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Fluctuations in sound transmission through sound speed profiles that are periodic in range and geotime

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Abstract: Acoustic propagation through sound speed profiles (SSPs) that are periodic in range and geotime is studied using the parabolic equation model and the WKB/adiabatic model. Fluctuations are shown to be significantly reduced when acoustic propagation is through a periodic range-dependent SSP, compared to a range-independent SSP. For a fixed source-receiver range, the number of tidal wavelengths contained within the range is an important factor that determines the fluctuations. The relationship of this factor to the direction of transmission relative to the tidal direction is presented. A useful criterion for predicting the extent of fluctuations under different environments has been developed by examining the range integral of the horizontal wave number for each mode.

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1. Introduction

The purpose of this paper is to demonstrate that significant reduction in fluctuations occurs when sound propagates through range-dependent sound speed profiles (SSPs) that are periodic in range and geotime. Such SSPs are generated by the lunar M2 tides in coastal waters with a period of 12.42 h, as well as by other weaker tides.[1] The reduction in fluctuations is in comparison with propagation through range-independent SSPs.

It is well known that energy transfer to higher modes caused by scattering and subsequent absorption by the bottom reduces variability.[2,3] Studying that mechanism is not the intended purpose of this paper. The focus here is on the reduction of variability resulting from adiabatic transmission through multiple tidal wavelengths.

A measured SSP from the Shallow Water Acoustic Technology (SWAT) experiment that was conducted in the New Jersey Bight has been used in the numerical calculations below.[4] The parabolic equation (PE) model[5] and the WKB/adiabatic model[6] have been used in the analysis.

2. The model

The chosen SSP from the SWAT experiment corresponds to one location on the continental shelf off the coast of New Jersey, where the water depth is 132 m. Time-averaged sound speed $c0$ as a function of depth is shown in Fig. 1 (left). Bottom sound speed values are 1544, 1571, 1635, 1692, and 1772 m/s, at depths of 132, 133.7, 137, 143.8, and 150 m, respectively. Bottom relative density values are 1.2, 1.6, 1.8, 1.9, and 2.1, at depths of 132, 133.7, 135, and 150 m, respectively. Bottom attenuations are 0.05, 1, 2, and 5(dB/λ), at depths of 132, 150, 250, and 300 m, respectively. The geoacoustic parameters were obtained from chirp sonar inversions that were done within the SWAT experimental area.[7]

The range and time dependence of the sound speed has been assumed to be sinusoidal, given by the equation

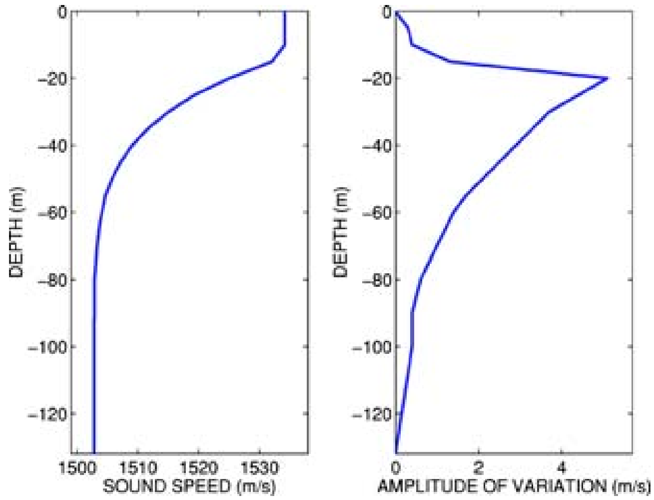

Fig. 1. The mean sound speed profile (left) and the amplitude of variation in sound speed as function of depth (right). Data are from the SWAT experiment.

$$c(r,z,t) = c0(z) + c1(z)^{*}\cos\left[2\pi\left(\frac{r}{\Lambda} - \frac{t}{T}\right)\right], \tag{1}$$

where r=range, Λ=tidal wavelength, t=geotime, and T=12.42 h (for M2 tides). The variation of $c1$ with depth is shown in Fig. 1 (right).

Since baroclinic tidal wavelengths are typically 20–30 km, the source-receiver range was chosen to be R=30 km. Source depth=30 m, and source frequency=100 Hz.

The tidal wavefront is given by r=const in Eq. (1); the tidal direction is perpendicular to this. In the range-dependent case, the acoustic transmission is along the tidal direction, or at an angle θ (between 0 and $\pi/2$) from it. In the range-independent case, the acoustic transmission is perpendicular ($\theta=\pi/2$) to the tidal direction.

When the transmission is along the tidal direction (θ=0), let there be $N=(R/\Lambda)$ tidal wavelengths contained within the source-receiver range R. Then for transmission at an angle θ from the tidal direction, the effective tidal wavelength would be ($\Lambda/\cos\theta$), and there would be $M=(N\cos\theta)$ tidal wavelengths contained within the source-receiver range R.

Since an acoustic transmission lasts only for a few seconds, and the geotime t in Eq. (1) is typically measured in days, the sound speed field is assumed to be constant during each acoustic transmission.

3. Variation of transmission loss

For a fixed source-receiver range, R=30 km, the received power (in dB) was calculated using the PE model[5] as a function of depth and geotime. The calculation was repeated for values of the parameter M=0.1, 0.3, 0.5, 0.7, 0.9, 1.1, 1.3, 1.5, and 1.7. The pattern does not change significantly for higher values of M. The results are summarized in Fig. 2, which shows the cases of M=0.1 (top), 0.7 (middle), and 1.7 (bottom). Blue represents the lowest power value in each color plot, and dark red represents the highest.

The top color plot in Fig. 2, for M=0.1, very nearly corresponds to the range-independent case and reveals complex power variations indicating overtones of the fundamental period 12.42 h all along the water column.[4,8] As the value of M increases, the overtone structure decreases, as seen in the middle plot for M=0.7. For M=1.7 (bottom plot), the fluctuation is essentially that of the fundamental sound speed variation. The higher concentration of acoustic energy in the lower part of the water column is consistent with the downward refracting SSP.

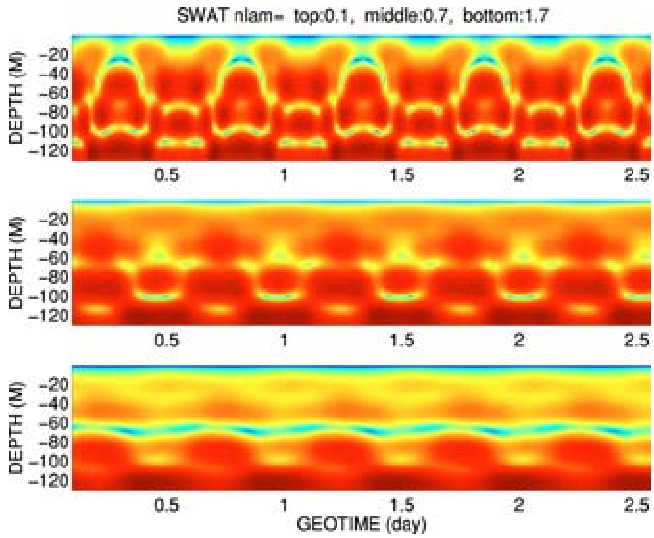

Fig. 2. Received power as a function of depth and geotime. The number of tidal wavelengths M, contained within the source-receiver range, are 0.1 (top), 0.7 (middle), and 1.7 (bottom). Tidal period $T=12.42$ h, range $R=30$ km, water depth=132 m, source depth=30 m, and source frequency=100 Hz.

Figure 3 shows the power spectra for the three cases in Fig. 2. The time series of power values at each depth had its mean subtracted and then was normalized before the FFT was calculated. This was done so that the spectrum is clearly seen in the color plot even for depths where the variation is relatively small. As M increases, the overtone structure progressively decreases, leaving only the fundamental variation of approximately 2 cpd.

4. Variation of phase integral

In the WKB/adiabatic model,[6] the phase integral $\int k_n(r)\, dr$ takes the place of the phase factor $k_n r$ in the range-independent case and is the dominant factor that determines fluctuations. Here

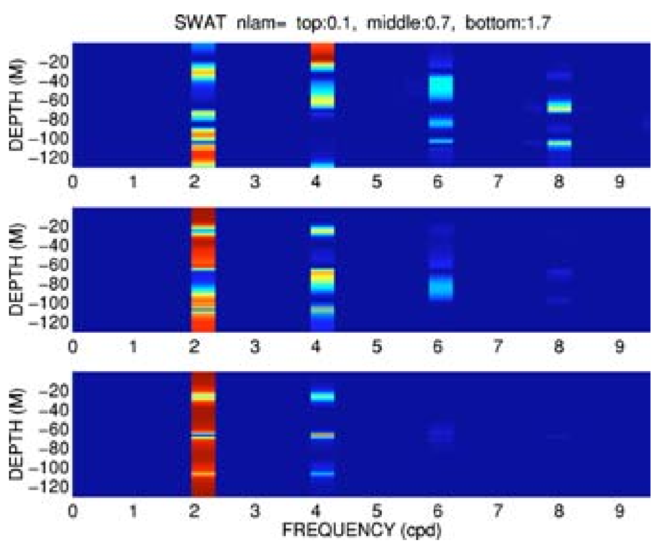

Fig. 3. Power spectra of the three cases in Fig. 2.

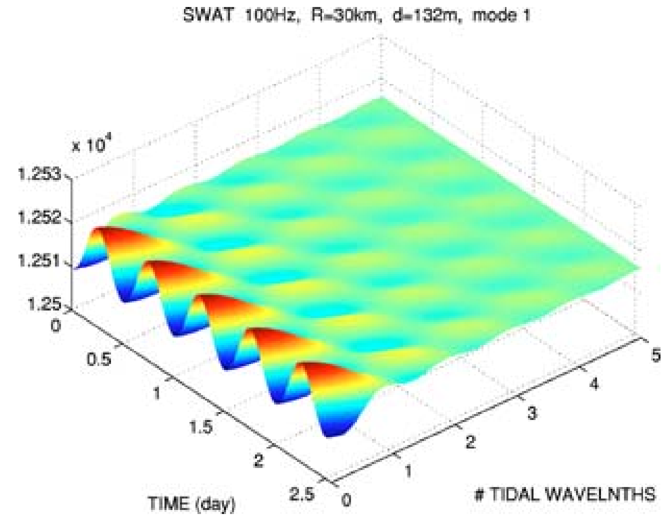

Fig. 4. The phase integral $\int k_n(r)\,dr$ as a function of M (the number of tidal wavelengths contained within the source-receiver range), and geotime, for mode 1 for the sound speed profile in Fig. 1.

$k_n(r)$ is the horizontal wave number for mode n. This integral has been calculated for the example described above in Sec. 2. The source-receiver range R is fixed at 30 km, and the number of tidal wavelengths contained within the source-receiver range M has been treated as a variable.

Figure 4 shows the phase integral $\int k_n(r)\,dr$ for mode 1 as a function of geotime and M. The figures for the remaining modes 2–6 are very similar to Fig. 4. The line along the time axis with the parameter M set equal to zero represents the range-independent case. Corresponding to each nonzero value of M, the line along the time axis represents a range-dependent case.

For a fixed value of M, variation of the phase integral $\int k_n(r)\,dr$ as a function of geotime is approximately sinusoidal with the tidal period $T=12.42$ h. For a fixed value of geotime corresponding to a peak (valley), as the value of M increases, successive peaks (valleys) with progressively smaller magnitudes appear with a period of unity.

While the fluctuation of the phase integral $\int k_n(r)\,dr$ for $M>2$ is only a small fraction of its maximum fluctuation at $M=0$, it is the actual value of the fluctuation that determines the variation in the acoustic pressure (or the power). For example, for $M=2.5$, the phase integral varies between 12 515 and 12 516 rad. Cosine (or sine) of these phases would differ significantly.

It is easy to confirm the above results analytically for the rigid-bottom waveguide. In that case the horizontal wave number $k_n=[(\omega/c)^2-\{(n-1/2)\pi/h\}^2]^{1/2}$, where the sound speed c is given by Eq. (1), but is independent of depth. Using binomial expansion keeping only first-order terms, the integral $\int k_n(r)\,dr$ can be written as

$$\int_0^R k_n(r)\,dr = k_{n0}R-(1/M)(\omega^2 c_1 R/\pi k_{n0}c_0^3)\sin(\pi M)\cos[2\pi(M/2-t/T)], \qquad (2)$$

where $k_{n0}\equiv k_n(c=c_0)$. For a fixed source-receiver range R, when M increases the second term becomes smaller.

For the example in Sec. 2, the first term in Eq. (2) is 12 561. The coefficient of the second term, viz. $\omega^2 c_1 R/\pi k_{n0}c_0^3$, for mode 1 is 2.67. (Here an average value for the perturbation $c1=1$ m/s was used, considering that in the rigid bottom case the sound speed is constant throughout the water column.) In this case for $M>27$, the second term would be less than 0.1

even for maximum values (unity) of the sine and cosine functions. If the source-receiver range R is 10 km instead of 30 km, the second term would be similarly small for $M > 9$.

5. Conclusions

Model calculations have been done to demonstrate that fluctuations in geotime are significantly reduced when acoustic propagation is through a periodic range-dependent sound speed profile (SSP), compared to a range-independent SSP. For a fixed source-receiver range, the number of tidal wavelengths contained within the range is an important factor that determines the fluctuations. The relationship of this factor to the direction of transmission relative to the tidal direction is presented. A useful criterion for predicting the extent of fluctuations under different environments has been developed by examining the range integral of the horizontal wave number for each mode.

Acknowledgments

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