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Abstract: Plane waves and the Pochhammer-Chree solutions are both used to describe longitudinal axially symmetric wave propagation in solid cylinders. For interpreting experiments, plane waves provide a physical explanation for signals with multiple distinct arrivals. For signals without distinct arrivals the Pochhammer-Chree solutions are often used for interpretation, but the complexity of the solutions makes accurate interpretations difficult. This paper discusses some previous misinterpretations and shows how the Pochhammer-Chree solutions relate to signals with and without distinct arrivals by considering more than just the dispersion curves from the solutions.

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1. Introduction

Wave propagation in solid cylindrical rods can be described by two methods. The first method considers plane waves and ray tracing, which provide a physical explanation of the wave propagation and the arrival times of wave fronts. The second method considers the solutions to the differential equations, which provide a quantitative means to calculate the amplitudes as well as the arrival times of the wave fronts. However, the Pochhammer-Chree solutions to the differential equations are quite complex and provide little physical insight into wave propagation in solid cylinders. Interpreting experimental results in terms of these solutions can be difficult, and incorrect conclusions have been made. This paper provides clarification about the contributions of individual propagating modes in observed experimental signals in solid cylindrical waveguides for axially symmetric wave propagation.

2. Plane waves in solid cylinders

Plane waves are a simple means for predicting the arrivals of pulses in large diameter cylinders. Only longitudinal waves and shear waves are considered, so there are only two wave speeds. If the initial direction of a wave is known, then the subsequent path, amplitude, and generation of new waves can be determined at each reflection from plane wave theory (for example, Kolsky, 1954). For waves at glancing incidence, traveling parallel to the surface of the cylinder, a simple plane wave interpretation breaks down, and the amplitude of the reflected waves cannot be calculated. However, experiments have shown that a longitudinal wave at glancing incidence excites a shear wave that trails the longitudinal wave (Christie, 1955). The excited shear wave generates a new longitudinal wave at the opposite side of the cylinder, and the process is repeated. The new longitudinal waves appear as trailing pulses in experimental signals (Mason and McSkimin, 1947). Figure 1 is an example of observed trailing pulses in a solid cylinder from a pulse excitation. The time between the trailing pulses has been used to calculate the shear velocity of a material and subsequently the elastic constants with reasonable agreement

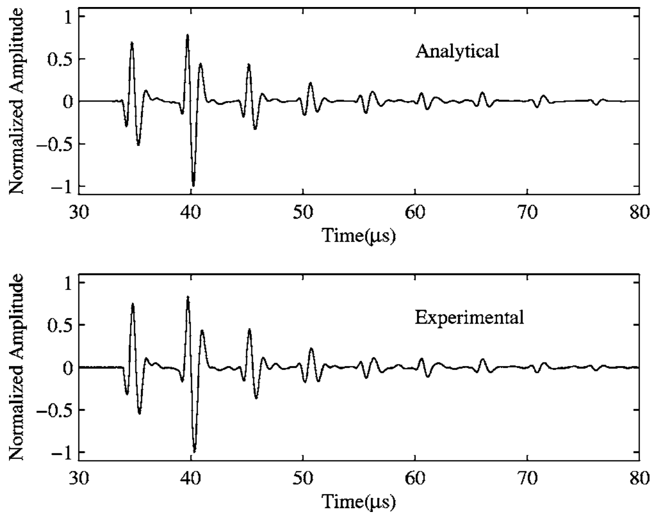

Fig. 1. Comparison of experimental and analytical signal for a 1-MHz pulse excitation propagated through a 0.2-m-long, 25-mm-diam fused quartz rod.

(Hughes *et al.*, 1949; Reynolds, 1953). However, a simple plane wave interpretation does not provide complete information for many signals in cylindrical rods, so the Pochhammer-Chree solutions are used.

3. Pochhammer-Chree solutions

The Pochhammer-Chree solutions describe axially symmetric wave propagation in an infinite solid cylindrical bar with a traction-free surface (Achenbach, 1973). These time harmonic solutions describe the wave propagation in terms of dispersive propagating modes. Dispersion curves are typically the only information about the modes used to interpret an experimental signal. However, the curves provide no amplitude information about the propagating modes, and the propagating modes themselves have little physical meaning that is identifiable in an experimental signal. Thus, misinterpretations have been made about the Pochhammer-Chree solutions.

From experimental observations of narrow-band high-frequency excitation signals in waveguides (a radius to wavelength ratio, a/λ, of the order of 10) Redwood (1959) found the first pulse in the series of trailing pulses travels with a velocity close to that of longitudinal waves in an infinite medium, c_L. If the pulse were to be described by the Pochhammer-Chree solutions, then the signal would have to be propagating in one of the higher modes. However, Redwood noticed the dispersion curves predict the arrivals of the pulses will change as the phase velocity and group velocity change with frequency, while in the experiments, the pulses arrived at the same time independent of frequency. Redwood concluded the Pochhammer-Chree solutions could predict neither the observed loss in amplitude of the main signal nor the presence of any trailing pulses and developed a modified solution.

Kolsky (1954) also found the Pochhammer-Chree solutions to be inadequate for describing the wave propagation in short large diameter waveguides. Based on the group velocity curves of the first two modes Kolsky believed that the Pochhammer-Chree solutions could not predict the arrival of waves traveling at the longitudinal wavespeed in short cylinders.

These ideas were formed by only considering the dispersion curves, which were at the time the primary source of information from the Pochhammer-Chree solutions. However, since then, there have been several advances. A complete mapping of all of the real, imaginary, and complex modes was made (Onoe *et al.*, 1962). Techniques for determining the relative ampli-

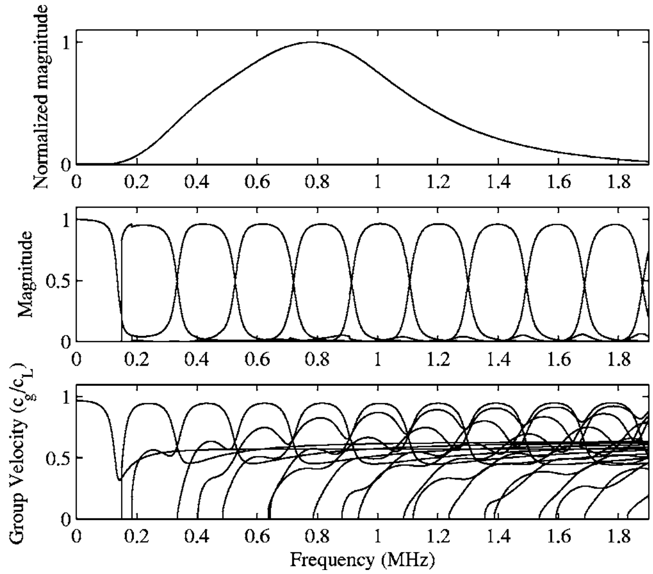

Fig. 2. Comparison of the magnitude of the frequency spectrum of the 1-MHz pulse excitation (top) with the transfer functions of the modes (middle) and the group velocity curves (bottom) for a 25-mm-diam fused quartz rod.

tudes of propagating modes have been developed (Zemanek, 1972; Gregory and Gladwell, 1989). Finally, the current availability of computing allows more information to be extracted from the Pochhammer-Chree solutions quickly and easily.

These advances combined with a phase shift from the phase velocity and the length of the rod provide the means for a semi-analytical model of wave propagation in finite solid cylindrical rods, which allows a more in-depth analysis of experimental signals (Puckett and Peterson, 2005). The linear nature of the operations in the model allows the temporal and spectral contribution of each propagating mode to be determined individually. Therefore, the contributions of individual modes in an experimental signal can be described more accurately.

4. Modal decomposition

The signals calculated using a semi-analytical model based on dispersion curves compare well to experimental signals (see Fig. 1). For a specific length bar, the semi-analytical model generates a transfer function for each propagating mode, which is an indication of the relative amplitude of the mode at each frequency. Figure 2 compares the frequency spectrum of the pulse excitation signal used for Fig. 1 (top) with the transfer functions of the modes of the cylindrical bar (middle) and the group velocity curves (bottom). In the graphs of the transfer functions and the group velocity curves, each line corresponds to a different propagating mode. The middle graph indicates the magnitude of the transfer function of each mode is largest over a small range of frequencies. In effect a mode dominates over a specific frequency range. This observation is consistent with previous findings (Zemanek, 1972; Lee *et al.*, 1995).

A closer look at the excitation signal and the transfer functions of the propagating modes reveals the relationship of the Pochhammer-Chree solutions to the observed trailing pulses. The frequency spectrum of the excitation pulse overlaps the transfer functions of multiple modes. This overlap implies that the excitation pulse excites multiple propagating modes in the bar. But how do the trailing pulses relate to the propagating modes? Qualitatively the shape of each of the trailing pulses is similar to the excitation pulse. This implies the range of the frequency spectrum of each of the trailing pulses is similar to that of the excitation pulse. Therefore, each of the trailing pulses contains energy from each of the propagating modes.

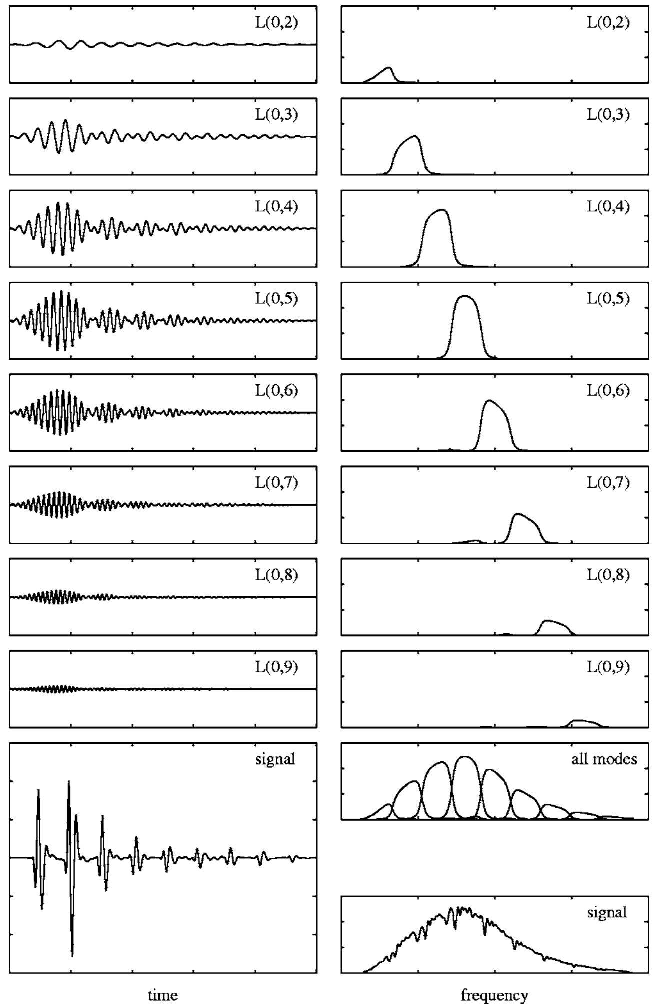

Fig. 3. Contributions of the individual modes for the analytical signal in Fig. 1. Temporal signals are on the left and spectral signals are on the right.

Additionally, each mode has a similar range of group velocities, so all of the modes should arrive at nearly the same time. The nature of the semi-analytical model allows these behaviors to be verified.

Figure 3 shows the time representation and the frequency representation of the propagating modes for the analytical signal in Fig. 1. In Fig. 3, the temporal signals appear on the left and the magnitudes of the spectral signals appear on the right. The second through the ninth modes are dominant in the frequency spectrum of the excitation signal and contribute to the dispersed signal. The bottom left graph is the summation of the temporal signals, and the bottom right graph is the magnitude of the sum of the transfer functions, which is the transfer function of the bar. The spectral signals of all of the modes are plotted together in the penultimate graph on the right.

The temporal signals in Fig. 3 confirm that each of the propagating modes contributes energy to each of the trailing pulses. It is the interference pattern of the modes that produces the observed trailing pulses. The relationship of trailing pulses to the Pochhammer-Chree solutions is more apparent, but considerably more complicated than a pulse corresponding to a single mode.

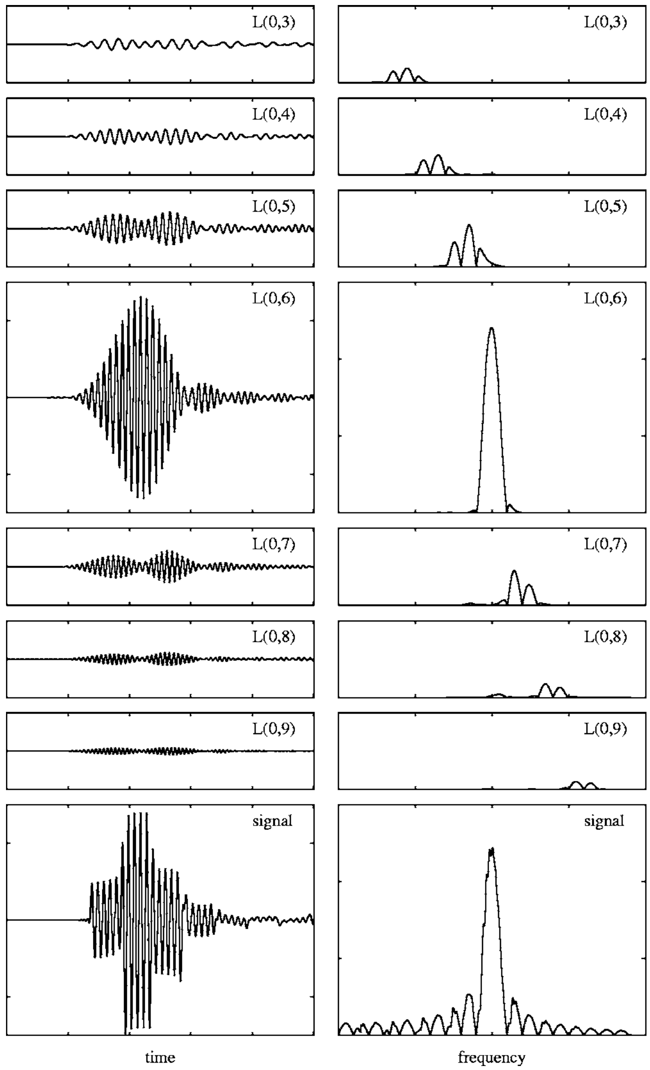

Fig. 4. Contributions of the individual modes for a 10-cycle sine burst propagated through a 0.25-m-long 25-mm-diameter quartz bar. Temporal signals are on the left and spectral signals are on the right.

Another common excitation is the tone burst or sine burst, on which Redwood based his conclusions. A similar series of figures as Fig. 3 are shown for a ten-cycle 1-MHz sine burst excitation in a 0.25-m-long, 25-mm-diam quartz bar in Fig. 4. Although more of the energy of the signal is focused at the center frequency, there is still a sizeable contribution from the modes that are dominant away from 1.0 MHz. An increase in the number of cycles will generate a narrower bandwidth signal; however, the absolute magnitude of the energy away from the center frequency does not change and multiple modes still contribute to the shape of the signal (Puckett, 2004).

5. Discussion and conclusion

A decomposition of the propagated signal into the modal components reveals that trailing pulses are a result of the superposition of multiple propagating modes. For any broadband signal many modes are excited, and each individual arrival will be the superposition of many modes. Even for some narrow-band signals several modes are excited, which contribute to the signal.

There are special cases where individual arrivals do correspond to individual propagating modes. Meitzler (1965) used a 2.5-MHz Gaussian excitation with a bandwidth of 10 kHz in a 2.12-m-long 1.44-mm-diam wire of Isoelastic alloy to demonstrate the backward wave motion of the third axially symmetric mode. For the wire, this was a sufficiently narrow-band signal to exhibit the variation of the group velocities of the first three modes over the narrow frequency range predicted by the Pochhammer-Chree solutions. In this case, although multiple modes are excited, over the narrow frequency range the modes do not have the same group velocities. As such, the pulses in the received signals correspond to individual modes. However, for the modes to be separated in time, long cylinders are required due to the large time signatures of the narrow-band signals. Meitzler's cylinder had a length to diameter ratio, L/d, of almost 1500 where as Redwood based his conclusions on bars with an L/d of less than 10. In all cases, care must be exercised when interpreting signals in terms of the Pochhammer-Chree solutions, and it is best to consider more than just the phase or group velocities.

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