K Meson-Pion Interaction and $K^- + p \rightarrow K^0 + \pi^- + p$ Reaction

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$K^-$ meson-pion interaction is studied by taking into account the effects of $K^*$. Both the value of coupling constant for $K\pi K\pi$-interaction and the magnitude of cross section of $K\pi$ scattering are estimated under the assumption that the isotopic spin of $K^*$ is equal to 1/2. Making use of the value of coupling constant we try to investigate the reaction $K^- + p \rightarrow K^0 + \pi^- + p$ at 1.15 Bev/c and 2 Bev/c. The total cross section of this reaction, the angular distributions and momentum spectra of protons and pions are examined. It is shown that the assumption of $S=1$ ($S$ indicates the spin of $K^*$) rather than that of $S=0$ is consistent not only with the experimental results for the angular distribution of $K^- + p \rightarrow K^0 + \pi^- + p$ process but also with those for $K^- + p \rightarrow K^0 + \pi^- + p$ reaction at 1.15 Bev/c, that is, the magnitude of cross section and the energy spectrum of pion.

§ 1. Introduction

The recent experimental results for $K^- + p \rightarrow K^0 + \pi^- + p$ reaction1) (1.15 Bev/c $K^-$ meson) have shown the following remarkable facts.

(i) The observed distribution for momentum of proton in the final state can be best explained by the reaction through the following process,

$$K^- + p \rightarrow K^0 + \pi^- + p.$$ 

(ii) The angular distribution for the reaction $K^- + p \rightarrow K^0 + \pi^- + p$ is consistent with isotropy.

(iii) The total cross section for $K^- + p \rightarrow K^0 + \pi^- + p$ is about 2 mb.

(iv) The mass of $K^*$ is nearly equal to 884 Mev.

(v) The half width of this resonant state is about 10 Mev.

(vi) The decay angular distribution seems to be consistent with $S=0$ or $S=1$, where $S$ is the spin of $K^*$.

(vii) The assumption of $I_{K*}=1/2$ ($I_{K*}$ indicates the isotopic spin of $K^*$) rather than that of $I_{K*}=3/2$ seems to be consistent with the experimental results for the branching ratio of the decay of $K^*$.

In advance of the study for $K^- + p \rightarrow K^0 + \pi^- + p$ reaction, we investigate the problems of $K^- \pi$ scattering.* In order to obtain the matrix elements (or cross

* In this paper, only the scattering process that gives rise to the resonance is taken into account. In order to satisfy the condition of crossing symmetry it is of course necessary to estimate the contribution from other processes. But these processes have not large effects on the scattering.
sections) for $\bar{K}\pi$ scattering, the propagation function of $K^*$ is modified by taking into account the damping effects. But it must be noted that, as will be shown later, the magnitude of the cross section $\sigma(\bar{K}\pi)$ at the resonance energy does not depend on the value of coupling constant for $\bar{K}\pi K^*$-interaction. Since its magnitude can also be estimated from kinematical point of view, it is possible to see whether the modified propagation function of $K^*$ is correct or not. Based both on the correct expression for the modified propagation function and on the experimental result for the width of $K^*$-resonance level, the value of coupling constant for $\bar{K}\pi K^*$-interaction is estimated.

From the experimental result (vii), it is impossible to expect the strong $\bar{K}^0\pi^+$ or $K^-\pi^-$ interaction and there are of course the following relations between the cross sections,

$$\sigma(K^-+\pi^+ \rightarrow K^-+\pi^+)/4 = \sigma(K^-+\pi^+ \rightarrow \bar{K}^0+\pi^0)/2 = \sigma(\bar{K}^0+\pi^0 \rightarrow \bar{K}^0+\pi^0)/1$$

Therefore only the scattering process $K^-+\pi^+ \rightarrow K^-+\pi^+$ is studied in this paper.

The problems of $s$-wave and $p$-wave $K\pi$ scattering are treated in §2 and §3 respectively.

Making use of the values of coupling constants obtained in §2 and §3, we should like to study the reaction $K^-+p \rightarrow \bar{K}^0+\pi^-+p$ from field theoretical point of view. First of all we pay attention to the reaction $K^-+p \rightarrow K^0+p$. In §4 the angular distribution for this reaction is examined and the magnitude of cross section is estimated. Based on the results, it is shown that the assumption of $S=1$ rather than that of $S=0$ is consistent with the experimental results (ii) for the interaction of 1.15 Bev/c $K^-$ meson in hydrogen. The angular distribution of proton for the reaction at 2 Bev/c has a remarkable peak in the backward direction (cf. Fig. 4 and Fig. 5). This tendency is more conspicuous in the case $S=0$ than in the case $S=1$.

In §5 the magnitude of cross section, the angular distributions and momentum spectra of pions and protons for the reaction $K^-+p \rightarrow \bar{K}^0+\pi^-+p$ at 1.15 Bev/c and 2 Bev/c are investigated in order to see the energy dependence of them. It is shown that the angular distribution and momentum spectrum of pion depend strongly on the spin of $K^*$. Comparing our theoretical results for the cross section and energy spectrum of pion with the experimental ones, we may conclude that $S=1$ is more promising than $S=0$.

§2. $s$-wave $\bar{K}\pi$ scattering

In the case of $S=0$, the interaction Hamiltonian is given as follows:

* With respect to the isotopic spin of $I_{K^*}$, some discussion will be made in the last section.
where $\phi, \varphi$, and $\phi_\alpha$ are the wave functions of the $K^*$, $\bar{K}$ and $\pi$, and $M$, $m_K$ and $\mu$ are the masses of $K^*$, $\bar{K}$ and $\pi$ respectively. For the purpose of estimating the value of coupling constant $G_s^2/4\pi$ from the experimental result for the width of $K^*$, the propagation function of $K^*$ is modified by means of damping theory. Of course it is necessary to get the correct expression for the modified propagation function in order to study the $\bar{K}$ meson reactions in which $K^*$ plays the important role. In our description of $\bar{K}\pi$ scattering, the damping effects may be introduced if we take into account the sum of contributions from a series of graphs $(a^1)$, $(a^2)$, $(a^3)$, $\cdots$ in Fig. 1.

![Fig. 1.](https://example.com/fig1.png)

The propagation function $D_F(k)$ written in the momentum space is modified as follows,

$$D_{F'}(k) = D_F + D_F ND_F + D_F ND_F ND_F + \cdots$$

which vanishes for $k^2 + (m_K + \mu)^2 > 0$. Here it should be noted that the damping effects due to the occurrence both of $(K^- + \pi^+)$ and of $(\bar{K}^0 + \pi^0)$ in the intermediate state must be involved in the expression for $N(k)$. The coefficients of 2 and 1 in the factor $(2+1)$ in Eq. (5) show the effects of the former and those of the latter respectively.

* The same way as this was employed previously in order to obtain the modified propagation function of the particle with spin 3/2.
The integration in Eq. (5) yields
\[ N(k) = -\frac{3}{2}m_K^2 \left( \frac{G_s^2}{4\pi} \right)^\pi \sqrt{-k^2} A, \] (6)
\[ A = \sqrt{\left[ k^2 + (m_K + \mu)^2 \right] \left[ k^2 + (m_K - \mu)^2 \right]}. \] (7)
Then
\[ \bar{D}_F(k) = \frac{1}{2\pi i} \frac{1}{k^2 + M^2} \frac{1}{1 - (2\pi i)^{-1} N(k) (k^2 + M^2)^{-1}} = \frac{1}{2\pi i} \frac{1}{k^2 + M^2 - (3/4) im_K^2 (G_s^2 / 4\pi) A / \sqrt{-k^2}} \] (8)
Such a method as this has been adopted usually in order to estimate the width of resonant state. However, we now should like to point out that the imaginary part of denominator of this modified propagation function $2\pi i \bar{D}_F(k)$ should be multiplied by factor 2 as will be seen from the following consideration.

By means of the relation (8), the scattering matrix $R$ for this process can be written as
\[ R(K^- + \pi^+ \rightarrow K^- + \pi^+) = 2R(\bar{K}^0 + \pi^0 \rightarrow \bar{K}^0 + \pi^0) \]
\[ = \frac{1}{2\pi i} \frac{-m_K^2 (G_s^2 / 4\pi)}{[\hat{p} + \hat{q}]^2 + M^2 - (3/4) im_K^2 (G_s^2 / 4\pi) A / \sqrt{-(\hat{p} + \hat{q})^2}} \] (9)

where $\hat{p}$ and $\hat{q}$ are the four momenta of the incident $\pi$ and $\bar{K}$ respectively. In the center of mass system, the scattering amplitude for the process $K^- + \pi^+ \rightarrow K^- + \pi^+$ is given by
\[ f(K^- + \pi^+ \rightarrow K^- + \pi^+) = \frac{1}{(\sqrt{\hat{p}^*^2 + m_K^2} + \sqrt{\hat{p}^{*2} + \mu^2})} \times \frac{m_K^2 (G_s^2 / 4\pi)}{[\hat{p} + \hat{q}]^2 + M^2 - (3/4) im_K^2 (G_s^2 / 4\pi) A / \sqrt{-(\hat{p} + \hat{q})^2}} \] (10)
because
\[ \frac{d\sigma(K^- + \pi^+ \rightarrow K^- + \pi^+)}{d\Omega} = \frac{4\pi^2}{(\sqrt{\hat{p}^*^2 + m_K^2} + \sqrt{\hat{p}^{*2} + \mu^2})^2} |R(K^- + \pi^+ \rightarrow K^- + \pi^+)|^2. \] (11)

$p^*$ is the momentum of the incident pion in the center of mass system.

In the scattering at the resonance energy
\[ \sqrt{\hat{p}^*^2 + m_K^2} + \sqrt{\hat{p}^{*2} + \mu^2} \rightarrow M, \]
\[ (\hat{p} + \hat{q})^2 + M^2 \rightarrow 0, \]
\[ A \rightarrow \sqrt{[(M^2 - m_K^2 - \mu^2)^2 - 4m_K^2 \mu^2] / 4M^2} = p^*. \] (12)
Thus the expression for scattering amplitude at the resonance energy is reduced to the following form (cf. Eq. (10)):

$$f(K^- + \pi^+ \to K^- + \pi^+)_{\text{resonance}} = \frac{4}{3} i \frac{1}{\rho^*}. \quad (13)$$

On the other hand, the scattering amplitude $f(\theta)$ can generally be written down as follows:

$$f(\theta) = \frac{\sqrt{2}}{i \rho^*} \sum_l \sqrt{2l + 1} R_{2l, l} Y_l(\theta, \varphi), \quad (14)$$

$$R_{2l, l} = \exp(2i\omega_{2l, l}) - 1, \quad (15)$$

where $R_{2l, l}$ means the $R$-matrix for the state of isotopic spin $I$ and angular momentum $l$. As we concentrate our attention to the resonance scattering in $I=1/2$, $l=0$-state, the scattering amplitude for the process $K^- + \pi^+ \to K^- + \pi^+$ at the resonance energy is reduced to

$$f(K^- + \pi^+ \to K^- + \pi^+)_{\text{resonance}} = \frac{2}{3} i \frac{1}{\rho^*}. \quad (16)$$

The factor $(2/3)$ in Eq. (16) comes from the consideration about the isotopic spin space. If the effects of the other reactions (for instance, $K^- + \pi^- \to K^- + \pi^+$) are taken into consideration, Eq. (16) should be modified as

$$|f(K^- + \pi^+ \to K^- + \pi^+)_{\text{resonance}}| \leq \frac{2}{3} i \frac{1}{\rho^*}. \quad (16)'$$

At any rate there is the difference of factor 2 between Eq. (16) and Eq. (13). Of course such a situation as this is reproduced also in the approach to pion-pion interaction\(^3\) although any remark on this point has not been made.* In the estimation of the damping effects it is necessary to pay attention to this fact.

The correct expression for the modified propagation function can be written down as follows:

$$\overline{D}_r(k) = \frac{1}{2\pi i} \frac{1}{k^2 + M^2 - 2(2\pi i)\gamma N(k)}$$

$$= \frac{1}{2\pi i} \frac{1}{k^2 + M^2 - (3/2)im \kappa^2 (G_2^f / 4\pi) A/\sqrt{v - k^2}}. \quad (17)$$

It is needless to say that the correct value of scattering amplitude at the resonance energy is given by the $\overline{D}_r(k)$-function. Since $M=884 \text{ Mev}$, the resonance scattering takes place at the pion energy of 382 Mev (lab.), and the cross section turns out to be

* We are grateful to Dr. Y. Miyamoto and Dr. Y. Fujii for their helpful discussions on this point.
In the center of mass system the denominator of $D_{p}(\hat{p}+\hat{q})$ can be rewritten as
\[ (\hat{p}+\hat{q})^2 + M^2 - (3/2)im_k^2(G_{s}^2/4\pi)A/\sqrt{-(\hat{p}+\hat{q})^2} \]
where $E_r$ and $E^*$ stand for the resonance energy and the total energy of the colliding system respectively. Paying attention to the scattering in the neighborhood of resonance and comparing the expression in Eq. (19) with the denominator in one level formula, we may get approximately the following relation,
\[ I' \approx \frac{3}{2} m_k^2 \left( \frac{G_{s}^2}{4\pi} \right) \frac{\Lambda}{2E_r^3}. \]

This approximation may be fairly well because the width $I'$ of the resonance is very narrow. Putting the experimental value $I'/2=(10\sim 13)\text{ Mev}$ into Eq. (20), we obtain
\[ (G_{s}^2/4\pi) = (0.15 \sim 0.2). \]

The energy dependence of $\sigma(K^- + \pi^+ \rightarrow K^- + \pi^+)$ in case of $(G_{s}^2/4\pi)=0.15$ is illustrated in Fig. 2.

\[ \text{Fig. 2.} \]

\section*{§ 3. $P$-wave $K^-\pi$ scattering}

Next let us deal with the problem of $K^-\pi$ scattering in the case where $K^*$ is a vector particle. As the interaction Hamiltonian, the following type may be adopted,
\[ H_2 = G_{\nu} \Phi_{\nu} \phi_{\nu} \left( \frac{\partial \phi_{\nu}}{\partial x_{\nu}} - \frac{\partial \phi_{\nu}}{\partial x_{\nu}} \right) \]
or
\[ H_2' = G_{\nu}' \Phi_{\nu} \phi_{\nu} \frac{\partial \phi_{\nu}}{\partial x_{\nu}}, \]
where $\Phi_{\nu}$ is the wave function of the $K^*$. When $K^-\pi$ scattering is described in terms of such an interaction as this, another partial wave besides $p$-wave contributes to the scattering.\(^{\ast}\) If the spin of $K^*$ is equal to one, the $K^-\pi$ resonance

\(^{\ast}\) If the mass of $K$ meson were equal to that of pion, the interaction Hamiltonian mentioned in Eq. (22) would mean the $p$-wave $K\pi K^*$-interaction.
scattering is of course attributed to the strong $p$-wave interaction. We here­after try to select the $p$-wave component out of the matrix elements obtained by employing the interaction Hamiltonian (22) or (22)'.

Our approach to $K^-\pi^+$ scattering is made along the same line with the study in § 2. The $N_{\mu\nu}(k)$ which corresponds to the $N(k)$ in Eq. (5) can be expressed as follows: For the $H_1$-interaction

$$N_{\mu\nu}(k) = -\frac{3}{2} \left( \frac{G^2}{4\pi} \right) \int (dt) (\partial_t t, 0) \delta[(k-t)^2 + m_k^2] \delta(t^2 + \mu^2).$$  (23)

For the $H_2$-interaction

$$N_{\mu\nu}'(k) = -\frac{3}{2} \left( \frac{G^2}{4\pi} \right) \int (dt) t, 0, t, 0 \delta[(k-t)^2 + m_k^2] \delta(t^2 + \mu^2).$$  (23)'

By virtue of the transformation property, $N_{\mu\nu}(k)$ or $N_{\mu\nu}'(k)$ can be expressed by the following form,

$$N_{\mu\nu}(k) = -\frac{3}{2} \left( \frac{G^2}{4\pi} \right) \left[ B(k) \delta_{\mu\nu} + C(k) \frac{k_\mu k_\nu}{-k^2} \right],$$  (24)

or

$$N_{\mu\nu}'(k) = -\frac{3}{2} \left( \frac{G^2}{4\pi} \right) \left[ B'(k) \delta_{\mu\nu} + C'(k) \frac{k_\mu k_\nu}{-k^2} \right].$$  (24)'

We can obtain the expression for $\bar{D}_\nu(k)$ putting the $N_{\mu\nu}(k)$ or $N_{\mu\nu}'(k)$ into the equation corresponding to Eq. (3). Thus the scattering matrix elements can be written down by using the $\bar{D}_\nu(\hat{p} + \hat{q})$ although they consist of infinitely many terms. In the expression for the scattering matrix elements, all the terms which contain $C(\hat{p} + \hat{q})$ are expressed in terms of $(\hat{p} - \hat{q})$, $(\hat{p} + \hat{q})^2$ and $(\hat{p} - \hat{q})$, and all the terms which contain $C'(\hat{p} + \hat{q})$ are expressed in terms of $(\hat{p}' + \hat{q}')$, $(\hat{p} + \hat{q})^2$ and $(\hat{p} + \hat{q})$, where $\hat{p}'$ and $\hat{q}'$ are the four momenta of the final $\pi$ and $K$ respectively. Therefore all the matrix elements which contain $C(\hat{p} + \hat{q})$ or $C'(\hat{p} + \hat{q})$ are responsible for the $s$-wave scattering because $\hat{p} + \hat{q} = \hat{p}' + \hat{q}' = 0$ in the center of mass system. Since $p$-wave scatterings are examined in our study, we have only to take into account the contributions from $B(\hat{p} + \hat{q})$ or $B'(\hat{p} + \hat{q})$. Then $\bar{D}_\nu(\hat{p} + \hat{q})$ can be expressed by a compact form. 

Tracing the similar consideration with that in § 2, we obtain the correctly modified propagation function $\bar{D}_\nu(\hat{p} + \hat{q})$. The $p$-wave scattering matrix $R(\text{or } R')$ for the process $K^- + \pi^+ \rightarrow K^- + \pi^+$ can be expressed by the following compact form,

$$R(K^- + \pi^+ \rightarrow K^- + \pi^+) = \frac{1}{2\pi i} \sum_{l=1} P_{l=1} \frac{-(G^2/4\pi)(\hat{p} - \hat{q})(\hat{p} - \hat{q})}{(\hat{p} + \hat{q})^2 + M^2 - (3/2\pi)i(G^2/4\pi)B(\hat{p} + \hat{q})}$$  (25)
Meson-Pion Interaction and $K^- + p \rightarrow \bar{K}^0 + \pi^- + p$ Reaction

\begin{equation}
\frac{1}{2\pi i} \left( \hat{p} + \hat{q} \right)^2 - M^2 - (3/2\pi) i (G_{r}^2/4\pi) B(\hat{p} + \hat{q})^2 \text{cos} \theta,
\end{equation}

or

\begin{equation}
R' (K^- + \pi^+ \rightarrow K^- + \pi^+) = \frac{1}{2\pi i} P_{l=1} \left( \hat{p} + \hat{q} \right)^2 + M^2 - (3/2\pi) i (G_{r}^2/4\pi) B' \left( \hat{p} + \hat{q} \right)^2 \text{cos} \theta.
\end{equation}

The $A$ in Eq. (27) is given by Eq. (7), and $P_{l=1}$ means the projection operator on $p$-state. If we replace $A(G_{r}^2/4\pi)$ in Eq. (26) by $(G_{r}^2/4\pi)$, the expression for $R(K^- + \pi^+ \rightarrow K^- + \pi^+)$ turns out to be the same with that for $R' (K^- + \pi^+ \rightarrow K^- + \pi^+)$.

The scattering cross section for this process at the resonance energy is

\begin{equation}
\sigma (K^- + \pi^+ \rightarrow K^- + \pi^+) = \frac{4}{3 \sqrt{1 - k^2}} A^2
\end{equation}

\begin{equation}
\sigma (K^- + \pi^+ \rightarrow K^- + \pi^+) = \frac{3 \times 4\pi}{9} \rho_{\pi^+} \approx 82 \text{ mb}.
\end{equation}

In the same way as in § 2, we can estimate the value of coupling constant for $K\pi K^*$-interaction. The result is as follows,

\begin{equation}
(G_{r}^2/4\pi) = (G_{r}^2/4\pi)/A = (0.35 \sim 0.45).
\end{equation}

The energy dependence of $\sigma (K^- + \pi^+ \rightarrow K^- + \pi^+)$ in case of $(G_{r}^2/4\pi) = 0.35$ is illustrated in Fig. 3.

§ 4. $K^- + p \rightarrow K^* + p$ process

In this section the angular distribution and the magnitude of cross section for $K^- + p \rightarrow K^* + p$ process are examined by means of perturbation theory. Since the so-called equivalence theorem for pion-nucleon interaction can be applied to our case, there is no need to examine the results in the case of pseudoscalar coupling and those in the case of pseudovector coupling separately. Hereafter we make our discussion about the former case only.
I) The case of \( S=0 \)

The differential cross sections for this reaction can be expressed as follows,

\[
\frac{d\sigma(K^-+p\rightarrow K^*+p)}{d\Omega} = \frac{k \left( \frac{m_K}{m} \right)^2 \left( \frac{g^2}{4\pi} \right) \left( \frac{G_\gamma^2}{4\pi} \right) \left[ -2 \left( \frac{\hat{I}\hat{F}}{m^2} \right) - 2 \right]}{q(v^2 + m^2 + \sqrt{q^2 + m_K^2})^3} \left[ -2 \left( \frac{\hat{I}\hat{F}}{m^2} \right) - 2 + \frac{v^2}{m^2} \right]^2 \left( \frac{4q^2 + k^2 + 4kq \cos \theta}{m^2} \right) \left( \frac{4q^2}{m^2} \right)
\]

where \( \hat{I} \) and \( \hat{F} \) are the four momenta of proton in the initial and final state respectively, \( q \) and \( k \) are the momenta of \( K^- \) meson and \( K^* \) respectively, and \( m \) is the mass of nucleon. The angular distributions of protons are expressed by the last term in Eq. (30) and have the character shown in Fig. 4. The tendency of backward peak for angular distributions of protons becomes remarkable with the increase of energy of incident \( K^- \) meson. This may be due to the following situation: i) In high energy region, a number of partial waves contribute to this reaction, ii) \( K^* \) will be emitted by the collision in periphery of the nucleon.

Putting the values \( g^2/4\pi=14.5 \) and \( G_\gamma^2/4\pi=0.15 \), we obtain the following results for total cross sections,

\[
\sigma(K^-+p\rightarrow K^*+p) \approx 0.18 \text{ mb} \quad \text{at} \quad 1.15 \text{ Bev/c},
\]

\[
\sigma(K^-+p\rightarrow K^*+p) \approx 0.16 \text{ mb} \quad \text{at} \quad 2 \text{ Bev/c}.
\]

II) The case of \( S=1^* \)

The differential cross sections for this process are given by

\[
\frac{d\sigma(K^-+p\rightarrow K^*+p)}{d\Omega} = \frac{k \left( \frac{g^2}{4\pi} \right) \left( \frac{G_\gamma^2}{4\pi} \right) \left[ -2 \left( \frac{\hat{I}\hat{F}}{m^2} \right) - 2 \right]}{q(v^2 + m^2 + \sqrt{q^2 + m_K^2})^3} \left[ -2 \left( \frac{\hat{I}\hat{F}}{m^2} \right) - 2 + \frac{v^2}{m^2} \right]^2 \left( \frac{4q^2 + k^2 + 4kq \cos \theta}{m^2} \right) \left( \frac{4q^2}{m^2} \right)
\]

where \( \theta \) is the angle between the direction of incident \( K^- \)-meson and that of proton in the final state. The angular distributions of protons for this process are shown in Fig. 5. Putting the values \( g^2/4\pi=14.5 \) and \( G_\gamma^2/4\pi=0.35 \), we obtain the following results,

\[
\sigma(K^-+p\rightarrow K^*+p) \approx 2.3 \text{ mb} \quad \text{at} \quad 1.15 \text{ Bev/c},
\]

\[
\sigma(K^-+p\rightarrow K^*+p) \approx 3.4 \text{ mb} \quad \text{at} \quad 2 \text{ Bev/c}.
\]

* Hereafter we adopt the interaction Hamiltonian \( H_2 \) with the value of coupling constant \( G_\gamma^2/4\pi=0.35 \) (cf. Eq. (22)).
As is seen from (31) and (33), the excitation function for this reaction depends on the spin of $K^*$. In the case of $S=0$ the magnitude of $\sigma(K^- + p \rightarrow K^* + p)$ at 2 Bev/c is nearly equal to that at 1.15 Bev/c, while in the case of $S=1$ its magnitude increases considerably with the energy of incident $K^-$ meson. By comparing our results illustrated in Fig. 4 and Fig. 5 with the experimental results (ii) for the angular distribution at 1.15 Bev/c, it may be said that the assumption of $S=1$ is more promising than that of $S=0$.

§ 5. $K^- + p \rightarrow K^0 + \pi^- + p$ reaction

According to the experimental result (i) this reaction mainly takes place through the process $K^- + p \rightarrow K^* + p \rightarrow K^0 + \pi^- + p$. Now we study this reaction in the case where $K^*$ is not observed directly in the course of collision although $K^*$ of course plays the most important role in the virtual state for this reaction. Fig. 6 shows the diagram corresponding to the lowest order perturbation for this process. When we denote the momenta of proton and $K$ meson in the initial state as $I$ and $q$ respectively and denote the momenta of proton, $K$ meson and pion in the final state as $F$, $l$ and $p$ respectively, the cross section for this process can be expressed by the following form,

$$
\sigma(K^- + p \rightarrow K^0 + \pi^- + p) = \frac{4\pi^2}{B} \int |\langle f | I | i \rangle|^2 \frac{d^2 F}{F_0} \frac{d^2 l}{l_0} \frac{d^2 p}{p_0} \delta(I + q - F - p - l) \delta(J_0 + q_0 - F_0 - p_0 - l_0)
$$

Fig. 4. Angular distribution of proton for $K^- + p \rightarrow K^0 + p$.

Fig. 5. Angular distribution of proton for $K^- + p \rightarrow K^0 + p$.

Fig. 6.
\[ B = q(\sqrt{q^2 + m^2} + \sqrt{q^2 + m_K^2}). \]

\( W \) is the total energy in the center of mass system, \( \theta \) is the angle between the direction of \( \hat{F} \) and that of \( \hat{p} \), \( \langle f|\langle i \rangle \rangle \) is the invariant matrix element for this process, and \( d\Omega \) is the element of solid angle for the direction of \( \hat{p} \). There is the following relation between \( p \) and \( \theta \) owing to the energy conservation law,

\[ \sqrt{p^2 + \mu^2} = \frac{W - F_0}{2} - \frac{(F^2 + m_K^2 - \mu^2)}{2(W - F_0)} - \frac{F_0}{W - F_0} \cos \theta. \]  

Making use of the modified propagation function of \( K^* \) which has been obtained in §2 and §3, we can write down the expression for \( |\langle f|\langle i \rangle \rangle|^2 \) as follows:

I) In the case of \( S = 0 \),

\[ |\langle f|\langle i \rangle \rangle|^2 = \frac{2}{4\pi^2} \left( \frac{m_K}{m} \right)^4 \left( \frac{G_{\delta}}{4\pi} \right)^2 \frac{1}{m^2} \left[ -2 \left( \frac{\hat{F}}{m^2} \right)^2 - 2 \right] \times \left[ \left( \frac{\hat{I} - \hat{F} + \hat{q}}{m^2} \right)^2 + \frac{9}{4} \left( \frac{m_K}{m} \right)^4 \left( \frac{G_{\delta}}{4\pi} \right)^2 - \frac{A^2}{\left( \hat{I} - \hat{F} + \hat{q} \right)^2} \right]. \]

II) In the case of \( S = 1 \),

\[ |\langle f|\langle i \rangle \rangle|^2 = \frac{2}{4\pi^2} \left( \frac{G_{\gamma}}{4\pi} \right)^2 \frac{1}{m^2} \left[ -2 \left( \frac{\hat{F}}{m^2} \right)^2 - 2 \right] \times \left[ -2 \frac{F}{m} \frac{p}{m} \cos \theta - \left( \frac{F}{m} \right)^2 - 2 \frac{p}{m} \frac{q}{m} \cos \theta \frac{F}{m} \frac{q}{m} \cos \theta \right] \times \left[ \left( \frac{\hat{I} - \hat{F} + \hat{q}}{m^2} \right)^2 + 4 \left( \frac{G_{\gamma}}{4\pi} \right)^2 - \frac{A^2}{\left( \hat{I} - \hat{F} + \hat{q} \right)^2} \right]. \]

where

\[ A^2 = \left[ \left( \frac{\hat{I} - \hat{F} + \hat{q}}{m^2} + (m_K + \mu)^2 \right) \left[ \left( \frac{\hat{I} - \hat{F} + \hat{q}}{m^2} + (m_K - \mu)^2 \right) \right], \]

\[ -4 \left( \frac{\hat{I} - \hat{F} + \hat{q}}{m^2} \right)^2 \]

\[ (38) \]

\[ (39) \]

\* The integral on \( d\hat{F} \) is performed at the last step of our calculation.

\** The factor 2 in the first term on the right-hand side of (36) or (37) is derived from the consideration about isotopic spin space.
\[
-(I - \hat{F} + \hat{q})^2 = 2m^2 + m_K^2 + 2[q^2 + Vq^2 + m_K^2 + 2m^2]
\]
\[
- V'F^2 + m^2(Vq^2 + m^2 + Vq^2 + m_K^2)
\]

\(\theta_1\) is the angle between \(\hat{q}\) and \(\vec{p}\), and \(\theta\) the angle between \(\hat{q}\) and \(\vec{F}\). The terms which contain \(\Lambda\) are derived from the damping effects. The magnitude of cross section for this process, the angular distributions and momentum spectra of pions and protons can be obtained by means of the relations (34), (36) or (37). But this work is considerably troublesome. We now try to examine them under the crude assumptions which will be mentioned below.

Let us examine the following function in Eq. (34),
\[
|\langle f | i \rangle|^2 \frac{p}{(W - F_0) + F(p_0/p) \cos \theta}.
\]
Although the term
\[
9 \left( \frac{m_K}{m} \right)^4 \left( \frac{G_s}{4\pi} \right)^2 \frac{A^2}{(I - \hat{F} + \hat{q})^2} - \frac{A^2}{4\pi} - (I - \hat{F} + \hat{q})^2
\]
\[
\times \left( \frac{A^2}{m^2} \right)^2 \text{ in } |\langle f | i \rangle|^2 \text{ depends on } F, \text{ its magnitude is very small in our case.}
\]

When \(F/m\) has the value \((F/m)_r\), corresponding to the resonance of \(K^*\), \((I - \hat{F} + \hat{q})^2 + M^2 = 0\). Therefore

\[
\mathcal{J}_s(F/m) = \left[ (I - \hat{F} + \hat{q})^2 + M^2 \right] + 9 \left( \frac{m_K}{m} \right)^4 \left( \frac{G_s}{4\pi} \right)^2 \frac{A^2}{(I - \hat{F} + \hat{q})^2}
\]

and

\[
\mathcal{J}_v(F/m) = \left[ (I - \hat{F} + \hat{q})^2 + M^2 \right] + 4 \left( \frac{G_v}{4\pi} \right)^2 \frac{A^2}{(I - \hat{F} + \hat{q})^2} \left( \frac{A^2}{m^2} \right)^2
\]

have large values only in the very narrow region, that is, only in the neighborhood of \((F/m)_r\). In addition, the other terms except \(\mathcal{J}_s(F/m)\) (or \(\mathcal{J}_v(F/m)\)) in Eq. (41) have not such a property as this with respect to the variable \(F/m\). In other words the resonance phenomena of \(K^*\) with the narrow width play the most important role in the reaction \(K^- + p \rightarrow K^0 + \pi^- + p\). This may make it possible to examine the characteristic property of this reaction by replacing approximately the variable \(F/m\) in all the other terms except \(\mathcal{J}_s(F/m)\) or \(\mathcal{J}_v(F/m)\) by the value of \((F/m)_r\).

In the case where this reaction takes place through the process \(K^- + p \rightarrow K^* + p \rightarrow K^0 + \pi^- + p\), the main feature of angular distribution of the final proton may be represented by the form of that of proton for the reaction \(K^- + p \rightarrow K^* + p\) mentioned in § 4. So far as the phenomena at 2 Bev/c (or at the energy higher than 2 Bev/c) are concerned, it may be said that the final protons in \(K^- + p \rightarrow K^* + p\) process are emitted mainly into the backward direc-

- Its value at the resonance energy may be expressed in terms of \((F/2)^2\).
in our study for the reaction $K^- + p \rightarrow \overline{K}^0 + \pi^- + p$ at 2 Bev/c we introduce another assumption of $\theta \approx \pi - \theta_1$.

§ 5.1. $K^- + p \rightarrow \overline{K}^0 + \pi^- + p$ reaction at 2 Bev/c

Under the assumption $\theta \approx \pi - \theta_1$, Eq. (35) can be written as follows:

$$\sqrt{\left(\frac{p}{m}\right)^2 + \left(\frac{\mu}{m}\right)^2} = \frac{\left(\frac{W}{m}\right) - \left(\frac{F_0}{m}\right)_r - \left(\frac{F_0}{m}\right)^2 + \left(\frac{m_K}{m}\right)^2 - \left(\frac{\mu}{m}\right)^2}{2\left[\frac{W}{m} - \left(\frac{F_0}{m}\right)_r\right]}$$

$$+ \left(\frac{\frac{F}{m}}{\frac{W}{m} - \left(\frac{F_0}{m}\right)_r}\right) \cos \theta_1. \quad (35)'$$

From this relation it follows that $(p/m)$ can be expressed in terms of $\cos \theta_1$. We can see the angular distributions of pions by examining the $\theta_1$-dependence of $|\langle f | I | i \rangle|^2$. They are expressed by the following forms:

1) In the case of $S=0$,

$$\left(\frac{p}{m}\right) = (W/m) - (F_0/m) - (F/m), (p/p_0) \cos \theta_1$$

$$= \mathcal{G}_S(p, \theta_1) = \mathcal{G}_S'(\theta_1).$$

(42)

because $|\langle f | I | i \rangle|^2$ does not depend on $\theta_1$.

2) In the case of $S=1$,

$$\left[2\left(\frac{F}{m}\right)_r, (\frac{p}{m}) \cos \theta_1, -\left(\frac{F}{m}\right)_r, \left(\frac{q}{m}\right) \cos \theta_1, +2\left(\frac{F_0}{m}\right)_r, \left(\frac{q}{m}\right)\right]^2$$

$$= \mathcal{G}_T(p, \theta_1) = \mathcal{G}_T'(\theta_1).$$

(43)*

The curves of $\mathcal{G}_S'(\theta_1)$ and $\mathcal{G}_T'(\theta_1)$ are illustrated in Fig. 7. If we refer to the rest system of $K^*$, the angular distributions of pions should be isotropic and of a form of $\sim \cos^2 \theta_1$ corresponding to the case of $S=0$ and $S=1$ respectively.

Under the assumption that the proton does not change its direction considerably during a collision, $K^*$ ought to be emitted in the forward direction. Therefore the angular distributions of pions in the center of mass system are enhanced remarkably in the forward direction compared with those in the rest system of $K^*$ (cf. Fig. 7).

Next let us examine the momentum spectrum of pion. Differentiating Eq. (35)', we get

* The assumption of $\theta \approx \pi$ is introduced in the expression of (43).
K Meson-Pion Interaction and $K^-+p\to\bar{K}^0+\pi^-+p$ Reaction

Fig. 7. Angular distribution of pion for $K^-+p\to\bar{K}^0+\pi^-+p$ at 2 Bev/c (where $F/m=(F/m)_r=0.6886$).

\[
\sin \theta_1 d\theta_1 = \left[ \cos \theta_1 - \frac{p}{p_0} \frac{(W)}{m} - \frac{(F_0)}{m} \right] \frac{dp}{p}. \tag{44}
\]

Through Eq. (44) we can see the momentum spectrum (or energy spectrum) of pion in the case where $F/m=(F/m)_r=0.6886$ (cf. Fig. 8).

Now we try to estimate the magnitude of cross section for this reaction and to examine the angular distribution and momentum spectrum of proton. As was mentioned above, in the expression of (34) we may approximately replace the variable $F/m$ in all the other terms except $\mathscr{J}_s(F/m)$ or $\mathscr{J}_v(F/m)$ by the value of $(F/m)_r$ and approximately replace $\theta$ by $(\pi-\theta_1)$. Then the
integral on $d\Omega$, can be carried out easily. Since $\mathcal{F}_s(F/m)$ and $\mathcal{F}_\nu(F/m)$ do not depend on the angle $\theta$, in our approximation the character of angular distribution of proton can be expressed by that of angular distribution in the case of $F/m=(F/m)_0$, (cf. Fig. 9).

![Graph showing angular distribution of proton for $K^-+p\rightarrow\bar{K}^0+\pi^-+p$ at 2 Bev/c.](https://example.com/graph.png)

Fig. 9. Angular distribution of proton for $K^-+p\rightarrow\bar{K}^0+\pi^-+p$ at 2 Bev/c.

So long as $K^*$ is not observed in the course of reaction $K^-+p\rightarrow\bar{K}^0+\pi^-+p$, the angular distribution of proton for this reaction may be more or less different from that for $K^-+p\rightarrow K^0+p$ (compare Fig. 9 with Fig. 5). The reason why the dotted line in Fig. 9 has the similar form with that in Fig. 4 is due to the following situation. Under the assumption $\theta\approx\pi-\theta_1$, the angular distribution of proton for $K^-+p\rightarrow K^0+\pi^-+p$ depends on $|\langle f|I|\rangle|^2$ only. In the case of $S=0$ the $\theta$-dependence of $|\langle f|I|\rangle|^2$ is the same with that of $d\sigma(K^-+p\rightarrow K^0+p)/d\Omega$ mentioned in Eq. (30) apart from a trivial factor.

The momentum spectrum of proton in the final state is shown in Fig. 10. Performing the integral on $dF$ and adopting the values of coupling constants $g^2/4\pi=14.5$ and $G_{\pi^0}/4\pi=0.15$ (or $G_{\nu^3}/4\pi=0.35$), we obtain the following results:

I) In the case of $S=0$,

$$\sigma(K^-+p\rightarrow\bar{K}^0+\pi^-+p)_{at \ 2 \ Bev/c} \approx 0.11 \text{ mb.} \tag{46}$$

II) In the case of $S=1$,

$$\sigma(K^-+p\rightarrow\bar{K}^0+\pi^-+p)_{at \ 2 \ Bev/c} \approx 2.2 \text{ mb.} \tag{47}$$

§ 5 2. $K^-+p\rightarrow\bar{K}^0+\pi^-+p$ reaction at 1.15 Bev/c

Although the approximation of $\theta\approx\pi-\theta_1$ may not be so useful in this case, we try to examine the angular distribution and energy spectrum (or moment-
K Meson-Pion Interaction and $K^-+p\rightarrow K^0+\pi^-+p$ Reaction

371

...tum spectrum) of pion by the same way as in § 5.1. The results are illustrated in Fig. 11 and Fig. 12.

First of all let us make some discussion about the angular distribution of pion. In the collision at 1.15 Bev/c, $K^*$ cannot be emitted with high momentum. Therefore the angular distributions of pions in the center of mass system ought to have nearly the similar forms with those in the rest system of $K^*$, that is, they are nearly isotropic and of the forms of $\cos^2 \theta_i$ corresponding to the case of $S=0$ and $S=1$ respectively. It may be said that the curves illustrated in Fig. 11 have these characteristic properties, although the angular distributions are slightly larger in the forward than in the backward direction because we assume that the proton does not change its direction remarkably during a collision.

Next let us make some discussion about the energy spectrum (or momentum spectrum) of pion. We show in Fig. 12 the energy spectrum of pion in the case where proton momentum in the final state is equal to $(F/m),_p=0.2091$ (proton energy $T_p \approx 20$ Mev). From these curves we can see the following characteristic properties:

i) In the case of $S=0$, the emitted pions have a uniform energy spectrum.

ii) In the case of $S=1$, the energy spectrum of pion has a minimum value in the neighborhood of $T_\pi \approx 190$ Mev.

Comparing these results with the experimental ones for the final proton energy $T_p \approx 20$ Mev, we may conclude that the assumption of $S=1$ is more promising than that of $S=0$.

Finally we try to estimate the magnitude of cross section for this reaction.

![Fig. 11. Angular distribution of pion for $K^-+p\rightarrow K^0+\pi^-+p$ at 1.15 Bev/c.](image1)

![Fig. 12. Energy spectrum of pion for $K^-+p\rightarrow K^0+\pi^-+p$ at 1.15 Bev/c, where kinetic energy of proton $T_p \approx 20$ Mev (corresponding to $(F/m),_p=0.2091$).](image2)
Since the approximation of $\theta \approx \pi - \theta_1$ may not be so useful in this case, it is necessary to examine the value of

$$\frac{(p/m)}{(W/m) - (F/m) + (F/m)\cos \theta} = \mathcal{H}_0(p, \theta).$$  

(48)

Using the relation of (35), in the case of $F/m = (F/m)_r = 0.2091$ we get

- $\mathcal{H}_0(p, \theta)_{\text{max}} = 0.515$, $\mathcal{H}_0(p, \theta)_{\text{min}} = 0.192$,

$$\langle \mathcal{H}_0(p, \theta) \rangle_{\text{mean}} = 0.331.$$  

Therefore

$$0.192 \cdot 4\pi \leq I_0 = \int \mathcal{H}_0(p, \theta) d\Omega_1 \leq (0.515) \cdot 4\pi,$$

$$\int \langle \mathcal{H}_0(p, \theta) \rangle_{\text{mean}} d\Omega_1 \approx (0.331) \cdot 4\pi.$$  

(50)

On the other hand, if we assume $\theta = \pi - \theta_1$,

$$I_0 = \int \mathcal{H}_0(p, \pi - \theta_1) d\Omega_1 \approx (0.317) \cdot 4\pi.$$  

(51)

Thus it may be said that the assumption $\theta \approx \pi - \theta_1$ is applicable to the case of $S=0$ in estimating the order of $\sigma (K^- + p \rightarrow K^0 + \pi^- + p)$. Performing the integral on $d\bar{F}$ and adopting the values of coupling constants $g^2/4\pi = 14.5$ and $G_s^2/4\pi = 0.15$, we obtain the following result. In the case of $S=0$,

$$\sigma (K^- + p \rightarrow K^0 + \pi^- + p)_{\text{at } t=0} \approx 0.12 \text{ mb}.$$  

(52)

Comparing this with the experimental value $(2.0 \pm 0.3) \text{ mb}$, we may draw the following conclusion. The assumption of $S=0$ is not consistent with the experimental results for cross section of $K^- + p \rightarrow K^0 + \pi^- + p$ reaction.

In the case of $S=1$ it is necessary to estimate the contributions from the following $\mathcal{H}_1(p, \theta)$ and $\mathcal{H}_2(p, \theta)$-terms in addition to $\mathcal{H}_0(p, \theta)$,

- $\mathcal{H}_1(p, \theta) = (p/m) \mathcal{H}_0(p, \theta)$,
- $\mathcal{H}_2(p, \theta) = (p/m)^2 \mathcal{H}_0(p, \theta)$.

Using the relation (35), in the case of $F/m = (F/m)_r = 0.2091$ we get

- $\mathcal{H}_1(p, \theta)_{\text{max}} \approx 0.197$, $\mathcal{H}_1(p, \theta)_{\text{min}} \approx 0.045$,
- $\langle \mathcal{H}_1(p, \theta) \rangle_{\text{mean}} \approx 0.106$,  

and

- $\mathcal{H}_2(p, \theta)_{\text{max}} \approx 0.075$, $\mathcal{H}_2(p, \theta)_{\text{min}} \approx 0.01$,
- $\langle \mathcal{H}_2(p, \theta) \rangle_{\text{mean}} \approx 0.035$.  

Therefore
\[ \langle I_0 \rangle = \int \langle \mathcal{K}_0(p, \theta) \rangle_{\text{mean}} d\Omega_1 = (0.331) 4\pi, \]
\[ \langle I_1 \rangle = \int \langle \mathcal{K}_1(p, \theta) \rangle_{\text{mean}} \cos \theta_1 d\Omega_1 = 0, \quad (55) \]
\[ \langle I_2 \rangle = \int \langle \mathcal{K}_2(p, \theta) \rangle_{\text{mean}} \cos^3 \theta_1 d\Omega_1 = (0.0117) 4\pi. \]

On the other hand, if we assume \( \theta \approx \pi - \theta_1 \),
\[ I_0 = \int \mathcal{K}_0(p, \theta) d\Omega_1 = (0.317) 4\pi, \]
\[ I_1 = \int \mathcal{K}_1(p, \theta) \cos \theta_1 d\Omega_1 = (0.025) 4\pi, \quad (56) \]
\[ I_2 = \int \mathcal{K}_2(p, \theta) \cos^3 \theta_1 d\Omega_1 = (0.0118) 4\pi. \]

Let us tentatively estimate the order of \( \sigma(K^- + p \rightarrow K^0 + \pi^- + p) \) at 1.15 Bev/c by using the values of (56). Although there is a considerable difference between \( \langle I_1 \rangle \) in (55) and \( I_1 \) in (56), this difference has not large effect on the final result. Adopting the values \( g^2/4\pi = 14.5 \) and \( G_{\gamma}^2/4\pi = 0.35 \), we obtain
\[ \sigma(K^- + p \rightarrow K^0 + \pi^- + p) \at \text{1.15 Bev/c} \approx 1.5 \text{ mb.} \quad (57)^* \]
This value agrees fairly well with the experimental one \((2.0 \pm 0.3) \text{ mb.} \) Thus it may be said that the assumption of \( S = 1 \) is consistent with the experimental result.

\section{6. On the isotopic spin of K*}

We have studied the \( K \) meson reaction with the assumption of \( I_{K^*} = 1/2 \). The description of the reaction in the case of \( I_{K^*} = 3/2 \) can be made along the same line with the above one. But it may be difficult to expect the case of \( I_{K^*} = 3/2 \) on the basis of the reason mentioned in § 1. We should like to make some additional discussion about this point.

Previously we made the following prediction about the \( \bar{K} \cdot N \) scattering at high energy.\(^4\) In the neighborhood of the energy 675 Mev (lab.) which corresponds to the threshold energy of \( \bar{K} + N \rightarrow K^* + N \), we may expect the large cross sections of \( \bar{K} \cdot N \) scattering. And this resonance is due to the strong interaction in the \( I = 0 \) state if the isotopic spin of \( K^* \) is equal to 1/2. On the other hand, if the isotopic spin of \( K^* \) is equal to 3/2, this resonance is due to the strong interaction in the \( I = 1 \) state.

According to the recent experiment,\(^5\) there seems to be some evidence of

\(^*\) The numerical value mentioned in (57) should not be interpreted so seriously because of our rough estimation.
resonance in $K^-p$ scattering in the neighborhood of the threshold energy of $\bar{K}+N\rightarrow K^*+N$, but such resonance phenomena are not observed in $K^-n$ scattering. Thus it may be said that not only the experimental results for the branching ratio of the decay of $K^*$ but also these experimental results for $\bar{K}N$ scattering are consistent with the assumption of $I_{K^*}=1/2$.

After completing this work the author saw the papers by Chia-Hwa Chan and M. A. B. Bég et al. in which the similar conclusion to ours is mentioned, that is, a vector $K^*$ fits the experimental data better than a scalar $K^*$.

References

3) For example, Y. Miyamoto, Prog. Theor. Phys. 24 (1960), 840.
5) Berkeley Conference (1960).