Design life of water transmission mains for exponentially growing water demand

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ABSTRACT

Gravity or pumping water transmission lines are planned for a predetermined time horizon generally called design period. For a static population the lines can be designed, either for a design period equal to the life of the pipes sharing the maximum cost of the system or for the perpetual existence of the supply system. For a growing population or water demand, it is always economical to design the mains in staging periods and then strengthen the system after the end of every staging period. In the absence of a rational criterion, the design period of water supply mains is generally based on the designer's intuition, disregarding the useful life of the component sharing maximum cost, pattern of population growth or increase in water demand and discount rate. Provided herein are numerical criteria for the design period of water transmission mains.

Key words | design period, optimal design, pipe networks, pumping main, water supply

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INTRODUCTION

Water is required to be carried over long distances through pipelines. Like electric transmission lines transmitting electricity, these pipelines transmit water. If the flow in a water transmission line is maintained by creating a pressure head by pumping, it is called a pumping main. On the other hand if the flow in a water transmission line is maintained through an elevation difference, it is called a gravity main (see Figure 1a and b). There are no intermediate withdrawals in a water transmission line. The pumping and the gravity-sustained systems differ in their constructional and functional requirements as described by Swamee & Sharma (2000).

Before designing a water supply main, a decision has to be made regarding the length of time the system should be designed for before it is strengthened further to meet the future demand. This length of time is termed the design period. Generally design periods are kept low due to uncertainty in population prediction and its implications on the cost of the water transmission mains.

For static populations, the design period should be the same as the life of components sharing the major portion of the system cost. Depending upon the material of construction, the pipes have a life ranging from 60 to 120 years. For a dynamic population, it is always economical to design the mains in stages and strengthen the system after every staging period. Hence designing the transmission mains for an optimal design period should be the main consideration. The extent to which the life cycle cost can be minimized would depend upon the planning horizon (design period) of the water supply mains. Kleiner *et al*. (2001) presented a methodology for rehabilitation planning of existing water distribution networks based on economic and hydraulic considerations. However, the question of initial planning remains unaddressed.

In the present design practices, disregarding the increase in water demand, useful life of pipes or future discount rate, the design period is generally adopted as 30 years on an ad hoc basis. With these considerations, an

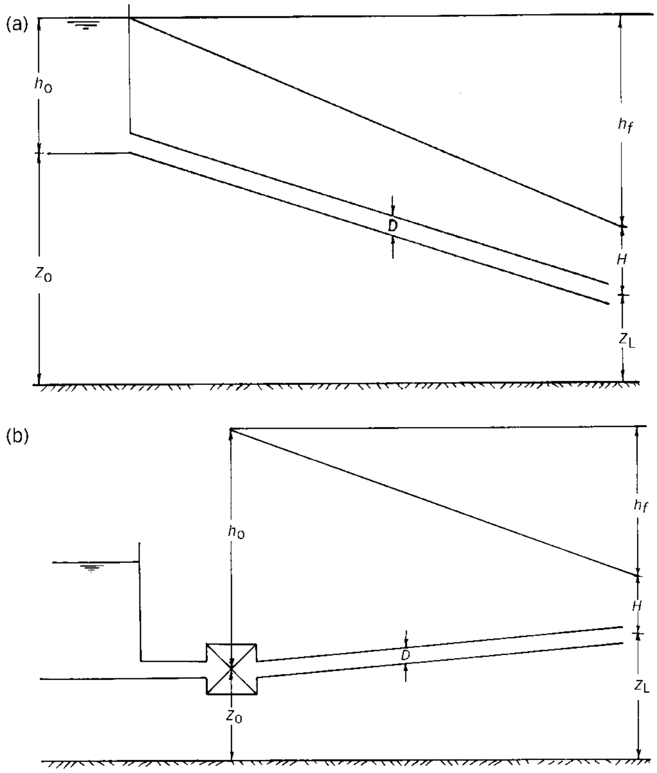

Figure 1 | Definition sketch: (a) gravity main; (b) pumping main.

investigation was undertaken to find a criterion for the design period of water supply mains.

DESIGN ATTRIBUTES

Water demand

Walski *et al.* (2001) indicated that the growth of cities and industries is hard to predict, and therefore it is also difficult to predict future water demand. Demand projections are only as accurate as the assumptions made and the methods used to extrapolate demand. On the other hand, Cesario (1995) is of the view that, although it is relatively easy to arrive at an estimate of total consumption in a water system, it is difficult to be very accurate. It raises the question of what is the acceptable level of accuracy. Several models (for example, McGhee 1991; Qasim *et al.* 2000) have been proposed by a number of researchers for population forecasting. Selection of any one model will depend on the available data and whether the projections are made for the short or long term. Considering a decreasing rate of increase model for population growth, and assuming the increasing rate of per capita demand increases with population, it is reasonable to consider that water demand increases at a constant rate. That is:

$$Q = Q_0 e^{\alpha t} \tag{1}$$

where Q = water demand at time t; Q_0 = initial water demand; and α = rate of increase.

Knowing the present and the projected water demands, the exponential growth factor α can be obtained irrespective of the model used for water demand projections.

Useful life of pipes

Pipes are the major component of a water supply system with a long life in comparison to other components of the system. Smith *et al.* (2000) reported the life of cast iron pipe as over 100 years. Alferink *et al.* (1997) investigated old polyvinyl chloride (PVC) water pipes laid 35 years ago and concluded that the new PVC pipes would continue to perform for considerably more than 50 years. The Plastic Pipe Institute (2003) has reported the considerable justification for assuming a 100-year or more design service life for corrugated polyethylene pipes. Exact information regarding the life of different types of pipe is not available. The PVC and asbestos cement (AC) pipes have not yet exceeded the life expectancy claimed by the manufacturers since being used in water supply mains. Based upon the available information from manufacturers and user organizations, Table 1 provides the average life, T_u, of different types of pipe used in water mains.

Head loss equation

Considering a transmission main of length L, and using Equation (1), the head loss due to pipe friction, h_f, at time t as given by the Darcy–Weisbach equation is:

Table 1 | Life of different types of pipe

Pipe material	Life, T_u (years)
Cast iron (CI)	120
Galvanised iron (GI)	120
Electric resistance welded (ERW)	120
Asbestos cement (AC)	60
Corrugated HDPE pipe	100
Polyvinyl chloride (PVC)	>50

$$h_f = \frac{8fLQ_0^2 e^{2\alpha t}}{\pi^2 g D^5} \tag{2}$$

where f = friction factor; g = gravitational acceleration; and D = pipe diameter.

Pipe cost function

The capitalized cost of pipeline C_m is given by:

$$C_m = k_m L D^m \tag{3}$$

where k_m and m = pipe cost parameters. The cost parameter k_m depends on the monetary unit used and m depends on the pipe material. Equation 3 is a general form of pipe cost. The values of k_m and m can be obtained by plotting variation of per metre cost of pipe with diameter on double logarithmic paper. The pipe constant k_m is equal to the cost of 1 m diameter pipe and m is the slope of the line.

GRAVITY MAINS

As shown in Figure 1a, the pressure head, h_0 (on account of the water level in the collection tank), varies from time to time and much reliance cannot be placed on it. For design purposes this head should be neglected. The entrance and the exit losses are also neglected as they would be negligible in comparison to the total head loss. Moreover, the water column, h_0, and entry/exit losses would counterbalance each other, thus the overall impact should be minimal. The available head loss, h_L, can be written as:

$$h_L = z_0 - z_L - H \tag{4}$$

where H is terminal head, z_0 elevation at source point and z_L elevation at supply point.

Considering the design period, T_d, equal to the life of the pipes, T_u, and using Equations (2) and (4), the pipe diameter, D_1, was found to be:

$$D_1 = \left[\frac{8fLQ_0^2 e^{2\alpha T_u}}{\pi^2 g(z_0 - z_L - H)}\right]^{0.2} \tag{5}$$

Using Equations (3) and (5), the cost function C_1 as:

$$C_1 = k_m L \left[\frac{8fLQ_0^2 e^{2\alpha T_u}}{\pi^2 g(z_0 - z_L - H)}\right]^{0.2m} \tag{6}$$

Non-dimensionalizing Equation (6), the following equation was obtained:

$$\overline{C_1} = e^{0.4\alpha m T_u} \tag{7}$$

where $\overline{C_1}$ is given by:

$$\overline{C_1} = \frac{C_1}{k_m L D_0^m} \tag{8}$$

where D_0 = diameter for discharge Q_0 was given as:

$$D_0 = \left[\frac{8fLQ_0^2}{\pi^2 g(z_0 - z_L - H)}\right]^{0.2} \tag{9}$$

If the design period is taken as $T_u/2$, the system is initially designed in the first phase for $T_u/2$ and then strengthened for another $T_u/2$ time span in the next phase. The cost function of the first phase C_{21} for a discharge $Q_0 e^{0.5\alpha T_u}$ is:

$$C_{21} = k_m L \left[\frac{8fLQ_0^2 e^{\alpha T_u}}{\pi^2 g(z_0 - z_L - H)} \right]^{0.2m} \tag{10}$$

Similarly C_{22} for another $T_u/2$ time span in the second phase is:

$$C_{22} = k_m L \left[\frac{8fLQ_0^2 \left(e^{\alpha T_u} - e^{\frac{1}{2}\alpha T_u}\right)^2}{\pi^2 g(z_0 - z_L - H)} \right]^{0.2m} e^{-rT_u/2} \tag{11}$$

where investment made at a later date was given a lesser weight and discounted by a discount rate, r, which is called future discounting. It is a procedure for estimating the present value of costs and the benefits that will be realized in the future or otherwise converting costs and benefits to a common time basis. The discount rate should be based on economic and social considerations. The procedure should be subject to adjustment if and when this is found desirable as a result of continuing analysis of all the factors pertinent to selection of a discount rate for these purposes (Kuiper 1971). Combining Equations (10) and (11), the total cost of the system is:

$$C_2 = C_{21} + C_{22} \tag{12}$$

Equation (12) can be rewritten as:

$$\overline{C_2} = e^{0.2\alpha m T_u} + \left(e^{\alpha T_u} - e^{\frac{1}{2}\alpha T_u}\right)^{0.4m} e^{-\frac{1}{2}rT_u} \tag{13}$$

and also

$$\overline{C}_3 = e^{0.4\alpha m T_u/3} + \left(e^{2\alpha T_u/3} - e^{\alpha T_u/3}\right)^{0.4m} e^{-rT_u/3} + \left(e^{\alpha T_u} - e^{2\alpha T_u/3}\right)^{0.4m} e^{-2rT_u/3} \tag{14}$$

Similarly cost functions $\overline{C}_4$, $\overline{C}_5$ and $\overline{C}_6$ were obtained by taking design periods as $T_u/4$, $T_u/5$ and $T_u/6$, respectively. Computing $\overline{C}_1$, $\overline{C}_2$, $\overline{C}_3$ . . . to $\overline{C}_6$ and comparing them, the minimum cost function and corresponding optimal design period T_d^* can be obtained. It has been found that T_d^* is very sensitive to variation of α and r and is less sensitive to the hydraulic parameters of the problem.

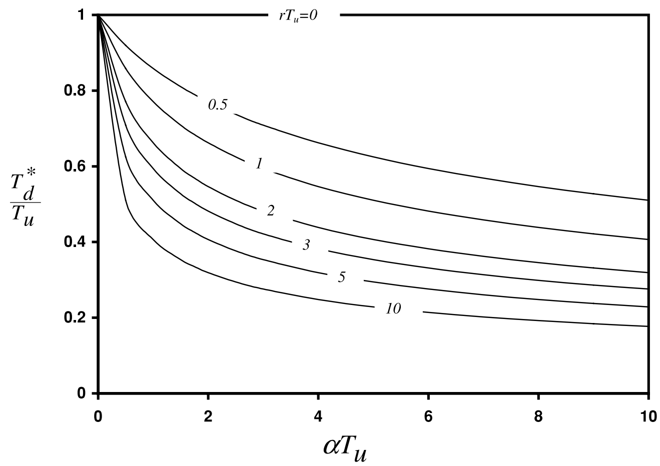

Figure 2 | Variation of design period of a gravity main.

The following non-dimensional groups were formed for this problem: T_d^*/T_u, αT_u, rT_u and m. As can be noted, αT_u and rT_u vary in a wide range, whereas m varies in the narrow range $1 \nless m \leq 1.67$ for various pipe materials. It was found that in this range m has little effect on T_d^*/T_u. Thus, the variation of T_d^*/T_u with the group of two independent variables αT_u and rT_u was tabulated. These tabulated values are shown plotted in Figure 2. It can be seen from Figure 2 that $T_d^*/T_u = 1$ for both $\alpha T_u = 0$ and $rT_u = 0$. Further, T_d^*/T_u gradually decreases with both αT_u and rT_u. This trend is represented by:

$$T_d^*/T_u = \left[1 + k_1 \left(\alpha T_u\right)^{k_2} \left(rT_u\right)^{k_3}\right]^{-k_4} \tag{15a}$$

where k_1–k_4 are unknown positive constants. Considering the tabulated values and minimizing the sum of absolute proportionate errors, $k_1 = 2$, $k_2 = 1$, $k_3 = 1$ and $k_4 = 0.375$ were obtained. Thus, Equation (15a) changes to:

$$T_d^* = T_u \left(1 + 2\alpha r T_u^2\right)^{-0.375} \tag{15b}$$

PUMPING MAINS

As depicted in Figure 1(b), the pressure head on account of the water level in the collection tank varies from time to time and for design purposes this head, along with the entrance and the exit losses, should be neglected as it would be very small in comparison to surface resistance losses in a pipe. Considering a horizontal pumping main of length L, and using Equation (2), the pumping head h_0 at time t was given as:

$$h_0 = \frac{8fLQ_0^2 e^{2\alpha t}}{\pi^2 g D^5} \tag{16}$$

In the first instance the design period T_d was considered to be equal to the life of the pipe T_u. With the increase in discharge, the energy cost progressively increases during the design period. Discounting investment made at a later date by a discount factor, and using Equation (1), the capitalized annual pumping cost C_e is given by:

$$C_e = \rho g k_e \int_0^{T_u} Q_0 e^{\alpha t} h_0 e^{-rt} \mathrm{d}t \tag{17}$$

where ρ = mass density of water; and k_e = annual pumping cost coefficient, which is given by:

$$k_e = \frac{24 \times 365 F_A F_D R_E}{1000\eta} = \frac{8.76 F_A F_D R_E}{\eta} \tag{18a}$$

where F_A = annual averaging factor; F_D = daily averaging factor; R_E = cost of energy per kWh; and η = combined efficiency of pump and prime mover. Equations (17) and (18a) have been derived from the basic formula for power P in kWh required to pump a discharge Q_0 with pumping head h_0 as:

$$P = \frac{\rho g Q_0 h_0}{1000\eta} \tag{18b}$$

Using Equations (16) and (17), the cost of energy C_e, is written as:

$$C_e = \frac{8\rho k_e f L Q_0^3}{\pi^2 D^5} \int_0^{T_u} e^{(3\alpha - r)t} \mathrm{d}t \tag{19}$$

Integrating Equation (19) and adding the pipe cost given in Equation (3), the pumping main cost F_1 can be written as:

$$F_1 = k_m L D^m + \frac{8\rho k_e f L Q_0^3 e^{2\alpha t}}{\pi^2 D^5} \delta_1 \tag{20}$$

where

$$\delta_1 = \frac{e^{(3\alpha - r)T_u} - 1}{3\alpha - r} \tag{21}$$

The fact that the cost of pumping plant is much lower than the cost of pumping energy has been neglected in Equation (20). Optimal diameter D_1^* was obtained by differentiating F_1 in Equation (20) with respect to D_1, setting the differential coefficient equal to zero and solving the resulting equation for D_1^*. Thus:

$$D_1^* = \left(\frac{40 f \rho k_e Q_0^3}{\pi^2 m k_m} \delta_1 \right)^{\frac{1}{m+5}} \tag{22}$$

The optimal cost function F_1^* is obtained by combining Equations (20) and (22) yielding:

$$\bar{F}_1^* = \left(1 + \frac{m}{5} \right) \delta_1^{\frac{m}{m+5}} \tag{23}$$

where the non-dimensional optimal cost function $\bar{F}_1^*$ is given by:

$$\bar{F}_1^* = \frac{F_1^*}{k_m L D_0^{*m}}. \tag{24}$$

where:

$$D_0^* = \left(\frac{40 f \rho k_e Q_0^3}{\pi^2 m k_m} \right)^{\frac{1}{m+5}} \tag{25}$$

If the design period is taken as $T_u/2$, the system is initially designed in the first phase for $T_u/2$ and then strengthened for another $T_u/2$ time span in the second phase. The cost function of first phase, F_{21}, includes the pipe cost and the energy cost of varying discharge over a time from 0 to $T_u/2$ and then for a constant discharge $Q_0 e^{0.5\alpha T_u}$ up to time T_u as:

$$F_{21} = k_m L D_{21}^m + \frac{8 f L \rho k_e Q_0^3}{\pi^2 D_{21}^5} \delta_{21} \quad (26)$$

where:

$$\delta_{21} = \int_0^{T_u/2} e^{(3\alpha - r)t}\,\mathrm{d}t + \int_{T_u/2}^{T_u} e^{1.5\alpha T_u - rt}\,\mathrm{d}t \quad (27)$$

After integrating, Equation (27) can be expressed as:

$$\delta_{21} = \frac{e^{0.5(3\alpha - r)T_u} - 1}{3\alpha - r} + e^{1.5\alpha T_u} \frac{e^{-0.5rT_u} - e^{-rT_u}}{r} \quad (28)$$

Similarly, the optimal diameter D_{21}^* is obtained as:

$$D_{21}^* = \left(\frac{40 f \rho k_e Q_0^3}{\pi^2 m k_m} \delta_{21} \right)^{\frac{1}{m+5}} \quad (29)$$

and the non-dimensional optimal cost function $\bar{F}_{21}^*$ as:

$$\bar{F}_{21}^* = \left(1 + \frac{m}{5}\right) \delta_{21}^{\frac{m}{m+5}} \quad (30)$$

The cost function for the second phase, F_{22}, includes the pipe cost and the energy cost of varying discharge over time from $T_u/2$ to T_u as:

$$F_{22} = k_m L D_{22}^m + \frac{8 f L \rho k_e Q_0^3}{\pi^2 D_{22}^5} \delta_{22} \quad (31)$$

where:

$$\delta_{22} = \int_{T_u/2}^{T_u} \left(e^{\alpha t} - e^{0.5\alpha T_u}\right)^3 e^{-rt}\,\mathrm{d}t \quad (32)$$

Equation (32) can be expressed after integrating as:

$$\delta_{22} = \frac{e^{(3\alpha - r)T_u}}{3\alpha - r} - 3\frac{e^{(2.5\alpha - 0.5r)T_u}}{2\alpha - r} + 3\frac{e^{(2\alpha - r)T_u}}{\alpha - r} + \frac{e^{(1.5\alpha - r)T_u}}{r} - e^{(1.5\alpha - 0.5r)T_u}\left(\frac{1}{3\alpha - r} - \frac{3}{2\alpha - r} + \frac{3}{\alpha - r} + \frac{1}{r}\right) \quad (33)$$

In a similar way the optimal diameter D_{22}^* is obtained as:

$$D_{22}^* = \left(\frac{40 f \rho k_e Q_0^3}{\pi^2 m k_m} \delta_{22}\, e^{-0.5rT_u} \right)^{\frac{1}{m+5}} \quad (34)$$

and the non-dimensional optimal cost function $\bar{F}_{22}^*$ as:

$$\bar{F}_{22}^* = \left(1 + \frac{m}{5} e^{-0.5rT_u}\right) \delta_{22}^{\frac{m}{m+5}} \quad (35)$$

Total system cost in this case is obtained by combining Equations (30) and (35) as:

$$\bar{F}_2^* = \bar{F}_{21}^* + \bar{F}_{22}^* \quad (36)$$

Similarly, cost functions $\bar{F}_3^*$, $\bar{F}_4^*$, $\bar{F}_5^*$ and $\bar{F}_6^*$ can be obtained by taking design periods as $T_u/3$, $T_u/4$, $T_u/5$ and $T_u/6$, respectively. Computing $\bar{F}_1$, $\bar{F}_2$, $\bar{F}_3$ . . . to $\bar{F}_6$ and comparing them, the minimum cost function and corresponding optimal design period T_d^* is obtained. It has been found that T_d^* is very sensitive to variation of α and r and only a little sensitive to the hydraulic parameters of the problem. Adopting a methodology similar to the gravity mains, the variation of T_d^*/T_u with the group of two independent variables, αT_u and rT_u, was tabulated. These tabulated values are shown plotted in Figure 3. It can be seen from Figure 3 that $T_d^*/T_u = 1$ for $\alpha T_u = 0$. Further, T_d^*/T_u gradually decreases with both αT_u and rT_u. Such a trend is represented by the following equation:

$$T_d^*/T_u = \left[1 + k_1 (\alpha T_u)^{k_2} (rT_u)^{k_3} + k_4 (\alpha T_u)^{k_5}\right]^{-k_6} \quad (37a)$$

where k_1–k_6 are unknown positive constants. Considering the tabulated values and minimizing the sum of absolute proportionate errors, $k_1 = 0.417$, $k_2 = 1$, $k_3 = 1$, $k_4 = 0.01$, $k_5 = 2$ and $k_6 = 0.5$ were obtained. Thus, Equation (37a) changes to:

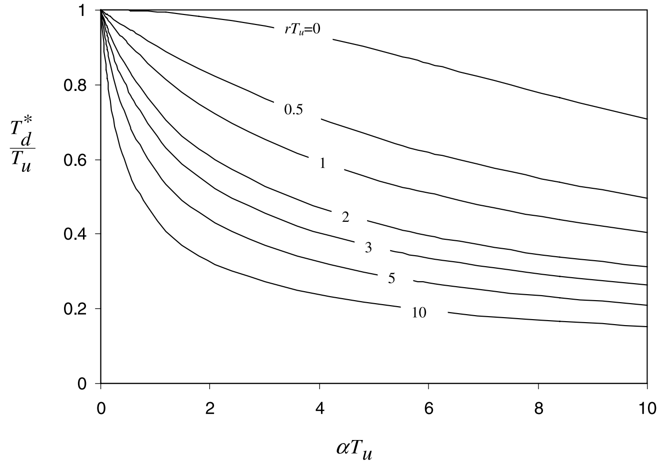

Figure 3 | Variation of design period of a pumping main.

$$T_d^* = T_u \left(1 + 0.417\alpha r T_u^2 + 0.01\alpha^2 T_u^2\right)^{-0.5} \quad (37b)$$

DESIGN EXAMPLE

It is necessary to ascertain the design period for a water supply gravity main as well as a pumping main using CI pipes, $\alpha = 0.04$ year^{-1} and $r = 0.05$. Using Equation (15b) the design period for a gravity main is obtained as:

$$T_d^* = 120\left(1 + 2 \times 0.04 \times 0.05 \times 120^2\right)^{-0.375}$$
$$= 26.07 \text{ yr} \approx 26 \text{ yr} \quad (38)$$

Similarly, using Equation (37b), the design period for a pumping main is obtained as:

$$T_d^* = 120\left(1 + 0.417 \times 0.04 \times 0.05 \times 120^2\right.$$
$$\left. + 0.01 \times 0.04^2 \times 120^2\right)^{-0.5} = 32.98 \text{ yr} \approx 33 \text{ yr} \quad (39)$$

Thus, the water supply gravity main should be designed initially for 26 years and then re-strengthened after every 26 years. Similarly the pumping main should be designed initially for 33 years and then re-strengthened after every 33 years.

CONCLUSIONS

Equations for the design period of water supply transmission mains have been obtained. These equations involve the life of the pipe, the rate of increase in water demand and the future discount factor. It is hoped that the developed methodology will be useful for water supply designers and planners to select a rational design period for gravity and pumping transmission mains.

NOTATION

C gravity main cost function
$\bar{C}^*$ optimal gravity main cost function factor
D pipe diameter
f friction factor
F pipe cost function
$\bar{F}^*$ optimal pumping main cost function factor
F_A annual averaging factor
F_D daily averaging factor
g gravitational acceleration
H terminal pressure
h_L available head loss
h_0 pumping head
h_f head loss due to pipe friction
k_e pumping cost coefficient
k_m pipe cost factor
L pipe length
m pipe cost exponent
Q pipe link discharge
Q_0 initial discharge
r discount rate
R_E cost of electricity per kWh
t time
T_d design period
T_u pipe life
z_0 elevation at source point
z_L elevation at point of supply
α rate of increase of water demand
ρ mass density of water
η efficiency
$*$ optimal

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