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The Movement of a Continuously Growing Body of Oil on the Ground Water Table

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Rudolf Greulich and Henrik Kærgaard,

TERRAQUA ApS, Kalundborg, Denmark

Approximative formulas for the movement of a continuously growing body of oil on the ground-water table are presented and illustrated. The approximations are many, but the formulas are believed to produce realistic results for a major flow of crude oil from a leaking pipeline. The main objective is a simple hydraulic flow description, where also capillary forces at the rim are considered.

Introduction

The presented work was initiated through evaluations of a ground-water pollution incident, resulting from a continuously active leak from a crude oil pipeline across Jutland from Kærgaard Plantation to Fredericia. The "worst case" studied was defined as a leakage amount of $Q \cong 1.3$ m^3/hour, which is considered to be the maximum leak not to show on the surface after a short time under high permeability ground conditions. However, the leak will be detected by "intelligent pig"-inspections, the necessary frequency of which was the aim of the study.The hydrogeologically "worst situation" along the pipeline was defined as a high-permeability, water-table aquifer with an unsaturated zone at least 10 m above the water table, Fig. 1. An important part of the study was the evaluation of the area of the ground-water table, that would be covered with oil as a function of time. The principles in this part of the study are presented in the following.

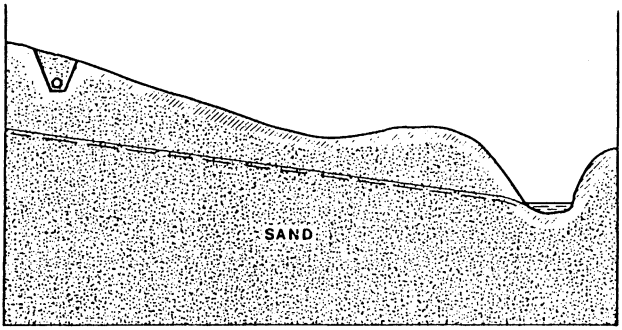

Fig. 1. Sketch, showing critical, hydrogeological situation: High permeable water-table-aquifer with a depth of about 10 m to the water table.

Physical Behaviour of the Oil Body

The oil is lighter that the water but has a higher viscosity and surface tension. A small part of the oil will go into solution in the ground water and be transported with it, although not with the same velocity as the water. The lighter fractions of the oil will gradually evaporate at the contact with the unsaturated zone, which will increase volume weight as well as viscosity of the oil slightly with time. However, inside shorter periods of time the main body of oil is supposed to be largely unaffected by these processes and move roughly in the following way:

Close to the leak the oil will move spherically outwards under practically saturated conditions. In a short distance gravity will create a downward movement and a few metres below the leak the oil will move vertically downwards inside a circular infiltration area. The downward gradient of the vertical flow will be practically 1 (m's of oil column per m) and the saturation close to 100%. The radius of the infiltration area, r_i, is determined by Darcy's law

$$V = \frac{Q}{\pi r_i^2} = K_o \times 1 \quad \text{or} \quad \pi r_i^2 = \frac{Q}{K_o} ,$$

where Q is the leakage amount (m^3/s) and K_o the hydraulic conductivity with respect to the oil (m/s), Fig. 2.

On the ground-water table the oil body will grow under normal, hydraulic conditions, displace the water according to Archimedes law and start moving outwards in a circular lens-like shape (provided the ground-water table is horizontal). This part of the movement is the one sought described mathematically in the present study.

The movement on the (horizontal) ground water will take place roughly in the following way:

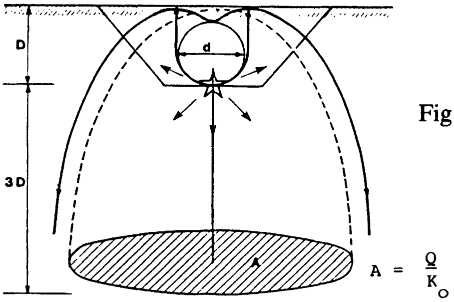

Fig. 2. Sketch, showing the flowpattern around the leak and the pipeline in a cross-section perpendicular to the pipeline. At a certain, rather small depth below the leak, the flow will take place inside a cylinder with an area of cross-section A.

In the central main body of the oil the oil layer thickness will be rather large. The driving forces of the movement here will be the pressure forces induced by the continuous infiltration of oil in the center. The capillary forces at the oil/water interface and the interface with the unsaturated zone will be unimportant apart from the fact that they will add the thicknesses of the capillary fringes to the oil body. This main body of the oil is called flow regime I.

At the rim of the oil body the horizontal capillary forces will tend to "drag" the oil further out than the main body of oil in regime I. A transition zone, where the horizontal capillary forces (blotting paper effect) gradually take the lead over the pressure forces in regime I, will be present but of small extension. In the outer regime II, where the capillary forces are governing the flow, the thicknesses of the oil will be practically constant, equal to the hight of the capillary fringe, and the oil will lie on top of the ground water without this being displaced (except from the capillary forces between oil and water). The active force will be the capillary drag at the rim which can be set equal to the head loss which is of the order of the height of the capillary fringe, h_c, measured in metres of oil column. The outward drag force is therefore constant per m circumference of the regime II body.

Both regimes I and II will continuously expand with time, but with decreasing velocity. This is evident for regime I; for regime II the decreasing velocity results from the fact that the dragging force is constant while the hydraulic resistance to be overcome by it increases with the outward expansion of the regime (the amount of fluid to be "dragged" increases).

In the following section a simple approximative, mathematical formulation of the above described process is presented.

Mathematical Formulation

The formulation takes place according to the description in the preceding section. The various symbols are defined when used; see also Fig. 3. As both regimes I and II are growing continuously

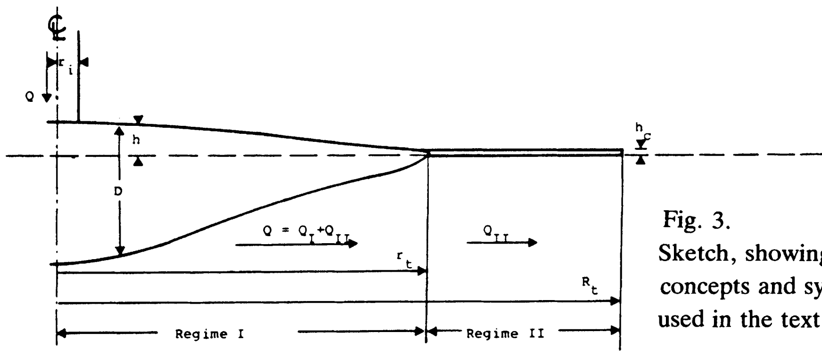

Fig. 3.
Sketch, showing some of the concepts and symbols, used in the text.

$$Q = Q_I(t) + Q_{II}(t) = \text{const},$$

where Q_I (t) and Q_{II} (t) are the volume growths per second at time t in regimes I and II, being functions of time.

Regime I

Crossection of infiltration (Fig. 2)

$$\pi r_i^2 = \frac{Q}{K_o}$$

K_o – hydraulic conductivity of oil = $\varrho g/\mu$ in m/s
ϱ and μ – the density and dynamic viscosity of the oil

When oil is spreading a little away from the infiltration area changes with time of oil-properties are ignored. Horizontal flow is presumed in the oil-body (Dupuit's assumption), the capillary fringe included. The underlying water is assumed stationary. According to Archimedes law the following relation holds

$$D\gamma_o = (D-h)\gamma_w \quad , \quad \text{or} \quad D = h\frac{\gamma_w}{\gamma_w - \gamma_o} = h\,\Delta$$

γ_o – is equal to ϱg, i.e. the volume weight of the oil. γ_w is for water
h – hydraulic potential in the oil-body, in m's of oil-column
D – thickness of the oil-body in m.

During the time interval from t to $t + dt$, a quasi steady state is assumed, with the radius of regime I equal to r_t. Darcy's law yields

$$v = -K_o\frac{dh}{dr}$$

and the continuity equation gives

$$Q = 2\pi\, r\, D(r)\, v$$

which is combined to

$$2h\frac{dh}{dr} = \frac{dh^2}{dr} = -\frac{Q}{\pi\Delta K_o r} \tag{1}$$

At $r=r_t$, the thickness of the oil-body is considered equal to the capillary fringe thickness, h_c. With this boundary condition, and D = h Δ Eq (1) can be solved for D under quasi-steady-state conditions at time t

$$D^2(r) = -\frac{\Delta Q}{\pi K_o}\ln\left(\frac{r}{r_t}\right) + h_c^2 \text{ , or}$$

$$D(r) = \sqrt{h_c^2 - \frac{\Delta Q}{\pi K_o}\ln\left(\frac{r}{r_t}\right)} \text{ , } r_i \le r \le r_t \tag{2}$$

A further equation is needed for determination of r_t. An approximative volume balance at time t is used

$$\int_0^t Q_I(t)dt \simeq n\pi r_i^2(h_u + D(r_i,t)) + n\int_{r_i}^{r_t} 2\pi r\, D(r,t)dr \tag{3}$$

The volume of the infiltration-regime in the unsaturated zone above ground-water level is incorporated in regime I. h_u is the height of the unsaturated zone, n is porosity, $D(r,t)$ is given by Eq. (2). The combination of Eq. (2) and Eq. (3) yields

$$\int_0^t Q_I(t)dt = n\pi r_i^2\left(h_u + \sqrt{h_c^2 - \frac{\Delta Q}{\pi K_o}\ln\left(\frac{r_i}{r_t}\right)}\right) + n\int_{r_i}^{r_t} 2\pi r\sqrt{h_c^2 - \frac{\Delta Q}{\pi K_o}\ln\left(\frac{r}{r_t}\right)}\, dr \tag{4}$$

If $Q_I(t)$ is assumed to be a constant part of Q, the left side reduces to $Q_I t$, approximatively.

The volume integral of the right side can be evaluated as follows

$$I = 2\pi n\int_{r_i}^{r_t} r\sqrt{h_c^2 - \frac{\Delta Q}{\pi K_o}\ln\left(\frac{r}{r_t}\right)}\, dr$$

$$h_c^2 + \frac{\Delta Q}{\pi K_o}\ln r_t = a \text{ , } \quad \frac{\Delta Q}{\pi K_o} = b \Rightarrow \frac{a}{b} = c = \frac{\pi K_o h_c^2}{\Delta Q} + \ln r_t \Rightarrow$$

$$I = 2\pi n\sqrt{b}\int_{r_i}^{r_t} r\sqrt{c - \ln r}\, dr$$

The following substitution is used

$$\sqrt{c-\ln r} = \frac{u}{\sqrt{2}}, \ln r = -\frac{u^2}{2} + c, r = e^c e^{-u^2/2}, dr = -e^c u e^{-u^2/2} du \Rightarrow$$

$$I = -\sqrt{2}\,\pi n\sqrt{b}\,e^{2c}\int_{\sqrt{2c-2\ln r_i}}^{\sqrt{2c-2\ln r_t}} u^2 e^{-u^2} du$$

The function under the integral is practically zero when u is close to zero and is ignorable too when $u \geqslant 2$. As the upper limit of the integral is close to zero

$$\frac{2\pi K_o h_c^2}{\Delta Q} \cong 0$$

the condition

$$\sqrt{2c-2\ln r_i} \simeq \sqrt{\ln\left(\frac{r_t^2}{r_i^2}\right)} > 2 \quad \text{or} \quad \frac{r_t}{r_i} > 10$$

allows the approximation

$$I \simeq \sqrt{2}\,\pi n\sqrt{b}\,e^{2c}\int_0^\infty u^2 e^{-u^2} du =$$

$$\sqrt{2}\,\pi n\sqrt{\frac{\Delta Q}{\pi K_o}}\,e^{(2\pi K_o h_c^2/\Delta Q)+2\ln r_t}\,\frac{\sqrt{\pi}}{4} = \frac{\pi n}{4}\sqrt{\frac{2\Delta Q}{K_o}}\,e^{2\pi K_o h_c^2/\Delta Q}\,r_t^2$$

Eq. (4) is now reduced to

$$Q_I t = n\pi r_i^2\left(h_u + \sqrt{h_c^2 - \frac{\Delta Q}{\pi K_o}\ln\left(\frac{r_i}{r_t}\right)}\right) + \frac{\pi n}{4}\sqrt{\frac{2\Delta Q}{K_o}}\,e^{2\pi K_o h_c^2/\Delta Q}\,r_t^2 \qquad (4a)$$

which is a relation between t and r_t in flow regime I under the assumptions and approximations made.

After a while, the first term on the right side may be ignored, resulting in

$$r_t^2 = \frac{4}{\pi n}\sqrt{\frac{K_o Q_I^2}{2\Delta Q}}\,e^{-2\pi K_o h_c^2/\Delta Q}\,t \qquad (4b)$$

Regime II

Here the thickness is constant, approximately equal to h_c, and the regime is bounded by an inner radius, r_t, and an outer radius R_t, which is the total radius of the oil-body. The driving force is the capillary drag at the rim (R_t), which is also considered equal to h_c (in m's of oil column).

Under these conditions a quasi-steady-state consideration at time t yields

$$v = -K_o \frac{dh}{dr}, \quad Q_{II}(t) = 2\pi r h_c v \Rightarrow \frac{dh}{dr} = -\frac{Q_{II}(t)}{2\pi K_o h_c}\frac{1}{r}$$

$$h = 0 \quad \text{for } r = r_t \text{ yields} \quad h = \frac{Q_{II}(t)}{2\pi K_o h_c}\ln\left(\frac{r_t}{r}\right)$$

$$h = -h_c \text{ at } r = R_t \text{ yields} \quad \ln\left(\frac{R_t}{r_t}\right) = \frac{2\pi K_o h_c^2}{Q_{II}(t)} \quad \text{or}$$

$$\frac{R_t}{r_t} = e^{2\pi K_o h_c^2/Q_{II}(t)} \tag{5}$$

This implies, that if $Q_{II}(t)$ is constant in time, the same is true for R_t/r_t.

A total volume consideration at time t for regime II yields

$$\int_t^o Q_{II}(t)dt \equiv n\,h_c\pi(R_t^2-r_t^2) \equiv n\,h_c\pi(e^{4\pi K_o h_c^2/Q_{II}(t)}-1)r_t^2 \tag{6}$$

After a while Eq (4b) is valid, $Q_I \sim Q(Q_{II}(t)$ is small) and r_t^2 will be proportional to t. The same will be true for the integral in Eq. (6) and therefore both Q_I and Q_{II} may be assumed constant, Q_{II} given by the implicit expression (Eqs. (4b) and (6))

$$Q_{II} \equiv 4h_c\sqrt{\frac{K_o Q}{2\Delta}}\; e^{-2\pi K_o h_c^2/\Delta Q}\,(e^{4\pi K_o h_c^2/Q_{II}}-1) \tag{7}$$

A computation can now take place in the following way: All the physical constants etc. are chosen. r_t-values greater than 10 r_i are chosen, and corresponding times are computed from Eq. (4a) or Eq. (4b), using $Q_I = Q$. A value of Q_{II} is computed from Eq. (7). The importance of Q_{II} with regard to the previous computations may be tested by putting $Q_I = Q\text{-}Q_{II}$ in Eq. (4b). When a final Q_{II} is obtained, R_t/r_t is computed from Eq. (5). The radial profiles of regime I may be computed from Eq. (2), and the geometry of the oil-body will then be defined.

Example Q – 1 m^3/t, $K_o = 10^{-4}$ m/s, r_i – 0.95 m
ϱ_o – 830 kg/m^3, $\Delta = 5.88$
h_c – 0.1 m, $h_u = 8$ m
n – 0.30
r_t – 10, 20, 40, 80 m
t – 5.6, 22.4, 89.8, 359 days
Q_{II} – $1.87 \times 10^{+5}$ m^3/s = 0.067 m^3/t, this is 0.067 Q

which is unimportant to the previous computations inside 5-10% of t

$$\frac{R_t}{r_t} = 1.40$$

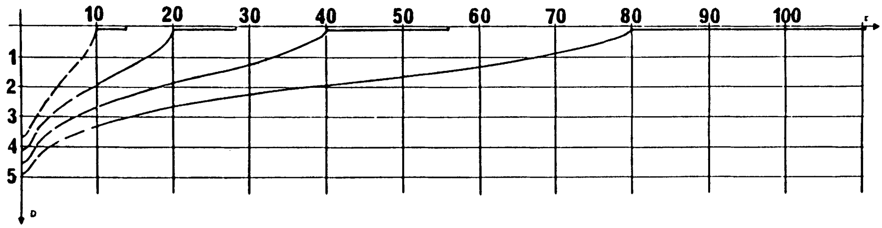

Fig. 4. Example, showing the profile of the oil body at 4 values of t (days): 5.6, 22.4, 89.8, 359, corresponding to r_t (m): 10, 10, 40, 80. For further conditions used, see the text.

Fig. 4 shows thickness of the oil-body, D, versus radius, r, for the 4 values of t and r_t listed above. Both regime I and II are shown.

Concluding Remarks

The validity of the formulas is of course dependent on the representativeness of the chosen parameters, i.e. on the presumptions made in the computations. Especially the assumption of "practically saturated flow" in the oil body may be argued. In practice the flow will not be saturated, especially not in the capillary fringe, i.e. regime II. This will reduce the actual value of K_o considerably compared to the value $\varrho_o g/\mu_o$, and thus also the spread of the oil (especially in regime II) will be reduced compared to the illustrative computations in the example. The formulas are of course still applicable with appropriate values of K_o inserted.

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Address: TERRAQUA ApS,
155A Slagelsevej,
4400 Kalundborg, Denmark.