determines the maximum traction which can be transmitted in an EHD contact. The variation of that property with temperature and pressure will be important to contact traction behavior.

The results presented show how the transition to plastic yield influence the traction as the yielded region in the contact spreads. It was seen that the transition spreads toward the inlet region and therefore one would expect that as the transition moves into the inlet zone it may also influence the film thickness. The effect would most likely be to decrease the film thickness compared to the values predicted for the usual viscous material model. This has been studied by Wilson and Aggrawal [21] for metalworking and needs to be explored for elastohydrodynamic lubrication.

Acknowledgment

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DISCUSSION

J. L. Tevaarwerk[2]

The authors are to be congratulated with their success in attaining realistic traction data and answers from measurements independent of EHD experiments. This indicates that the interpretation of the traction curves and the relevant material parameters and constitutive equations is essentially correct. It is also very pleasing to me that the authors arrived at the same form of the constitutive equations as Johnson and Tevaarwerk (1976) did; nl. in one-dimensional form

$$\dot{\gamma} = \frac{1}{G}\frac{d\tau}{dt} + F(\tau) \tag{1}$$

where $F(\tau)$ is a suitable dissipative function that describes the experimentally observed data. The authors of this paper choose, from their independent laboratory experiments, to let

$$F(\tau) = \tau_L/\mu_0 \ln\left(\frac{1}{1 - \tau/\tau_L}\right) \tag{2}$$

Johnson and Tevaarwerk take, based upon experimental traction data;

for low pressures $F(\tau) = \tau_0/\mu_0 \sinh(\tau/\tau_0)$ (3)

for high pressures $F(\tau) = \dot{W}/\tau$

where $\dot{W} = \tau_L\dot{\gamma}$ (4)

(The latter equation, combined with the elastic component, gives rise to the so called Prandtl-Reuss equation as used in plasticity.) Equation (2) may be rewritten in the following form

$$\tau/\tau_L = 1 - \exp(-\mu_0\dot{\gamma}/\tau_L) \tag{5}$$

This equation is evaluated and plotted in Fig. 16, together with the ideally plastic-model. The figure shows that for $\mu_0\dot{\gamma}/\tau_L > 5$ the two models yield identical results. Now, for typical experiments we obtain values of $\tau_L \simeq 10^8$ Pa, $\dot{\gamma} \simeq 10^4$ s^{-1} or greater which means that if $\mu_0 > 5 \times 10^4$ Pas the ideally plastic approximation of the authors model may be used. Certainly fluids in EHL contacts are not considered to be behaving elastic until $\mu_0 > 10^6$ Pas. For practical purposes then,

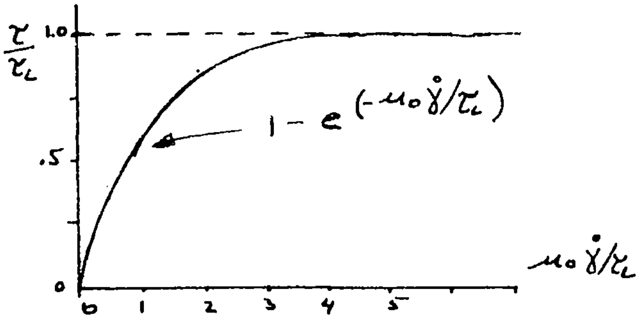

Fig. 16

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the elastic/plastic J+T model may be used with parameters derived by the new methods of Bair and Winer. This now provides a real tool to the Traction Drive designer because a closed form solution of the elastic/plastic J+T model for the traction is known for all aspect ratios (see for example Tevaarwerk 1976, Tevaarwerk 1978, and Paper 78-Lub-10 at the 1978 Lubrication Conference).

The remaining problem is now one of explaining what causes the difference between EHL shear moduli and limiting strength and those observed by the authors. For example for the best fit of the J+T elastic/plastic model (see the authors' Fig. 14) *J+T* obtain $\overline{G} \cong .10$ GPa; $\overline{\tau}_L \cong .80$ GPa while at the same conditions the above authors obtain $\overline{G} \cong .15$ GPa; $\overline{\tau}_L \cong 0.092$ GPa. Some of the differences may in fact be due to compressional heating effects, as hinted at in the paper, and also compliance enters into the picture. The other problem may be one of time delay effects.

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C. W. Allen[3]

The authors have extended their previous work on traction in the elastohydrodynamic region in an attempt to relate the traction coefficient to primary fluid data.

In reviewing the paper, however, it appears that the primary measurements are not easily obtained but require the apparatus and procedures outlined in the companion paper, ASME Paper No. 78-Lub-8. In many cases the traction data may be more easily obtained directly than by first measuring the primary data.

The equivalent viscosity versus pressure curve (Fig. 8) is somewhat similar in form to that presented by the discusser and his colleagues in reference [25]. In our model, however, the secondary part of the curve continues to increase with pressure. The authors show this secondary portion of the curve as being almost flat. Using the authors' Fig. 8 would probably result in a traction coefficient which would decrease with pressure at high pressures. This does not agree with much of the published data, for example, that given by Trachman and Cheng (reference [26]).

It would have been helpful if the authors would have published comparisons of their model with some of the experimental results mentioned in references [1, 13, 16, 17, 18, and 19]. The one comparison that is given (Fig. 14) shows fair agreement throughout the range of experimental values, but the experimental values continue to increase at the high slide/roll ratios whereas the authors' isothermal values remain constant above a slide/roll ratio of 5×10^{-3}. Presumably at the higher slide roll ratios, the heat generation would increase and result in higher temperature, therefore the authors' values corrected for temperature would appear to actually decrease.

It is hoped that the authors will extend this work, particularly in the area of obtaining primary data for a large number of lubricants and hence use these data to compare their model with the many traction experiments already published.

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Authors' Closure

The authors appreciate the discussions of Professors Tevaarwerk and Allen and believe that they contribute to the content of the paper.

We believe the difference in the G_∞ values between those measured in EHD experiments such as Tevaarwerk's and, on the other hand, those measured ultrasonically or by our techniques is mainly due to the extreme difficulty in evaluating and interpreting the results from the EHD experiments. It is well known that very small alignment difficulties, knowledge of film thickness, careful traction and speed measurements and adequate accounting for disk elastic compliance all contribute to the difficulty of inferring shear moduli from EHD measurements. I think these experimental difficulties are at the root of the difference in shear modulus values reported.

With respect to Professor Allen's comments, we have performed both traction and limiting shear stress measurements and have found the limiting shear stress to be much easier to determine. The measurement of primary material properties is not only desirable from the point of view of being able to generalize results, but also from the fact that very small samples are required. We believe that Professor Allen may have misinterpreted Figure 8. The ordinate is a log scale, and the effective viscosity nearly proportional to pressure for a given shear rate. Because of the log scale of the ordinate the small variation of effective viscosity with pressure is difficult to see. Since the presentation we have measured the limiting shear stress for three of the Johnson and Tevaarwerk fluids on samples received from them.

These measurements have indicated that the predicted maximum traction value is closer than that shown in this paper and no illusion to temperature is necessary. The limiting shear stress measurements on these fluids also very correctly ranks the fluids with respect to maximum traction coefficient.

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