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DISCUSSION

Paul C. Paris[3]

The authors are to be complimented on the obvious care they have taken in examining the strength of cracked aluminum-foil laminates. The discusser can take no deference to their work but would like to indicate some additional points which may be examined in future studies of a similar nature.

Irwin[4] has discussed the influence of the crack-tip plastic zone size on the cracked strength of sheets. An estimate of the width of the plastic zone, δ, in the direction of cracking is:

$$\delta = \frac{K^2}{2\sigma_{y.p.}^{\;2}} = \frac{E\dfrac{dW}{dA}}{2\pi\,\sigma_{y.p.}^{\;2}}$$

The data in Tables 1 and 2 of the paper indicate plastic zone sizes from 0.05 to 0.13 in. using the above equation. The influence of the plastic zone is two-fold, i.e.,

1 The effective crack length in static fracture tests should be taken to be $x + 2\delta$ for use in equations based on elastic stress analysis such as equations (1), (2), and (3) in the paper.

In the tests reported in the paper this correction of the crack length would change the calculated values of dW/dA slightly but would not change the trends observed.

2 The stress state near the crack-tip tends from plane stress toward plane strain as the plastic zone size to thickness ratio of test specimens varies from above to below a value near unity.

In the tests reported the ratio was always well above unity indicating crack-tip stress conditions of a plane stress nature.

In future tests investigators might include thicker panels which would allow investigation of plane strain crack-tip stress states. An interaction between laminations would be induced by such a state of stress which might cause local delamination and other effects. Since local delamination would tend to produce plane stress conditions, rather than plane strain, the result might be an apparent increase in strength over single sheets of a thickness equal to the total thickness of the laminate. The effect described here is an increase in strength beyond those found in the paper which might be possible with laminates.

Finally, some attention should be drawn to the disturbing phenomena of buckling of cracked thin sheets subject to tension. Mote and Frisch point out that buckling did occur in their tests which raises the question of whether the increase in strength observed with a greater number of laminations is due to inhibiting buckling or some other cause. If buckling is not a serious factor in these tests one would wonder why the dW/dA values reported are so unusually low for aluminum alloys. This matter of buckling bears further investigation since it arises in many other phases of fracture testing as well as laminate studies.

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[4] G. R. Irwin, "Dimensional and Geometric Aspects of Fracture," Conference on Fracture of Engineering Materials, August 23–25, 1959, Rensselaer Polytechnic Institute.

Authors' Closure

The authors wish to thank Mr. Paris for his discussion and his comments regarding the plastic zone width at the crack tip. In this investigation, the correction, as pointed out, would have a very slight effect on the numerical value of dw/dA. The experimental data have been presented to make further analyses possible, such as plastic zone effects on crack stability. The buckling stability of the laminate specimens increased with the number of plys; however, the exact dimensions bounding the critical effects of buckling on fracture strength are difficult to determine. Comparing Figs. 9, 11, and 12, it can be seen that the two-ply specimens always sustain a lower maximum gross area stress. Except for the cross grain tests shown in Fig. 12(b), the 4, 6, and 10-ply specimens are grouped quite close together. The 10-ply laminate specimen is either not rigid enough to negate the buckling effect on maximum gross area stress or buckling has little effect on specimens thicker than four plys. The authors agree with Mr. Paris that this area of fracture studies deserves further investigation.