should precede transitional boiling were seriously inhibited on the top. Since the thermal resistance of the ribbon is low, heat transfer is achieved almost entirely from the upper surface; the resulting heat flux based on both pressures is reduced by about half as is shown in Fig. 8.

The correlation equations, equations (2b) and (3b), thus appear to provide means for extending Zuber's \( q_{\text{max}} \) and \( q_{\text{min}} \) predictions to application in a variety of real situations. In the case of \( q_{\text{min}} \) they demonstrate a strong—but as yet incompletely defined—sensitivity to geometrical configuration. A more qualitative indication of the less severe effect of geometry upon \( q_{\text{max}} \) is also shown. In the future, additional correlation curves should be formed experimentally over all pressures for a larger variety of configurations, in order to provide relatively complete design information.

### References


### DISCUSSION

Y. P. Chang

This writer has found this to be an interesting paper and, in general, in agreement with the authors' connotation that the peak and minimum heat fluxes are to be expressed in terms of the same type of equation, such as equations (2b) and (3b). It is remarkable that the Cichelli-Bonilla's pioneer correlation of the peak heat flux for organic liquids has been found to be consistent with the hydrodynamic and thermodynamic consideration. However, the writer feels that certain statements and the equation for the minimum heat flux, equation (3), on which the authors' correlation was based are worthy of further discussion.

Equation (2) is practically identical with the one recommended by Kutateladze [7], because \( \rho_p/\rho_l \) can be neglected if the pressure is not too close to the critical. It predicts fairly well the pressure effect for a given system under the standard gravitational field. For different systems, the numerical coefficient is subject to change in order to take into account the surrounding fluid, surface combination, orientation, and geometry of the heater [16]. If the radius of curvature and the size of the heating surface is sufficiently large, the heater's geometry may not exert any significant effect. Equation (2b) indicates, however, a reduced-pressure dependent coefficient. This is probably owing to the fact that the Van der Waals relations, \( \frac{\rho_p}{\rho_l} = \frac{3}{RT/M} \), does not always hold true.

Equation (3) for the minimum heat flux from which the author's correlation has been made does not predict the test data, as shown in Fig. 2, no matter what value the constant coefficient may take, unless this coefficient is a function of pressure. However, equation (2) still predicts well provided that the coefficient is multiplied by a factor of 0.71, because this coefficient should depend on the system [16].

To test the general validity of equation (3) by the test data of the authors, a simple comparison of equations (2) and (3) may be made, because the validity of the type of equation (2) has been checked by many investigators, such as references [5] and [7]. Dividing equation (2) by (3) gives (for pressures not too close to the critical),

\[
\frac{q_{\text{max}}}{q_{\text{min}}} \approx C \sqrt{\frac{\rho_p}{\rho_l}}
\]

where \( C \) is a constant. Since \( \rho_p \) changes rapidly with the change of pressure, \( q_{\text{max}}/q_{\text{min}} \) is far from being a constant. The measured values of \( q_{\text{max}}/q_{\text{min}} \) in Table 1, however, are approximately constant, namely,

\[
q_{\text{max}} = 5.75 \pm 20 \text{ percent for water, benzene, methanol, and cyclohexane}
\]

The existence of an approximately constant ratio between \( q_{\text{max}} \) and \( q_{\text{min}} \) for saturated pool boiling has also been observed by Kutateladze [17].

Furthermore, if the numerical coefficient in equation (3) is determined according to the test value of \( q_{\text{min}} \) at pressures lower than one atmosphere, calculated values of \( q_{\text{min}} \) at reduced pressures larger than 0.7 will be several hundred percent larger than the peak heat flux. This is obviously not possible.

It was proposed by the writer in reference [16] that the occurrence of \( q_{\text{max}} \) and \( q_{\text{min}} \) is due to the role of critical flow of vapor bubbles away from the heater and of quasi-liquid droplets toward the heater. Here the word "critical" means that the vapor bubble or liquid droplet just starts to disintegrate in the proximity of the heater. According to this postulation, the peak heat flux was obtained as

\[
q_{\text{max}} = C_1 \sqrt{\frac{\rho_p}{\rho_l}} h_s \delta \frac{\sigma}{\mu} \left[ 1 - \exp \left( -C_2 \sqrt{V/(\beta a/\mu)} \right) \right]
\]

where \( V = [\rho_p (\rho_p - \rho_l)]/\rho_l \), \( C_1 = 0.0206 \) and \( C_2 \) and \( \beta \) are coefficients depending on the system. For pressures not too close to the critical, equation (12) can be simplified to

Numbers 16 and 17 in brackets designate References at end of the discussion.
are the functions \( f_z \) and \( f_4 \) which the authors have adopted in equations (26) and (36) in this paper. Equations (13) and (17),

\[
q_{\text{min}} = 0.0206 C_z^{1/4} \sqrt{\rho_0 \rho_f} h_{fs} V = C \sqrt{\rho_0 \rho_f} h_{fs} V
\]

(13)

In Fig. 9 is shown a schematic sketch of the condition where \( q_{\text{min}} \) may occur. By assuming that there exist a statistical-mean spacing and a statistical-mean size of the liquid droplets and by considering the droplet as a hemisphere or an equivalent sphere under equilibrium condition between the gravitational and surface tension forces, the droplet diameter can be obtained as

\[
D = \frac{K_s}{K_B} \left[ \frac{\sigma}{\rho_f (\rho_f - \rho_v)} \right]^{1/4}
\]

(14)

where \( K_s \) and \( K_B \) denote the coefficients of the surface tension and gravitational forces, respectively. If \( V_s \) is the velocity of vapor moving with respect to the droplet, the dynamic force is proportional to \( \rho_f V_s^2 D^2 \) and the surface tension force to \( \sigma D \). To be compatible with the disintegration criterion, there should be a critical Weber number, such that

\[
\rho_f V_s^2 D / \sigma = \text{We}^*
\]

(15)

Combining equations (14) and (15) gives

\[
V_s = \left( \frac{K_B}{K_s} \text{We}^* \right) \left[ \sigma (\rho_f - \rho_v) / \rho_f \right]^{1/4}
\]

(16)

Since the volume of vapor immediately next to the heater is displaced by the same volume of liquid and the lost volume and liquid during evaporation is negligibly small in comparison with the volume of the liquid droplets (at pressures not too close to the critical), \( V_s \) is essentially proportional to the vapor-flow velocity away from the heater. The minimum heat flux can then be estimated by an equation as

\[
q_{\text{min}} = K_s K_3^{1/4} \sqrt{\rho_0 \rho_f} h_{fs} V = K \sqrt{\rho_0 \rho_f} h_{fs} V
\]

(17)

where \( K = K_3 K_4 \), \( K_1 \) being a universal constant and \( K_3 \) a coefficient depending on the condition of the heater as well as the coolant. Equations (13) and (17) are identical in form, different only in the coefficients.

If the droplet is extremely small (such as at very high pressures), the viscosity of vapor is to be considered. Following the same procedure as obtaining equation (12) in reference [16] a similar equation for the minimum heat flux may be obtained as

\[
q_{\text{min}} = K_1 \rho_0 \rho_f \beta' \frac{\sigma}{\mu_2} \left[ 1 - \exp \left\{ -K_3 \sqrt{V} / (\beta' \sigma / \mu_2) \right\} \right]
\]

(18)

where \( V' = [\sigma (\rho_f - \rho_v) / \rho_f]^{1/4} \) and \( \beta' \) correspond to \( C_z \) and \( \beta \) in equation (12).

Now, if equation (17) is used for the calculation of the minimum heat fluxes, not only fairly good agreement with test results can be obtained but also the coefficient is independent on the pressure. Note that equations (13) and (17) are of the same type as are equations (26) and (36) in this paper. Equations (13) and (17), however, are simpler and more convenient for practical use.

The final question the writer thinks worthy of asking is what are the functions \( f_z \) and \( f_4 \) which the authors have adopted in their curves.

References


W. L. Owens

Several items in this paper merit comment concerning the factors governing the maximum nucleate boiling heat flux and the proposed method of correlation.

The effect of surface orientation on the maximum nucleate boiling heat flux has been extensively investigated by Morozov [18] for methyl alcohol and normal propyl alcohol over a wide range of pressures. McNab, et al. [19] reported the influence of wire diameter on the maximum nucleate boiling heat flux for water boiling at one atmosphere. The influence of height for boiling from a flat vertical surface was reported in reference [20] for water at atmospheric pressure.

In general, a more important influence than geometry on the maximum critical heat flux appears to be the method used to clean the heat-transfer surface. This was investigated and reported in reference [20] for water boiling on stainless-steel tubes and ribbons at atmospheric pressure. A 1/4-in. diameter tube "cleaned" with carbon tetrachloride gave a maximum heat flux of 341,000 BTU/hr ft² while a tube cleaned with acetone gave a value of 221,000 BTU/hr ft². A 1/4-in. diameter stainless-steel tube cleaned with a commercial detergent gave a value of 338,000 BTU/hr ft² which is consistent with the low value reported by the authors for water at atmospheric pressure. The manner in which test sections are prepared may provide one reason for the wide range of values reported in the literature for seemingly similar conditions.

The "absence of a nucleate boiling regime at low pressures" was observed and discussed by van Stralen [21].

It is this writers feeling that the authors' proposed method for correlating maximum and minimum boiling heat-transfer data is in fact that of Cichelli and Bonilla obscured by a few pseudo-reductions. That this is so may be demonstrated by dividing their \( \lambda \) values listed in Table 3 by the critical pressure.

<table>
<thead>
<tr>
<th>Substance</th>
<th>( \lambda / P_c )</th>
<th>( \times 10^{-2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Isopropanol</td>
<td>1.65</td>
<td>1.05</td>
</tr>
<tr>
<td>Methanol</td>
<td>0.83</td>
<td>0.98</td>
</tr>
<tr>
<td>Acetone</td>
<td>0.89</td>
<td>0.98</td>
</tr>
<tr>
<td>Benzene</td>
<td>1.01</td>
<td>1.01</td>
</tr>
<tr>
<td>Cyclohexane</td>
<td>1.02</td>
<td>1.02</td>
</tr>
<tr>
<td>Water</td>
<td>1.03</td>
<td>1.03</td>
</tr>
<tr>
<td>Ethanol</td>
<td>0.92</td>
<td>0.98</td>
</tr>
<tr>
<td>n-Pentane</td>
<td>0.98</td>
<td>0.98</td>
</tr>
<tr>
<td>Propane</td>
<td>1.05</td>
<td>1.05</td>
</tr>
<tr>
<td>Butanol</td>
<td>1.10</td>
<td>1.10</td>
</tr>
</tbody>
</table>

Their "different—but extremely pleasing—correlation curve" for the data of Cichelli and Bonilla shown in Figs. 5 and 6 is nothing more than Cichelli and Bonilla's correlation divided by a constant. Almost without exception an even more pleasing correlation would have been obtained had they used the critical pressure alone. By a coincidence their correlation of Kazakova's data shown in Fig. 7 is Kazakova's correlation which is shown in reference [22]. Fig. 14 of reference [23] has been mislabeled and the dotted curve represents the data of Cichelli and Bonilla.

The exact reproduction of Kazakova's correlation arises from the fact that \( q/A \) (max/(const) \( P_c \), correlates maximum nucleate boiling heat flux data and in the present case the value of \( \lambda \) for water is fortuitously the same as the maximum nucleate boiling heat flux at atmospheric pressure.

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* Numbers 18 to 23 in brackets designate References at end of the discussion.
The advantage of using the authors' \( \lambda \) values to correlate minimum and maximum boiling heat flux data would not appear to be obvious.

References

N. Zuber

The authors should be complimented for making an interesting application of thermodynamic similarity to an analysis of the hydrodynamic transitions in pool boiling. For pool boiling from horizontal plates these transitions are described by equations of the form of (2) and (3).

Fig. 2 shows a comparison of values predicted by (2) with the experimental data for methanol measured by the authors and reported in [11]. As the authors note Fig. 2 shows "the worst representation of the present \( q_{\text{max}} \) data obtained with equation (2)." The writer thought that a comparison with all of the experimental data reported in [11] would be of general interest. Figs. 10, 11, 12, 13, 14, 15, and 16 which are reproduced from Professor Lienhard's dissertation, i.e., from reference [11], show this comparison. These figures are reproduced here in order to make available in the literature a more complete comparison of values predicted by (2) with the experimental data of [11].

For comparison, on these figures the writer has plotted the dashed curves which are the values predicted by the authors correlation (i.e., by the curve faired through data) shown in Fig. 3. The

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\( ^{10} \text{The writer is indebted to the authors for giving him a copy of reference [11].} \)
authors noted that "Equation (2) failed, however, to give values of \( q_{\text{max}} \) that were consistently greater, or less than, the measured values for the several substances." Figs. 10 through 16 tend to indicate that a similar statement can be made also with respect to the correlation proposed by the authors. The agreement of the values predicted by equation (2) with the data shown in these figures is rather surprising if one keeps in mind that 1) equation (2) was derived for a horizontal plate by considering a rather idealized model, whereas 2) the experimental data of the authors were obtained with 0.020-in. dia wires, and 3) the authors' correlation is a curve fairied through the data.

The writer agrees with the authors discussion relative to the effect of wire diameter on the value of \( q_{\text{min}} \) predicted by (3). This effect was clearly shown by Breen and Westwater in a paper presented at this conference. As the authors note a correction factor which includes the inverse fourth root of the heater diameter would account for this difference between values predicted by (3) (which was derived for a horizontal plate) and the experimental data. In the problem under consideration a difference between heat-transfer rates from horizontal plates and heat-transfer rates from horizontal thin wires is not surprising. Even in ordinary heat-transfer problems (i.e., with single-phase systems) one differentiates between horizontal plates and thin wires. Mr. J. Hench from this Laboratory and the writer have conducted experiments in the film boiling regime from a large horizontal plate. The results (which will be published shortly) show that the limiting value of the wavelength predicted from the theory is in agreement with the experimental results. Similarly, in agreement with the conceptual model, the results show that bubbles tend to be released in a square pattern. The writer was informed recently by Professor Lienhard that he has re-examined the experimental data of Hoeler and Westwater (which are discussed in footnote no. 3 of the paper) and that he has also detected a rectangular pattern for the release of bubbles. Perhaps Professor Lienhard could elaborate further upon his recent investigations.

The writer would like to compliment the authors for a very interesting contribution to the literature on boiling heat transfer.

**Authors' Closure**

The authors are grateful to Drs. Chang, Owens, and Zuber for their interest in this paper.

Dr. Chang has suggested that, because Zuber's equations and the present correlations do not vary with pressure in the same way, the numerical coefficients in Zuber's equations should actually be pressure dependent functions in order for both to be correct. This is not actually the case, since many possible equations lead to the present correlating factor, \( \lambda \). As Dr. Chang has noted, equations (13) and (17), for example, also reduce to equations (2b) and (3b). Since \( \lambda \) works as a correlating factor both for flat plates and for small horizontal wires, it follows that \( \lambda \) results from a different pair of equations (albeit unknown ones) in the latter case.

It is for this reason that equations (13) and (17) could not be used in place of equations (2b) and (3b) as Dr. Chang suggests. His and other predictions are restricted to particular geometrical configurations while equations (2b) and (3b) are intrinsically flexible. Because of this flexibility, analytical expressions for \( f_b \) and \( f_h \) have not been written for the present restrictive geometry.

That the Van der Waals critical compressibility is \( \frac{1}{4} \) instead of the real-gas value of about 0.29, has no bearing upon the present correlations. Very often it is taken to be unity, for convenience of computation, in applications of the law of corresponding states.

Dr. Chang has raised some issues relating to the ratio \( q_{\text{max}}/q_{\text{min}} \) which are best discussed with the help of the exact form of his equation (11). Division of equation (2) by equation (3) gives:

\[
\frac{q_{\text{max}}}{q_{\text{min}}} = \frac{\rho_{\text{f}} + \rho_{\text{g}}}{\sqrt{\rho_{\text{f}}/\rho_{\text{g}}}}
\]  

Equation (11a) has been plotted for water in Fig. 17. The data for liquids boiling on 0.020 in. dia wires, and the result of the present correlations for this configuration, have been included.

Equations (13) and (17) predict that \( q_{\text{max}}/q_{\text{min}} \) should be constant, but they include no information about the geometry for which this will be true. The present data differ from Zuber's flat plate prediction, as Dr. Chang has observed. Rather than
being constant, however, they exhibit a 40 percent rise, even in the narrow range: 0.001 < \( p_i < 0.025 \). Berenson's flat plate data, on the other hand, show that for clean, polished surfaces:

<table>
<thead>
<tr>
<th>Substance</th>
<th>( p_i )</th>
<th>Berenson</th>
<th>Zuber</th>
<th>( \frac{q_{\text{max}}}{q_{\text{min}}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>CC4</td>
<td>0.0222</td>
<td>29.4</td>
<td>12.25</td>
<td></td>
</tr>
<tr>
<td>n-pentane</td>
<td>0.0303</td>
<td>25.6</td>
<td>10.8</td>
<td></td>
</tr>
</tbody>
</table>

Berenson's experimental \( \frac{q_{\text{max}}}{q_{\text{min}}} \) is 2.4 times as large as predicted by equation (11a) at both reduced pressures. The form of Zuber's prediction is thus supported. The factor of 2.4 arises largely because, as Berenson has shown, the constant in Zuber's \( q_{\text{min}} \) prediction is large by a factor of about two.

It is also well to note that equation (11a) predicts that the flat plate \( q_{\text{max}} \) will still exceed the flat plate \( q_{\text{min}} \) at the critical point, just as one would anticipate. The alteration of equation (11a) by an arbitrary coefficient could change this, but no such alteration—particularly not one based upon a single value of \( q_{\text{min}} \) for a slender wire—would be meaningful.

The authors would like to thank Dr. Owens for pointing to prior observations which support certain aspects of the present study (paragraphs 2 and 4 of his discussion). His own observations of a 25 percent variation in \( q_{\text{max}} \) owing to differences in cleaning techniques are surprising, however. They are in opposition, for example, to Berenson's conclusion, based upon careful experimentation with a substantial variety of surface conditions, that:

"The maximum (burnout) heat flux is, to all practical significance, independent of the surface material, roughness, and cleanliness."

The first author has made the following supplementary experiment (using apparatus similar to that described in this paper), to check the effect of surface condition:

A 0.020-in-dia nichrome wire was carefully cleaned with soap and water, rinsed in hot water, and finally cleaned with acetone. It gave \( q_{\text{max}} = 120,000 \) Btu/ft/hr in isopropanol. The wire was then coated with a film of axle grease. \( q_{\text{max}} = 132,000 \) Btu/ft/hr in this case. Finally, the same greasy wire was allowed to glow bright red for several minutes in the flat plate boiling regime. The result was a badly discolored wire which gave \( q_{\text{max}} = 145,000 \) Btu/ft/hr. This gives a net increase of 10 percent in \( q_{\text{max}} \) which compares favorably with the 15 percent increase due to dirtiness observed by Cichelli and Bonilla [see equation (1)]. These minor variations in \( q_{\text{max}} \) do not arise from differences in cleaning techniques, but from observable deteriorations of surface cleanliness.

Dr. Owens' remarks concerning the similarity between the present correlation and Cichelli and Bonilla's correlation are very misleading: His comparison simply repeats the observations made by the authors in connection with the equation (2c). His claim that the present correlations "obscure" Cichelli and Bonilla's empirical correlation would indicate that the latter has fundamental significance when it is, in fact, the result of an intelligent guess. The present analysis has made perfectly straightforward use of thermodynamic similarity to show why it was successful, and that it is successful only under restrictive conditions.

Figs. 10 through 14 which Dr. Zuber has included are of interest and are welcome additions. Figs. 15 and 16 are based upon measurements made in impure fluids [as was noted in reference (11)] and should have been excluded.

The arguments which these curves serve to implement are somewhat unnecessary, however. The paper has shown that Dr. Zuber's \( q_{\text{max}} \) gives rather good results in geometries for which no one could reasonably expect it to do so. It has also shown that the \( q_{\text{min}} \) prediction fails as one would surely expect that it should.

That the present correlation is neither consistently above nor below the data is of no relevance since it is a correlation and is necessarily fired through the data.

Finally, the first author welcomes the opportunity to change his statement (made in footnote No. 3) that the Hosier-Westwater wavelengths were random.

A careful re-examination of Fig. 7 in [10] has been made. It was reasoned that bubbles must depart in a square pattern since only this array could favor a particular wave length and permit a site to release bubbles alternately with each of its neighbors as well. Such an array will appear disorderly because of:

(a) drift in the phase angle between bubble release sites.
(b) horizontal movements of the bubbles as they rise in a turbulent field.

Fig. 18 shows the Hosier-Westwater Fig. 7 which is a plan view sketch of the bubbles on a flat plate during film boiling in water, made from a photograph. On it have been drawn the square arrays that seemed most obvious to the eye. The basic wavelength of the grid is 0.96 in.—almost exactly equal to Hosier and Westwater's average value of 0.97 in. Two separate arrays have resulted, but their border points could be related to one another. Just one point is associated with each bubble, although one kidney shaped bubble is called two bubbles. This could have been done in a few additional places. About 40 percent of the grid points in the two arrays had bubbles associated with them in the instant of the photograph; this is a reasonable proportion.

While such a study of a single photograph is not conclusive, it nevertheless serves as a strong indication that Dr. Zuber's description of flat plate film boiling is a reasonable one.