Pion Production in Pion Nucleon Scattering and Pion-Pion Interaction

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The reaction $\pi+N \rightarrow 2\pi+N$ is treated under the assumption that the production process takes place mainly via $\pi-\pi$ interaction. As one of the methods for obtaining the knowledge of $\pi-\pi$ interaction, the extrapolation formula given by Chew and Low is useful. This formula for $\pi-2\pi$ process of Fig.1 is

$$\frac{d^3\sigma}{d^3\theta dw^2} \rightarrow \frac{\alpha^2}{2\pi} \frac{1}{q^2 \mu^4 (q^2 + \mu^2)^2} \times w \sqrt{\frac{w^2}{4} - \mu^2} \sigma_{ee}(w^2), \quad (1)$$

Fig. 1.
where \( \Delta \) is the 4-momentum transfer of nucleon and \( \Delta^2 = 2MT_L(T_L \text{ is the laboratory kinetic energy of recoil nucleon}) \), \( w \) is the total energy of final di-pion in their c.m. system. The Chew-Low method was applied by Anderson et al. and Bonsignori et al. but ambiguities due to the obscurity of \( \Delta^2 \)-dependence in physical region remain. Bonsignori and Selleri anticipated that Eq. (1) is also applicable even for physical process (\( \Delta^2 > 0 \)) in low \( \Delta^2 \) region and that \( \partial \sigma/\partial \Delta^2 \) should have a low \( \Delta^2 \) peak due to the factor \( \Delta^2/(\Delta^2 + \mu^2)^3 \), if the one-pion-exchange process dominates in the production process. Then it was confirmed by Derado and Rushbrooke and Radojicic that the experimental \( \partial \sigma/\partial \Delta^2 \) at 0.96 Bev has a low \( \Delta^2 \) peak. Recently, for obtaining \( \sigma \) from experimental \( \partial \sigma/\partial w^2 \) Eq. (1) was also used by Pickup et al. and Erwin et al. (Eq. (1) used in physical region is referred to as Eq. (1')). However, these applications of Eq. (1) are not appropriate even in the low \( \Delta^2 \) region for p-wave \( \pi\pi \) interaction which has strong \( \Delta^2 \) dependence. It is necessary to take into account the \( \Delta^2 \) dependence of off-the-shell \( \pi\pi \) scattering.

If only s- and p-wave \( \pi\pi \) interactions are included, \( T \) matrix elements for \( \pi\pi \) scattering part may approximately be written

\[
T(q_1, q_2; q_3, q_4) \rightarrow T_s(w^2) \quad \text{for s-wave}
\]

\[
\rightarrow (q_1 - q_2; q_3 - q_4) T_p(w^2) \quad \text{for p-wave.}
\]

(These approximations correspond to treating \( \pi\pi \) scattering by the chain approximation originated in the point interaction \( \hbar^4 \) or \( \hbar (\phi \times \phi)^4 \).) Then the contribution from off-the-shell \( \pi\pi \) scattering to the differential cross section is equal to \( \sigma_{l=\pi}(w^2) \) for s-wave, but for p-wave it is different from \( \sigma_{l=\pi}(w^2) \) by an additional factor \( \Delta^2/\Delta^2 \) in the di-pion c.m. system. Accordingly, instead of (1') we have

\[
\frac{\partial^2 \sigma}{\partial \Delta^2 \partial w^2} = \frac{\alpha f^2}{2\pi} \left( \frac{\Delta^2}{q_1 \mu^2 (\Delta^2 + \mu^2)} \right) \left( \frac{w^2 - \mu^2}{4} \right) \times \left\{ \sigma_{l=\pi}(w^2) \right\}
\]

\[
Q(\Delta^2, w^2) = \frac{(\Delta^2 + w^2 + \mu^2)^2}{4w^2} - \mu^2 / \frac{w^2}{4} - \mu^2
\]

for the s- and p-wave \( \pi\pi \) interactions respectively, where \( Q(\Delta^2, w^2) \) is the invariant form of \( \Delta^2/\Delta^2 \). Eqs. (2) and (3) are not so serious and the effect of \( Q(\Delta^2, w^2) \) may be partly damped in large \( \Delta^2 \) region on account of other \( \Delta^2 \) dependence neglected in the above approximation. However, the existence of a factor such as \( Q(\Delta^2, w^2) \) brings considerable consequences and plays an important role even in low \( \Delta^2 \) region. Compared with Eq. (1'), the effects of existence of \( Q(\Delta^2, w^2) \) are as follows:

1. The contribution from p-wave \( \pi\pi \) interaction is enhanced considerably. Even in the limited region \( \Delta^2 < 10 \mu^2 \) it is twice enhanced at 1.25 Bev incident energy.
2. The characteristic of low \( \Delta^2 \) peak due to the factor \( \Delta^2/(\Delta^2 + \mu^2)^3 \) is lost in Eq. (3). If we assume that the p-wave \( \pi\pi \) interaction contributes only through the resonance, it gives the rather flat distribution in \( \partial \sigma/\partial \Delta^2 \), or the plateau, the height and existence region of which depend on the resonance width \( \Gamma \) and the resonance energy \( \omega_r \), respectively. For example, if \( \omega_r \approx 30 \mu^2 \), the plateau extends from about \( 3 \mu^2 \) to \( 60 \mu^2 \) at 1.25 Bev, and its height is almost proportional to \( \Gamma \).

The interpretations for experimental results are changed in the following way:

(i) The low \( \Delta^2 \) peak appearing in experimental \( \partial \sigma/\partial \Delta^2 \) may be contribution from
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s-wave $\pi\pi$ interaction; (ii) If the flat part appears in $\partial\sigma/\partial d^2$, it is contribution from $p$-wave $\pi\pi$ resonance, and its height and existence region have to correspond to the width and position of a peak in $\partial\sigma/\partial\omega^2$. (iii) It is incorrect to obtain $\sigma_{-\pi}$ from $\partial\sigma/\partial\omega^2$ by using Eq. (1').

Such interpretations do not contradict the recent experiments. For example, in the report of Rushbrooke and Radojić [6] the events showing the peak at the neighbourhood of $\omega^2 \approx 22\mu^2$ in $\partial\sigma/\partial\omega^2$ have a flat distribution in $\partial\sigma/\partial d^2$. In the report of Pickup et al. [7] a plateau spreading up to about $60\mu^2 \approx 70\mu^2$ is found in $\partial\sigma/\partial d^2$ and its correspondence with the peak in $\partial\sigma/\partial\omega^2$ is consistent, if $\omega^2 \approx 30\mu^2$ and $\Gamma < 100$ Mev.

On the basis of the above considerations we give a program which gives a consistent explanation to $\pi\pi$ process ranging from low energy to high energy region (0.3 to 1.5 Bev). We take up the processes of Fig. 2 (a), (b) and (c). We expect that the contribution from $s$-wave $\pi\pi$ interaction of Fig. 2 (a) and (b) is considerably large judging from the observed steep peak at low $d^2$ in $\partial\sigma/\partial d^2$ of high energy experiments. We assume that the $p$-wave $\pi\pi$ interaction contributes only through the resonance ($\rho$ meson state) as shown in Fig. 2 (c). Its contribution to low energy process (not leading to $\rho$ meson state) is assumed to be small, because the $p$-wave $\pi\pi$ scattering length $a_1$ is expected to be considerably small ($a_1 \approx 0.05\mu^{-1}$), judging from the fact that the rather narrow $\Gamma$ is consistent with the experimental $\partial\sigma/\partial d^2$. Accordingly, in low energy region the processes (a) and (b) dominate, and the rapid rise of $\sigma$ in 300 to 500 Mev should be explained as due to the enhancement effect of final ($\frac{3}{2}, \frac{3}{2}$) state interaction of Fig. 2 (b). For the $s$-wave scattering lengths $a_0$ and $a_2$ of $I=0$ and $I=2$ di-pion states somewhat larger value than that given by Rodberg [9] and by Geobel and Schnitzer [10] should be required. (In their cases, $p$-wave contribution plays an important role in the rapid rise of $\sigma$.) The contribution from $p$-wave $\pi\pi$ resonance of Fig. 2 (c) would rise in the region 0.9 to 1.1 Bev, and would bring about increase of $\sigma$ and changes of shapes in $\partial\sigma/\partial d^2$ and $\partial\sigma/\partial\omega^2$. The consistency of the above anticipations to the observed branching ratio has to be examined. From such examinations it is expected that both $I=0$ and $I=2$ states of di-pion should contribute.

A more detailed discussions of this work will be reported subsequently.

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1) G. F. Chew and F. E. Low, Phys. Rev. 113 (1959), 1640.