Phenomenological Model for $\pi^-P$

Elastic Scattering near Resonances

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In the previous paper,¹ we have proposed the phenomenological model to treat the associated production in $\pi^-P$ collision by taking into account the known resonant states. Using the same method, we have analysed here the $\pi^-P$ elastic scattering process in the Bev region. There is a remarkable experimental feature in $\pi^-P$ elastic angular distribution in the energy range $600-1000$ Mev, that is, the forward diffraction peak and backward bump. The present paper is an investigation into whether it is possible to explain the above-mentioned feature at $900$ Mev by taking account of 2nd and 3rd $\pi^-N$-Nucleon resonances and recently established $\pi-\pi$ ($T=J=1$) resonant state.

$S$-matrix element for $\pi^-+P\rightarrow\pi^-+P$ process is given by

$$S_{fi} = \delta_{fi} - (2\pi)^4 \delta^4(p_i + q_1 - p_1 - q_1)$$

$$\times \frac{m}{\sqrt{4E_1E_2\omega_1\omega_2}}\bar{u}_f(p_2)Tu_i(p_1),$$

$$T = -A + i\frac{\gamma \cdot (q_1 + q_2)}{2}B,$$  \hspace{1cm} (1)

where we use the same notations as Bowcock and others.³

As previously done,¹ it is assumed that $A$ and $B$ can be written by

$$A^{(\alpha)}(s, t) = \frac{1}{\pi} \int_{m + \mu_a}^{\infty} \text{Im} A^{(\alpha)}_{ij}(s', t') \left[ \frac{1}{s' - s} \pm \frac{1}{s' - s_e} \right] ds'$$

$$+ \frac{1}{\pi} \int_{m}^{\infty} \text{Im} A^{(\alpha)}_{ij}(s, t') \frac{dt'}{t' - t}$$

and

$$B^{(\alpha)}(s, t) = g^2 \left( \frac{1}{m^2 - s} + \frac{1}{m^2 - s_e} \right)$$

$$+ \text{terms similar to (1), (2)}$$

where indices I, III denote elastic and $\pi+\pi\leftrightarrow N+N$ channels which we respectively call $s$ and $t$ channels. Superscript $+(-)$ refers to isospin even (odd) state. Assumed resonant states in $s$-channel are $N^*_1(D_{33})$ and $N^*_2(F_{51})$ in the $t$-channel $\pi-\pi(T=J=1)$ resonant state.

Imaginary parts of the partial wave amplitudes are written in Breit-Wigner formulas as

$$N^*; \text{Im} f'_j = -\frac{1}{4q} \left( \frac{\Gamma_1\sqrt{s-M^*} - \Gamma_1/4}{\sqrt{s-M^*}^2 + \Gamma_1/4} \right),$$

$$\pi-\pi; \text{Im} f'_{j'} = \pi N^*_1 \delta(t-M^*_1),$$  \hspace{1cm} (3)

where $\Gamma'_1 (\Gamma'_1)$ is half (partial) width of $N^*$, and $M^*$ and $M_{1s}$ are masses of $N^*$ and $\pi-\pi$ resonant state respectively.

As nonresonant states are considered to play an important role in diffraction peaks, we take account of $s$- and $p$-wave nonresonant amplitudes in $s$-channel according to the following formula:
\[
\Im f_2^{(\text{nonres.})} = -\frac{2}{2J+1} \frac{q}{4\pi} \sigma_2^{(\text{nonres.})},
\]

where \(\sigma_2^{(\text{nonres.})}\) is partial wave \(\pi^-P\) total cross section. In estimating \(\sigma_2^{(\text{nonres.})}\), we assumed that the nonresonant total cross section consisted of only \(p(2J+1)\) and \(s\)-waves, and took the values \(\sim 15\) mb for \(T = \frac{3}{2}\) state and \(\sim 25\) mb for \(T = \frac{1}{2}\) state. *)

Using the parameters \(\Gamma_1(D) = \Gamma_1(F) = 50\) Mev, \(\Gamma_1 = 100\) Mev, \(g^2/4\pi\) (renormalized \(ps-ps\) coupling constant) = 3, and \(t\)-channel parameters that are used in Bowcock's paper,*) we can calculate angular distribution and polarization of recoil proton at laboratory pion kinetic energy \(T = 900\) Mev that are shown in Figs. 1 and 2.

From the above analysis we can obtain the following conclusions:

1. A rather small coupling constant is necessary to obtain reasonable order of magnitude of cross section.
2. Backward bump in angular distribution is reproduced in our model.
3. In diffraction peak, nonresonant amplitudes play an essential role because the peak disappears without them.
4. Polarization of recoil nucleon is large and positive** (\(P \sim 50\%\)).

In contrast with the associated production process, large polarization is obtained in elastic scattering. This is due to the fact that in the latter process, the \(t\)-channel (\(\pi\pi\) resonance) contribution is relatively large so that the strong angular dependence caused by higher angular momentum states in \(s\)-channel is weakened.

Small coupling constant may suggest that in our procedures the higher order effects are not fully taken into account. And these situations might be improved by taking into account the subtractions in our dispersion relation (2).

As far as \(d\sigma/d\Omega\) and \(\sigma\) are concerned, aside from the small coupling constant, our model seems to give the consistent and unified phenomenological description of both associated production and elastic scattering processes.

*) \(\sigma_{\frac{3}{2}}^{p} \) and \(\sigma_{\frac{1}{2}}^{p} \) are estimated by assuming the relation \(\sigma_{\frac{3}{2}}^{p} = \frac{2J+1}{2(2J+1)} \sigma_{\text{tot}}^{(\text{nonres.})}\).

**) The polarization is positive if the spin is parallel to \(q_1 \times q_2\), and negative if anti-parallel, where \(q_1\) and \(q_2\) are 3-momenta of initial and final pions respectively.
Though it is not clear whether the different behaviors of $P(\theta)$ in both processes are significant or not, we have a hope that $P(\theta)$ may, in future, give some insights to the isobaric consideration of strong interactions. Measurement of $P(\theta)$ is desired in near future.

For completeness, similar investigations for pion photoproduction and kaon photoproduction processes are now in progress.

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