Practical and theoretical analysis of relationships between particle count data and turbidity

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ABSTRACT

Potential relationships between particle counts and turbidity have been examined by several researchers over recent years. This paper describes how a different approach to this issue was adopted. Particle count and turbidity data arising from the combined filtrate of a block of six rapid gravity filters are analysed for possible relationships. The results are compared with those of other researchers and it is demonstrated that no linear relationship exists between particle count and turbidity. This practical result is then corroborated by a theoretical analysis from first principles of the light scattering properties of suspended particles.

Key words | combined filtrate, light scattering, particle counts, turbidity

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INTRODUCTION

The use of particle counting as a technique to assess water quality has come to the fore in the UK following the publication of the Badenoch (1990, 1995) and, more recently, Bouchier (1998) reports. These reports have examined the water industry's approach to water quality monitoring and focused the attention of water undertakers on the health threats presented by the parasitic protozoan, *Cryptosporidium parvum*. The health threats associated with transmission of *Cryptosporidium* oocysts are well known and well documented. A detailed review of these issues is outside the scope of this paper; however, the interested reader is directed to the work of Gregory (1994), Badenoch (1990), Ives (1995), and Lisle & Rose (1995).

Recommendation 25 of the Badenoch Report (1995) stated that 'Strategies should be developed for each treatment plant whereby the optimum use can be made of turbidity and/or particle monitors to minimise passage of particles into supply at all stages in the filtration cycle'. The problem facing water utilities is to determine the required level of use of turbidimeters and particle counters at water treatment works and borehole sources. It is incumbent upon the undertakers to ensure that the risk of *Cryptosporidium*, and indeed any particulate, breakthrough is minimised, while not incurring excessive and abortive capital and revenue expenditure.

The research reported in this paper is based upon work undertaken at Severn Trent Water's direct river abstraction water treatment works at Mythe, near Tewkesbury, Gloucestershire. The research considered, *inter alia*, how data arising from a single particle counter and turbidimeter, measuring particle counts and turbidity of combined filtrate from a block of six filters, may be used to assess particulate breakthrough, both in the 2–5 μm range and in the total particle count (i.e. particles in the range 2–150 μm). This paper focuses on the derivation of relationships between particle counts and turbidity.

Particle counting

Particle size analysis has been used extensively in other fields for many years, but has only recently found an application in the water industry. The technique involves the sizing and counting of small particulate matter

suspended in water. Instruments are available to detect particles greater than 0.5 μm in size, and counts may be registered in several size ranges (i.e. 2–5 μm, 5–10 μm, etc.), allowing the construction of a detailed particle size distribution for the water sample being analysed. The ability to measure accurately and count particles in these size ranges is significant for the water industry in particular, as *Cryptosporidium* oocysts are of a size that is readily detected by particle counters. *Cryptosporidium* oocysts are typically between 4 and 6 μm in size, but are placed by particle counters in the 2–5 μm size range as a result of their low refractive index and the calibration procedures for particle counters. Thus, particle size analysis offers the industry a method of monitoring filtrate quality and using the results to optimise the filtration process. However, it is important to appreciate that although particle size analysis can be used for the counting and sizing of particles, the technique does not identify the exact nature of the particles. Rather, the concentration trends produced by the counters serve simply to indicate an increase of particulate breakthrough if levels suddenly rise.

There are currently three counting techniques employed by particle counters in the water industry; these being light scattering, light obscuration, and electrical resistance. Of these, the most commonly applied in the water industry is the light obscuration (or light extinction) method. The light-based techniques are based upon the theories of light scattering and absorption, both of which remove energy from the incident light, decreasing the intensity upon the receiving photodetector. They are analogous to turbidity measurement, but use individual particles as opposed to an entire sample.

Turbidity measurement

Turbidity is a measure of suspended insoluble matter in a sample of water and is measured either by light transmission according to the Beer-Lambert Law, or by light scattering (nephelometric) techniques. In the latter, more common, method, light scattered at an angle of 90 degrees to the incident light beam is measured and a turbidity value derived. This turbidity value is on a standard scale and is measured in nephelometric turbidity units (NTU).

Therefore, turbidity is not a direct measurement, rather a function of various particulate properties including size, shape, refractive index and incident light wavelength. Bridgeman and Bridgeman (1994) derived rigorous mathematical models for the scattering and absorption behaviour of particles in several environments.

Gregory (1994) presented a much simplified analysis of the particle scattering characteristics that are important for turbidity measurement, demonstrating the strong dependence of turbidity on particle size. Gregory demonstrated that large particles contribute less to the turbidity of a suspension than smaller particles, potentially causing difficulties when one attempts to use nephelometric techniques to classify a body of water. However, this result is only valid on the basis of a constant particle mass, or volume concentration. The intensity, I, of scattered light is proportional to d_p^2, where d_p is the particle diameter. With a constant mass, the number concentration, c_N, decreases with particle diameter; that is, c_N is proportional to d_p^{-3}. This implies that I is proportional to d_p^{-1} and, on this basis, a turbidimeter gives maximum turbidity for approximately 0.3 μm particles. Gregory (1994) also developed an argument that the light scattering potential of suspended particles is much reduced when their refractive indices approach that of water since they become almost transparent to the light. This led Gregory to conclude that turbidity measurement may not be appropriate for the measurement of *Cryptosporidium* oocysts as they have a low refractive index. However, in the absence of other techniques, turbidity has been accepted as an adequate measure of water quality since turbidity levels rise dramatically when filter performance deteriorates markedly.

As a result of the recommendations of the first Badenoch Report, many water treatment works now have turbidimeters installed on individual filters, on the basis that measurement of turbidity of combined filtrate from a bank of filters does not allow the risk of breakthrough of oocysts in individual filters to be identified.

Relationship between particle count and turbidity

Much work has been devoted by many researchers to the search for relationships between particle counts and turbidity for raw, clarified and filtered water. However, there

exists little agreement among researchers regarding any such relationships. Hargesheimer *et al.* (1992) examined the full range of particle counters available at the time, and concluded that no single conversion factor could be applied to relate turbidity to particle count. Hargesheimer *et al.* provided as support for this conclusion the fact that turbidity is a one-dimensional measurement of water quality whereas particle counting is two-dimensional (particle size and count). This additional information can be used to observe differences that may be otherwise undetected by turbidity measurement, and so water samples with identical clarity can be distinguished on the basis of particle size analysis.

Doyle (1998) presented coefficients of determination for total particle counts and turbidity in raw water, which varied from 0.570 for the 2–3 μm size range, to 0.984 for the 5–10 μm size range. Doyle also studied the relationship between treated water turbidity and particle counts, finding a good correlation between the two. Like several other researchers (Keay 1995; Murray 1995; Morse *et al.* 2000), Doyle found that small changes in particle numbers were not indicated by corresponding changes in turbidity, and it is now accepted that particle size analysis offers a much quicker rate of detection in quality changes than turbidity measurement. Hamilton *et al.* (2000) found good correlation between total particle counts and turbidity in the feed water at a 15 Ml day^{-1} groundwater treatment works, although the greater sensitivity of particle counters over turbidimeters was also demonstrated.

MYTHE WTW

Treatment capacity at Mythe WTW is 120,000 m^3 day^{-1}, with the facility to draw an additional 26,000 m^3 day^{-1} from a neighbouring water treatment works. Raw water is treated by coagulation, clarification, rapid gravity filtration, ozone, granular activated carbon and disinfection. The flow is separated into three streams prior to clarification and is combined following filtration. The flow is split unequally across the streams, which take approximately 50%, 30% and 20% of the total flow (see Figure 1). Turbidimeters are fitted to the outlet of each of the 28 filters, and an investigation is carried out on any filter that exceeds a prescribed turbidity action limit.

METHODOLOGY

A Particle Measuring Systems (PMS) Liquilaz particle counter was installed on-line on one filter. A further PMS Liquilaz counter was installed downstream of all six filters in the stream, monitoring the combined filtrate. This arrangement is shown schematically in Figure 2. Samples were taken at 5-minute intervals and the data from both counters were downloaded on to computer. Each counter was set to size particles into five different size bins (2–5 μm, 5–10 μm, 10–15 μm, 15–25 μm and 25 μm and above) up to a maximum size of 150 μm. Data from on-line turbidimeters monitoring turbidity from the combined filtrate were recorded at 2-hourly intervals and data were collected over a 12-week period.

RESULTS

Weekly data analysis

The research investigated possible relationships between combined particle count and corresponding combined turbidity values. The combined filtered water turbidity values were collated into 1-week batches and plotted against the combined particle count in the 2–5 μm range and also total particle count (i.e. particles greater than 2 μm). The standard Microsoft Excel linear regression tool was utilised to derive any linear relationship existing between the combined turbidity and combined particle count for both the individual size range and total count.

The results, shown in Table 1, indicate a wide variation in coefficient of determination for both the 2–5 μm size range and the total particle count. Considering the total count data, the coefficient of determination was found to vary from 0.076 to 0.907, while for the 2–5 μm size range, the corresponding range of coefficients was 0.166 to 0.769.
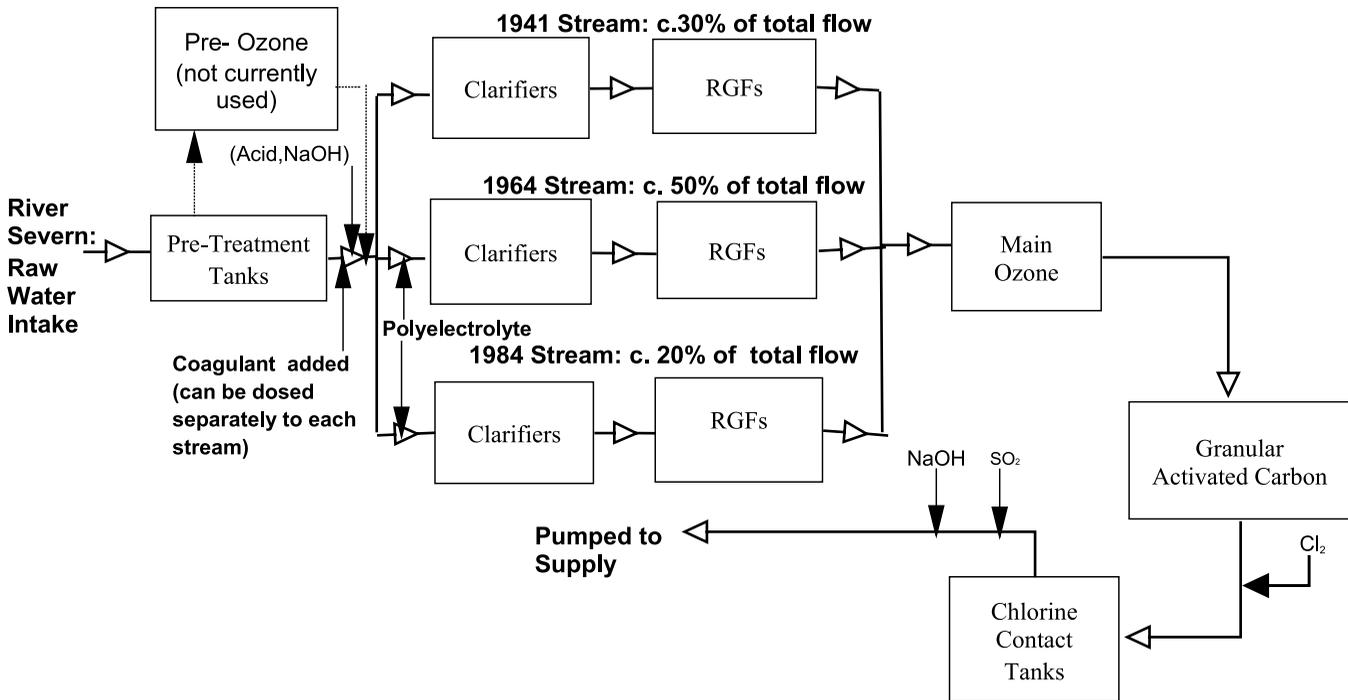
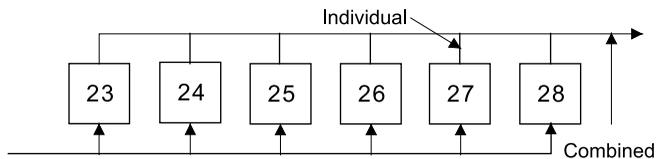**Figure 1** | Schematic diagram of treatment process.

Figure 2 | Schematic diagram showing sample points in relation to filters.

Total data analysis

In order to use the largest data set possible, all data collected over the 12-week period were combined and coefficients of determination derived for the individual size range and also the total count. The resulting plots are shown in Figures 3 and 4.

From Figure 3, there appears to be a reasonable relationship between the two parameters (total count and turbidity), with a coefficient of determination of 0.854. However, the same is not true of the relationship between particle count in the 2–5 μm size range and turbidity. In this instance there is a coefficient of determination of just 0.296. It is noticeable how the majority of the particle count values are distributed over a very narrow band of turbidity values, demonstrating clearly the increased sensitivity of particle counters compared with turbidimeters.

Summary

Practical analysis of the data collected has demonstrated that no robust relationships between combined turbidity and combined particle counts (in either a single size range or the total count) can be derived. The analysis has, however, demonstrated the increased sensitivity of particle counters compared with turbidimeters.

DISCUSSION

Particle counts at low turbidity values

Figure 4 corroborates the conclusion of Gregory (1994), who commented that 'conventional turbidity

Table 1 | Weekly linear coefficients of determination for relationship between total count per millilitre and turbidity, and 2–5 µm size range count per millilitre and turbidity

	Total count ml^{-1}				2–5 µm size range ml^{-1}			
No of points	r^2	Max	Min	Mean	r^2	Max	Min	Mean
47	0.667	2,157	173	882	0.347	665	130	456
61	0.584	995	137	579	0.57	705	206	404
58	0.8	1,485	329	875	0.769	807	205	518
58	0.076	1,199	595	804	0.166	750	379	510
56	0.53	1,574	623	943	0.735	386	356	518
43	0.344	1,558	561	906	0.236	713	240	404
46	0.907	7,478	260	2,063	0.625	823	176	451
49	0.521	360	129	208	0.457	250	80	152
64	0.423	188	58	105	0.42	146	44	77
52	0.207	207	63	118	0.23	105	46	75
44	0.245	291	156	213	0.498	170	90	129
49	0.632	633	80	255	0.633	459	54	188
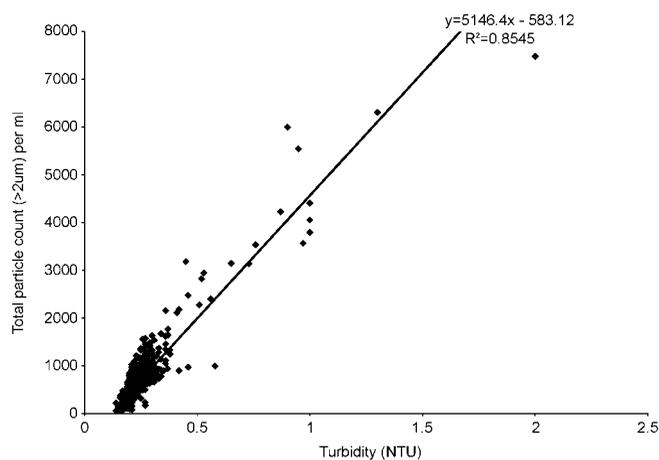
Figure 3 | Relationship between combined turbidity and combined total particle count.
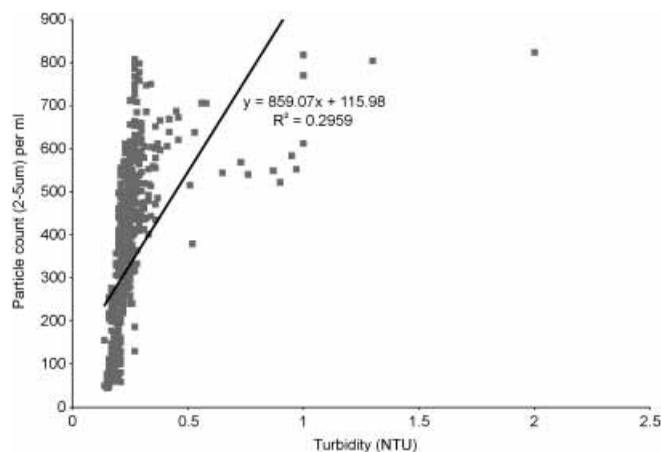
Figure 4 | Relationship between combined turbidity and combined particle count in the 2–5 µm size range.

measurement is not sufficiently sensitive to particles in the oocyst size range'. Gregory continued, 'A low turbidity value does not ensure that such particles (i.e. *Cryptosporidium* oocyst-sized) are absent'. This conclusion is clearly corroborated by the results presented in Figure 4, which shows particle counts in excess of 800 counts ml^{-1} at turbidity values of less than 0.2 NTU.

At lower turbidity values, Figure 4 may at first appear to indicate that there is a linear relationship between particle count in the 2–5 μm range and turbidity. However, the line is almost vertical indicating that there is no relationship between the two parameters. The results merely serve to reinforce the different degrees of accuracy of the instruments used.

Analysis of the complete dataset also shows that the highest particle count values shown in each figure occur at the same turbidity values (e.g. counts in the 2–5 μm range in Figure 4 and total count set in Figure 3 both exhibit maxima during the same event, when turbidity is at its maximum value of 2 NTU). This demonstrates that high turbidity value data points in Figure 4, which may at first appear anomalous, do in fact correspond to occurrences of high total count.

Theoretical analysis of relationship between turbidity and particle count

Having obtained such a wide variation in coefficients of determination, the actual relationship between turbidity and particle count was investigated from a theoretical basis. The analysis, which provides only an approximate solution to the problem, is given below.

Turbidity is measured in NTU using light scattering (nephelometric) techniques and is derived from consideration of the ratio of incident light to scattered light, and it is this relationship which is derived below from first principles.

In the following mathematical analysis, the essentially discrete system of particle suspension is replaced by a continuous material of constant composition.

Consider a homogeneous medium, with rectangular horizontal cross-section, of sides 2a and 2b, with all four sides vertical (Figure 5). Denote the vertical sides by f_1, f_2, f_3 and f_4 as shown.
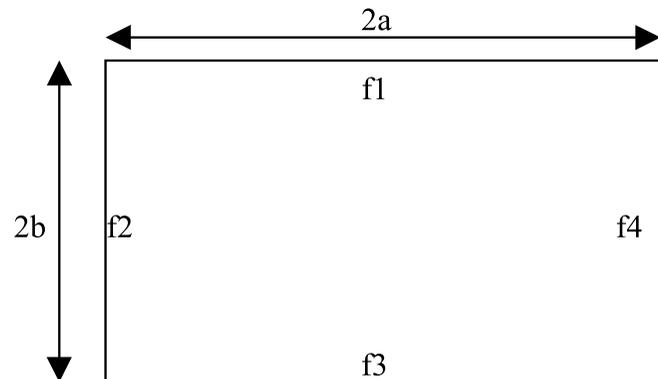
Figure 5 | Scattering and absorbing medium with horizontal cross-section. Note: 2a and 2b denote dimensions of scattering medium; fx denotes specific face of scattering medium.

The basic properties of the medium are its scattering coefficient, 4σ, and absorption coefficient, α, so defined that fractions $4\sigma\delta x$ and $\alpha\delta x$ are respectively scattered and absorbed from a beam of radiation in traversing an elementary path of length δx.

If the medium contains n constituents with concentrations c_1, c_2, c_3, ..., c_n, and scattering and absorption coefficients $4\sigma_1$, α_1; $4\sigma_2$, α_2, $4\sigma_3$, α_3; ...; $4\sigma_n$, α_n, then it is assumed that

$$\sigma = \frac{c_1\sigma_1 + c_2\sigma_2 + c_3\sigma_3 + \ldots + c_n\sigma_n}{c_1 + c_2 + c_3 + \ldots + c_n} \tag{1}$$

and

$$\alpha = \frac{c_1\alpha_1 + c_2\alpha_2 + c_3\alpha_3 + \ldots + c_n\alpha_n}{c_1 + c_2 + c_3 + \ldots + c_n} \tag{2}$$

The assumption that the scattering and absorption co-efficients combine linearly, as in Equations 1 and 2, is only valid for a medium of constant composition. For a non-homogeneous medium, such as when the medium is layered, it has been shown by Bridgeman (1964) that the combination is non-linear.

For the purposes of this example, it is assumed that any radiation only undergoes one scattering in its passage through the medium. Thus, if the incident light is a parallel beam which produces diffuse light due to scattering, the reduced intensity of the incident beam is scattered
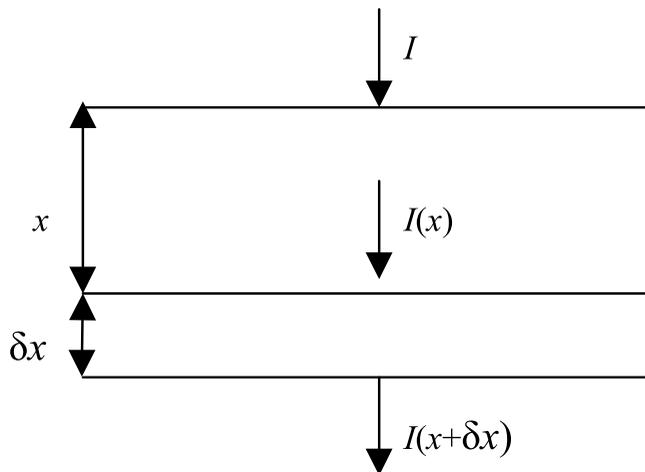
Figure 6 | Incident beam transmission. Note: I=incident beam transmission; Ix=incident beam transmission at depth x; $I(x+\delta x)$=incident beam transmission at depth $x+\delta x$.

and absorbed while the resulting diffuse radiation is absorbed but not scattered. Thus, 'single scattering' is assumed.

Consider a beam of radiation of intensity I_o to be incident normally on the side f_1. The collimated beam enters the medium and is scattered and absorbed. This reduces the intensity of the beam and produces scattered light, which is termed 'diffuse' light.

Therefore, within the medium, there is the reduced intensity of the incident beam (which will continue to be both scattered and absorbed), together with diffuse radiation (which will only be absorbed) produced from the scattering of the reduced incident beam.

Using the notation defined above, let the scattering coefficient be 4σ and the absorption coefficient be α.

Consider the reduced intensity of the incident beam. Let $I(x)$ denote the intensity when it has penetrated a distance x. If the intensity then passes through a thin layer of thickness δx its value becomes $I(x + \delta x)$ (Figure 6). In passing through this layer, it loses $4\sigma I(x)\delta x$ due to scattering and $\alpha I(x)\delta x$ due to absorption.

Hence,

$$I(x + \delta x) = I(x) - 4\,\sigma I(x)\,\delta x - \alpha I(x)\delta x$$

$$\therefore \frac{I(x + \delta x) - I(x)}{\delta x} = -(4\sigma + \alpha)I(x)$$

$$\text{As } \delta x \to 0, \frac{dI(x)}{dx} = -(4\sigma + \alpha)I(x)$$

$$\therefore I(x) = Ae^{-(4\sigma + \alpha)x}$$

and $I(x) = I_o$ when $x = 0$, so $A = I_o$

$$\therefore \frac{I(x)}{I_o} = e^{-(4\sigma + \alpha)x} \tag{3}$$

Assuming uniformly distributed scattered light, it can be seen that the $4\sigma I(x)\delta x$ of diffuse light that is produced as the reduced incident beam passes through the thickness δx is distributed as follows:

i. $\sigma I(x)\delta x$ will be directed back towards face f_1
ii. $\sigma I(x)\delta x$ will be directed forwards to face f_3
iii. $\sigma I(x)\delta x$ will be directed sideways to face f_2
iv. $\sigma I(x)\delta x$ will be directed sideways to face f_4.

Assume that the incident beam is directed at the centre of face f_1 (distance a from both f_2 and f_4).

For normal incidence and 90° collection, it is the light received at face f_4 which is of interest. (From symmetry, the light received at face f_2 will be the same as that received at f_4.)

The amount $\sigma I(x)\delta x$ travels a distance a towards face f_4. It is not scattered, but is absorbed. Therefore, by Beer's law, its intensity at face f_4 is given by

$$\sigma I(x)\delta x e^{-\alpha a}$$

Therefore,

$$\sigma e^{-(4\sigma + \alpha)x}e^{-\alpha a} = \sigma e^{-\alpha a}e^{-(4\sigma + \alpha)x} \tag{4}$$

This represents the contribution from scattering at depth x into the medium. It is necessary to include the contributions from all depths, i.e. from $x = 0$ to $x = 2b$.

Thus, if I_4 is the intensity of radiation emerging from face f_4,

$$\frac{I_4}{I_o} = \int_0^{2b} \sigma e^{-a\alpha} e^{-(4\sigma + \alpha)x} \, \mathrm{d}x$$

$$\therefore \frac{I_4}{I_o} = \sigma e^{-a\alpha} \frac{1}{(4\sigma + \alpha)} [1 - e^{-(4\sigma + \alpha)2b}] \tag{5}$$

Therefore, it is apparent that the light received at an angle of 90° is a non-linear function of the scattering and absorption coefficients. The scattering and absorption coefficients were defined above as functions of the number of total particles. As discussed above, turbidity is directly related to the ratio of received to incident intensity and so it follows that there can be no linear relationship between turbidity and particle count. This corroborates the conclusions of both Hargesheimer *et al.* (1992), who stated that no single conversion factor could be applied to relate turbidity to particle count, and Hall and Croll (1996), who found no correlation between the two parameters.

The above represents an approximate solution for the received light. Chandrasekhar (1960) has examined similar cases considering the problem of received light intensity from a semi-infinite medium (i.e. a medium with flat surface stretching to infinity in all directions and with infinite depth) and a plane parallel medium of finite thickness (i.e. a medium bounded by two parallel planes). Chandrasekhar has developed solutions for both cases; the former is defined by a non-linear integral equation, while the latter is defined by simultaneous non-linear integral equations, and to date they have only been solved numerically. Both cases considered by Chandrasekhar are simplified cases of the turbidimeter, which involves a medium of finite dimensions, and so the solution, which would be extremely difficult to define, would be complicated and most certainly non-linear.

Several researchers have expended much effort on the analysis of interrelationships between particle count data and turbidity data, and some (Doyle 1998; Hamilton *et al.* 2000) have presented data showing good correlation. While the results of these researchers are accepted, it is believed that they are fortuitous rather than representative. Both Hamilton and Doyle presented data from a groundwater source, which exhibits little variability in quality. For such a works, it is perhaps not surprising that

a correlation was found, since the quality is constant in terms of turbidity and counts.

In summary, a detailed theoretical analysis of the relationship between incident light, scattered light and particulate characteristics has demonstrated that no linear relationship exists between turbidity and particle count. The results obtained from this research clearly corroborate this conclusion.

CONCLUSIONS

Research has been undertaken to identify a possible relationship between turbidity and particle counts using data arising from the combined filtrate of a block of six rapid gravity filters. No robust relationships between combined turbidity and combined particle counts (in either a single size range or the total count) were derived from the data collected over a 12-week period. Linear correlation coefficients for weekly data ranged from 0.08 to 0.91 for total count, and from 0.17 to 0.77 for particles in the 2–5 μm size range. Combining all data yielded correlation coefficients of 0.85 and 0.30 for total count and the 2–5 μm size range, respectively. However, the results clearly demonstrate the increased sensitivity of particle counters when compared with turbidimeters.

Relationships between particle counts and turbidity were also examined from a theoretical perspective via an analysis of the relationship between incident light, scattered light and particulate characteristics. The analysis demonstrates that light scattered by a medium at an angle of 90 degrees is a non-linear function of that medium's scattering and absorption coefficients. It is therefore concluded that no linear relationship between turbidity and particle count can exist.

ACKNOWLEDGEMENTS

NOMENCLATURE

μm	microns
NTU	nephelometric turbidity units
Ml day^{-1}	megalitres per day
r^2	coefficient of determination ($=$ linear correlation coefficient^2)
ml^{-1}	per millilitre
4σ	scattering coefficient of a medium
α	absorption coefficient of a medium
x	thickness
δx	elementary small thickness
I	intensity of light beam
c	concentration
d_p	particle diameter

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