in the paper

$$\gamma=\left|\frac{\sigma_F-\sigma_D}{\sigma_F}\right|\times 100$$

where σ_F and σ_D are the bending stresses in the tested sections for the force and the displacement excitation case, respectively. The calculation is shown in Table 3. For comparison purposes, the γ values of the paper at 5 Hz are reported in Table 3; as we can see, the γ values differ seriously. At the fixed end (Section 1), it is not 0 percent, but 16 percent and at Section 12, 33 percent in lieu of 9.5 percent.

Until now, we used only one value of frequency of 5 Hz to establish the confidence level between the analytical and computer procedures. The same calculation was then repeated for the following cases using the piping computer code:

Run 1. Force load case for frequencies ranging from 5 to 145 Hz with increment of 20 Hz.

Run 2. Use Run 1's displacement at Node 8 as input for displacement excitation case for the same frequency range.

Run 3. Use Run 1's displacement at Nodes 8 and 16 as dual translational vibration measurements (TVM) input for the same frequency range.

For finite element analysis point of view, Runs 2 and 3 are treated as an excitation due to multi-base motion. The natural frequencies of the piping system is no longer the same as in Run 1. They are as follows:

Mode	Frequency (Hz) Run 2	Run 3
1	23.2	70.3
2	75.2	102.2
3	115.0	145.5
4	172.3	210.8

After stress runs completed, the γ values were then calculated with the attempt to reproduce the plots of Figs. 2 and 4(*a*) in the paper. The outcome is completely different; the results are shown in Tables 4 and 5, respectively, rather than plotted. This is because, during the load steps execution, the imposed displacement piping system will go through its natural frequencies and there will be resonance situations where the stresses become excessively large. In the stress output, some frequency related results have been eliminated or shifted because of the aforementioned resonance reason. In light of this noticeable discrepancy, the precision of stress calculation of the paper is seriously questionable and definitely not reliable, and only one conclusion could be drawn: the dynamic stresses are only identical if, and only if, the motion measurement is applied exactly at the same force position. When the loading positions are unknown or different between the force and displacement excitation case, all boundary conditions, mode shapes, as well as the stiffness matrix change accordingly, and the errors are unpredictable.