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A Numerical Model of Aquifer Tests in Multi-Layered Aquifer/Aquitard Systems

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Numerical model is presented, capable of computing drawdowns anywhere in a multiaquifer/aquitard system, resulting from pumping from a single well, screened in arbitrarily chosen aquifer(s) in the system. The model is of radial, symmetric type, in the sense that inhomogeneities in horizontal directions cannot be taken into account. The model was created primarily to compute the effects of any kind of stratification in an aquifer system on the results of pumping tests.

The computing principles are described, and drawdowns in a hypothetical system are computed to be compared with standard aquifer test formulas in order to illustrate the behaviour of the system.

The model is finally used for interpretation of two earlier reported aquifer tests from localities in Denmark.

Introduction

During dr. Mucha's visit in Denmark in the winter 1979-1980, a further treatment of some lately published work by Kærgaard on long-term behaviour of pumped geohydrologic systems (H. Kærgaard 1978) was carried out. A multilayer-model was formulated and computerized by dr. Mucha, with the purpose of describing short- as well as long-term behaviour of complexly stratified aquifer systems during pumping. Also the upper boundary condition of a free water table, being responsible for the occurrence of the leakage-steady-state, was taken into account.

Some typical Danish aquifer systems were theoretically test-pumped on the model and compared with aquifer test in Denmark. Hereby, the model proved its usefulness as interpretation tool in cases with only minor horizontal inhomogeneities. It is also recommendable for construction of typecurves for complicated systems, where none of the available standard models are sufficient. Some of these standard-models for leaky aquifer systems are briefly reviewed in the following.

Review for Formulas for Interpretation of Pumping Test Data in Leaky Aquifers

a) Leaky Artesian Aquifer without Storage in Aquitard

The equation for groundwater flow towards a well in a leaky aquifer when neglecting the storage in the aquitard is: (Hantush and Jacob 1955)

$$s = \frac{Q}{4\pi T} W\left(u, \frac{r}{B}\right) \tag{1}$$

where

$$B = \sqrt{\frac{T}{k'/b'}} \quad , \quad u = \frac{r^2 S}{4Tt} \tag{2}$$

s – drawdown at distance r from pumping well time t after start of pumping
Q – pumping discharge
T – transmissivity of aquifer
S – storage of aquifer
k' – hydraulic conductivity of aquitard
b' – thickness of aquitard.

Under steady state conditions, when the time-drawdown data follow the horizontal parts of the family of type curves constructed from Eq. (1), indicating that the discharge is balanced by leakage from the water table, the cone of drawdown is stationary and described by

$$s_{st} = \frac{Q}{2\pi T} K_0\left(\frac{r}{B}\right) \tag{3}$$

where K_0 is the modified Bessel function of the second kind and of zero order.

When the pumping test data are plotted on semilogarithmic paper the gradient i of the curve at the point of inflection determines the coefficient of transmissivity

$$T = \frac{0.1832\,Q}{i} \exp\left(-\frac{r}{B}\right) \tag{4}$$

b) Leaky Artesian Aquifer with Release of Water from Storage in Aquitard

If the aquitard is too thick and its storage cannot be neglected two different situations may occur

1) above the aquitard there is a zone from which the water can leak (a sufficiently permeable aquifer or a water table with sufficient storage),
2) above the aquitard there is an aquiclude. For the first, short period of pumping in both cases Hantush (1955) introduces an equation

$$s = \frac{Q}{4\pi T} H(u,\beta) \tag{5}$$

where

$$\beta = \frac{r}{4B}\sqrt{\frac{S'}{S}} = \frac{r}{4}\sqrt{\frac{S'k'}{T\,S\,b'}} = \frac{r}{4}\sqrt{\frac{S'_s\,k'}{T\,S}} \tag{6}$$

S'_s is the specific storage of the aquitard and the other parameters are previously defined. For long term pumping in case 1) some formulas are given by Boulton (1955) (Water table aquitard) and Neumann and Witherspoon (1969).

c) Water Table Conditions

One of the complications in unconfined aquifers is the "double storage", i.e. storage in both the elastic aquifer/aquitard, and the specific yield which is the storage resulting from the gravity drainage at the water table. To understand the effect of these two types of storage mechanisms it is useful to realize that the behaviour of piezometric heads during pumping from a water table aquifer initially conforms to that of the confined aquifer. Then it shows some of the characteristics of a leaky aquifer through the retardation of the drawdowns as the pumped water is gradually supplied from the specific yield at the water table. Hereafter follows gradual increase in the drawdowns with delay in gravity drainage. Once gravity drainage is efficient it behaves as a confined aquifer, but with storage coefficient equal to the specific yield. As the gravity drainage at the water table also occurs in the confined case after sufficiently long periods there are no major differences in the principles of function of confined and unconfined aquifers.

Principles of Numerical Simulation of Groundwater Flow towards a Well Using Finite Difference Analogs

Modelling of the groundwater flow towards a well is based on the equation of flow, Darcy's law, and the conservation principle, expressed as a water balance (continuity-) equation for each discretized block. The set of these equations for each block constitutes the finite difference analog.

Discretization of the equations is performed using the so-called backward difference implicit method. The result of such discretization is a system of equations which in cylindrical coordinates creates a five-diagonal matrix. This matrix is solved by the direct method (Mondkar and Powell 1974).

Discretization is carried out in the vertical direction by considering the geological stratification and the inflow interval of the well screen. Discretization in the horizontal direction is performed by cylindrical surfaces the axes of which are identical with the axes of the well.

When discretizing the aquifer in the horizontal direction the increase of radii of cylindrical surfaces (according to Stalmann – ex Luckner and Schestakow 1975) might be selected as

$$\rho_i = 1.78\,\rho_{i-1} \tag{8}$$

Node points of the finite difference grid (sometimes called the computing net) are placed in the middle of the blocks bordered by the cylindrical surfaces and are moved forward by the distance

$$\Delta r_i = \frac{(\rho_{i+1}-\rho_i)^2}{6(\rho_{i+1}+\rho_i)} \tag{9}$$

Transmissivity factors in the vertical direction are calculated in the following way

$$TFZ_{i,j} = \frac{k_{z,j,j+1}\,\Pi(\rho_{i+1}^2-\rho^2)}{\Delta Z_j} \tag{10}$$

where k_z is the vertical permeability in the discretized strata j.

Transmissivity factors in the horizontal directions are expressed by

$$TFH_{i,j} = \frac{2\Pi\, k_{h,i,i+1,j}\,\Delta b_j}{\ln\,(r_{i+1}/r_i)} \tag{11}$$

where Δb_j is the thickness of the discretized strata.

Storage factors are estimated as

$$SF_{i,j} = S_s\,\Pi(\rho_{i+1}^2-\rho_i^2)\,\Delta b_j$$

and for the upper discretized strata ($j = 1$) with the water table condition

$$SF_{i,1} = S_y\,\Pi(\rho_{i+1}^2-\rho_i^2)$$

where S_s and S_y are specific storage coefficients and specific yield.

Example of Modelling a Pumping Test in a Leaky Aquifer System

A typical Danish geohydrological system may consist of two aquifers, each closed between aquitards and with water table condition in the upper aquifer or aquitard. Such a fictitious modelled system with discretization in z and r is represented in Fig. 1. A pumping test was modelled with a constant discharge of 25 l/s pumped from the lower aquifer from a well of radius 0.1 m.

Table 1 – Input data used in Fig. 1.

Geol. description	thick-ness (m)	No. of grid points	coef. conductivity horizont. (m/s)	vert. (m/s)	Specific storage (l/m)	T (m^2/s)	Q (m^3/s)
Moraine till aquitard II	28	7	1×10^{-8}	1×10^{-8}	water table $S_y = 0.2$ $S_s = 5\times10^{-5}$		
sand aquifer II	12	3	1×10^{-4}	1×10^{-4}	5×10^{-5}	1.2×10^{-3}	
moraine till aquitard I	20	5	1×10^{-8}	1×10^{-8}	5×10^{-5}		
limestone/ chalk aquifer I	12	3	1×10^{-4}	1×10^{-4}	5×10^{-5}	1.2×10^{-3}	0.025
			non permeable bed				

Coefficient B from Eq. (2)
a) including aquitard I only B = 1,549 m
b) including aquitard I and II B = 2,400 m

Discussion of the Modelled Aquifer Test

The first step of evaluation of the modelled pumping test was to plot the output data of drawdown in points (1.18), (10.18), (18.18) and of the watertable (1.1) in a semilogarithmic scale, Fig. 2.

3 years after the start of pumping the horizontal part of the curves indicates that the discharge is balanced by leakage from the water table. This situation was used for computation of T and B values using Eq. (3), graphical method (Fig. 3). Resulting values $T = 1.243\times10^{-3}$ and B = 2,450 m show (consult Table 1) that in the period of stabilization of drawdowns due to influence of water table the

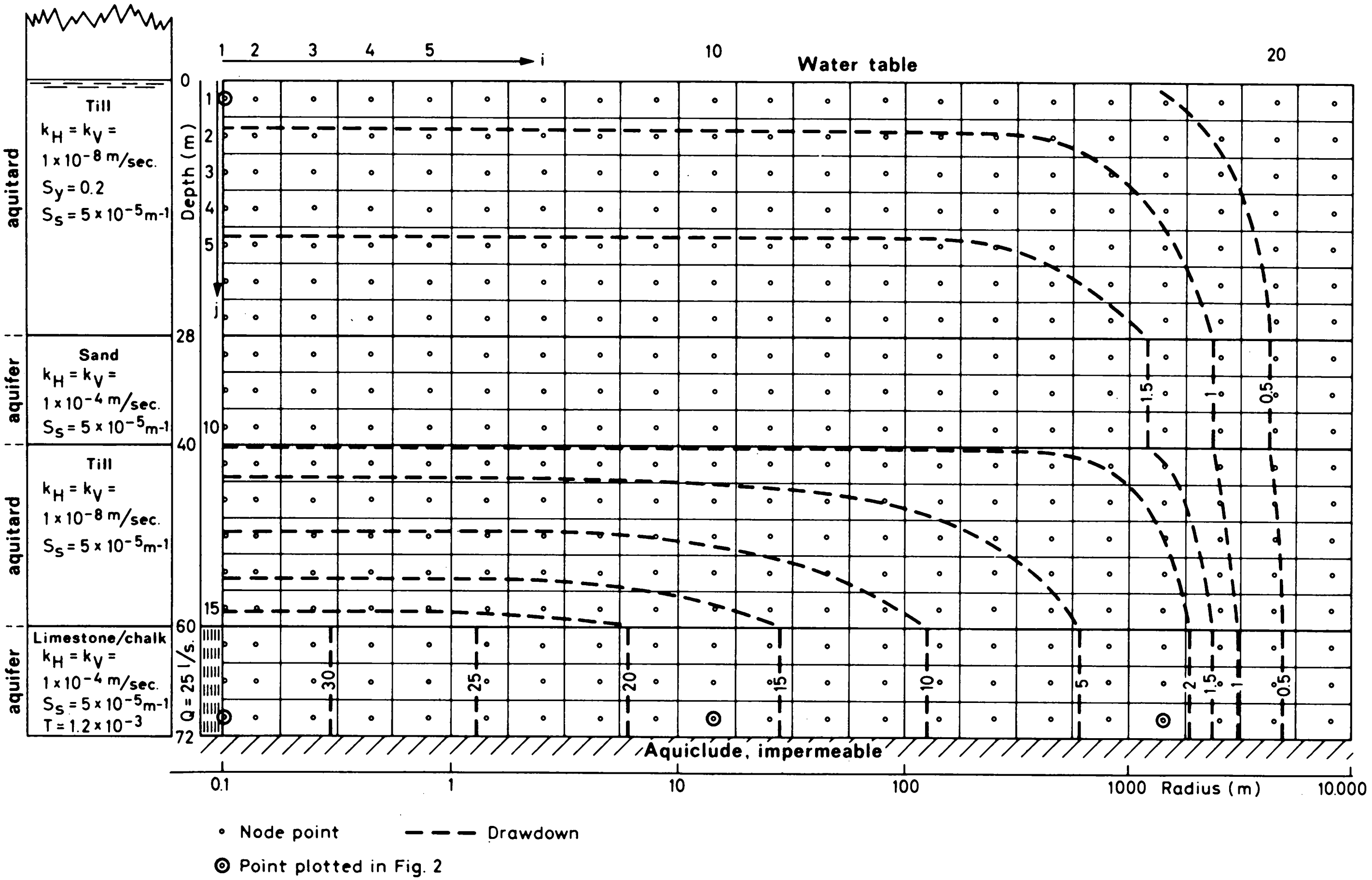

Fig. 1. Discretised geohydrologic system with radial symmetry.

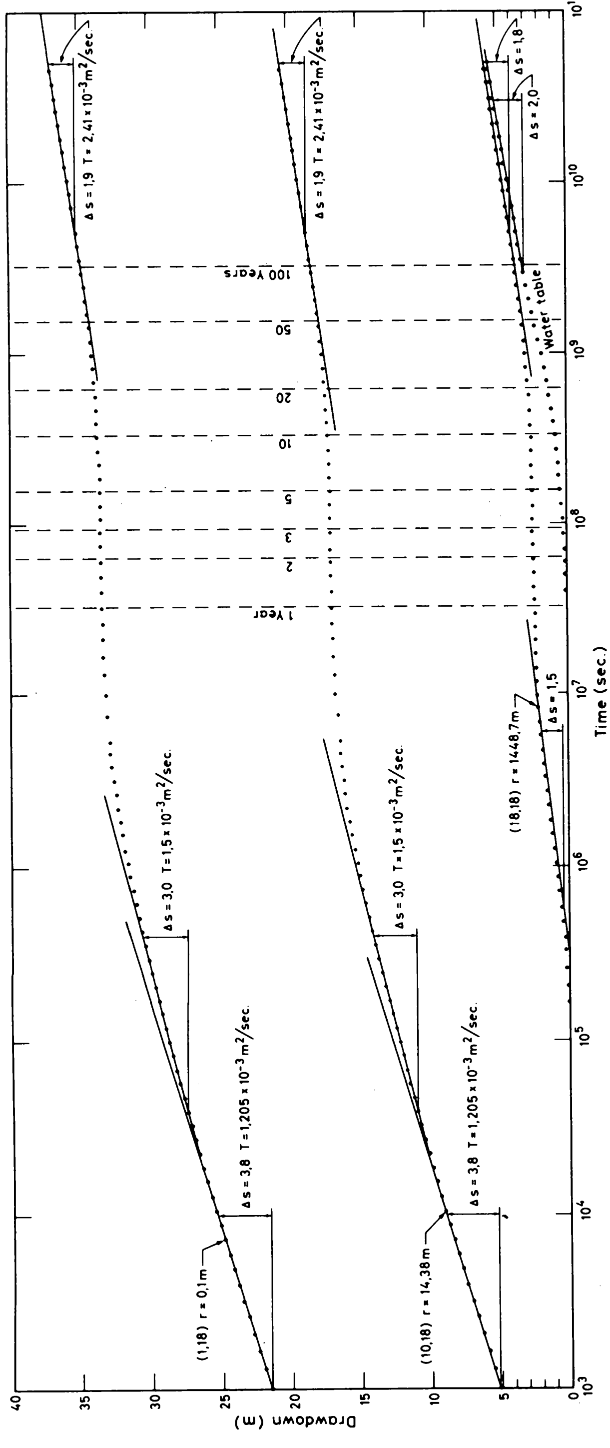

Fig. 2. Semilogarithmic plots of drawdown versus time in points (1,18), (10,18), (18,18) and (1,1) in the system in Fig. 1.

resulting value of T represents the pumped aquifer (I) and the value of B represents the whole system of aquitards above the pumped aquifer.

Evaluation of the semilogarithmic plotted drawdowns in aquifer I (Fig. 2.) using Eq. (4) showed that in the first straight line interval the resulting value of T corresponds to that of the pumped aquifer.

The first deviation from this straight line is due to the leakage from the aquitard and the second aquifer and seems to be a straight line, too, giving the value of $T = 1.5 \times 10^{-3}$ m²/s. Evaluation of this straight line using Eq. (4) or supposing some boundary condition will be erroneous.

After the retardation follows a gradual increase in drawdown due to gravity drainage at the water table. Coefficient of transmissivity computed from the corresponding (last) straight line has a value $T = 2.41 \times 10^{-3}$. This means that the resulting value equals the transmissivity of all aquifers (aquifers I and II) in the system influenced by the pumping test.

Computation (Eq. (4)) of T in the point (18.18) where $r = 1{,}448.7$ m using $B = 1{,}549$ gives the result $T = 1.2 \times 10^{-3}$ m²/s.

This indicates that T-values from wells situated a long distance from the pumped well in the pumped aquifer correspond to values of pumped aquifer and the first aquitard. Using B values from retardation part of pumping test (Eq. (3), Fig. 3) may create an error.

Similarly data from modelled pumping test using Theis' type curve method were evaluated (Eq. (1), Fig. 4). In Fig. 4 the Theis type curve and type curves related to the r/B parameter (Eq. (1)) where $B = 2{,}400$ m (both aquitards) are shown. The type curves are placed so that the values of T and S are $T = 1.2 \times 10^{-3}$ m²/s and $S = 6 \times 10^{-4}$ (aquifer I). For the observations in a long distance from the pumping well type curves related to Eq. (5) are shown.

From Fig. 4 it may be seen that when the distances of observation wells are small, the drawdown data in the beginning follow the Theis type curve and later the other type curves involving the leakage. For large distances the late-time drawdowns correspond to the r/B-curves but the drawdowns of early times follow more closely the type curves, Eq. (5). It can be seen that data from distant observation wells cannot be interpreted by a family of type curves based on Eq. (1).

We shall not discuss these problems into deeper details, but leave this to the reader. However, we want to point out the possibility of controlling interpretations of pumping tests by simulations of the tests to confirm the conclusions about the hydrogeological conditions and maybe improve the interpretations. Of course the model presented is possible to use in many other ways. Construction of type curves in multi-stratified systems is an example. Also the model will be able to yield the drawdowns in aquifer II resulting from pumping from aquifer I (or vice versa) in the system on Fig. 1. This could be useful in cases where observation wells in non-pumped zones of the systems are available.

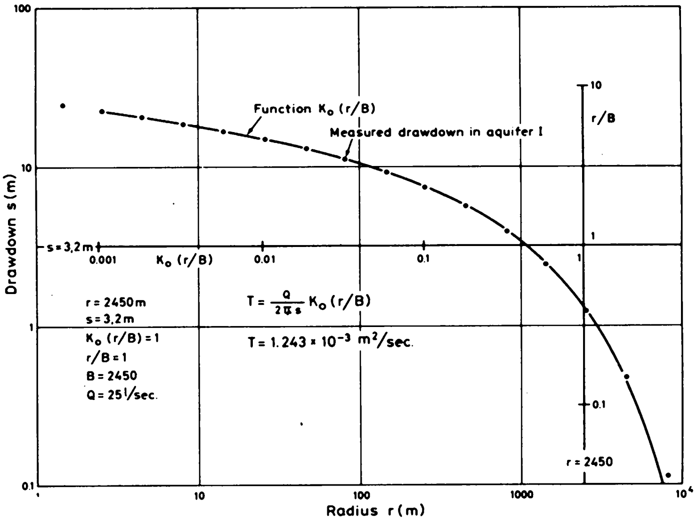

Fig. 3. Distance-drawdown plot (logarithmic) in the leakage steady state after 3 years of pumping (aquifer I).

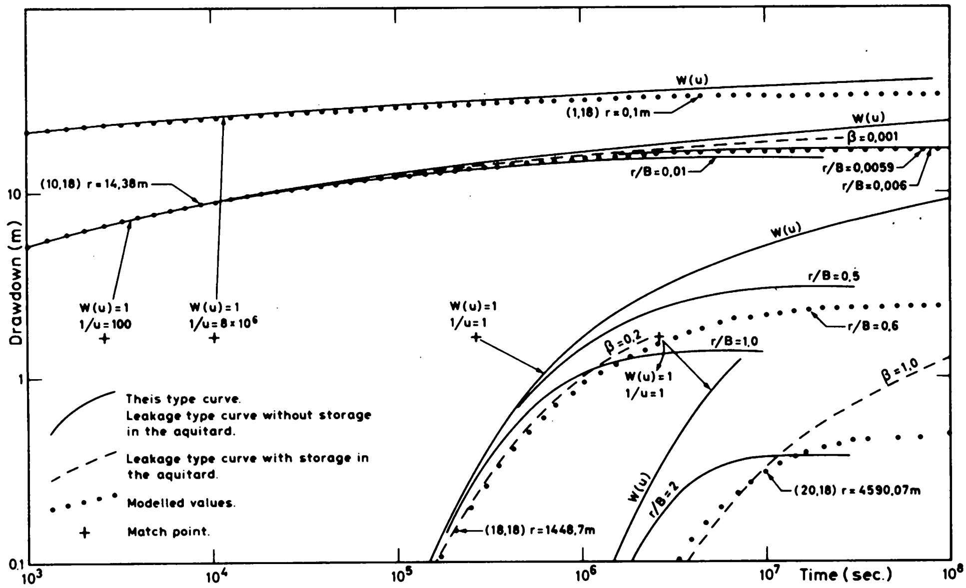

Fig. 4. Logarithmic plot of drawdowns in points (1,18), (10,18), (18,18) and (20,18) in the system in Fig. 1.
Type curves according to Theis' formula and Eqs. (1) and (5) are shown for comparison.

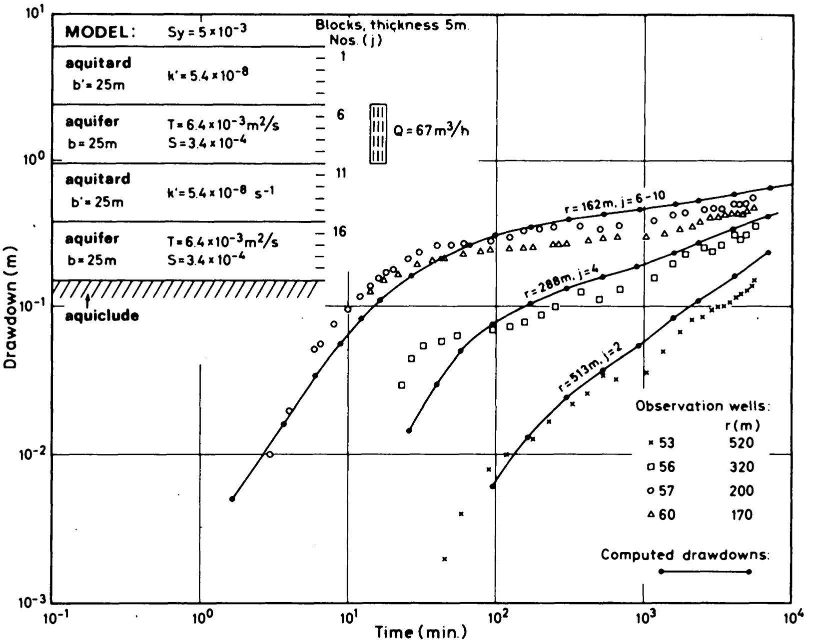

Fig. 5. Observed and modelled drawdowns versus time during pumping test on well 61, Horsens Municipal Water Supply, Egebjerg Waterworks. Eastern Jutland, Denmark. Test reported by Andersen and Haman (1970).

Examples of Interpretation of Aquifer Tests

In the following two previously published aquifer tests in Danish localities are simulated by use of the model.

1) Aquifer test on well 61, Horsens Municipal Water Supply, Egebjerg Waterworks, Denmark. The test was reported by Andersen and Haman (1970), and took place in a system with (at least) two aquifers. Pumping took place from the upper aquifer, all aquifers consisting of diluvial sands, aquitards of drift clay. The bottom of the system is tertiary clay with very low permeability. The representation of the system in Fig. 5 is considerably idealized, but is reasonably fit for the purpose.

   The simulation has proved to be fairly good though minor changes of parameters might improve the fit. In the direction of wells 56 and 53 the upper aquifer becomes gradually a water table aquifer, which cannot be accounted for in the model. However, the effect is qualitatively achieved by choosing node points closer to the water table (that is in the upper aquitard) for comparison with the data.

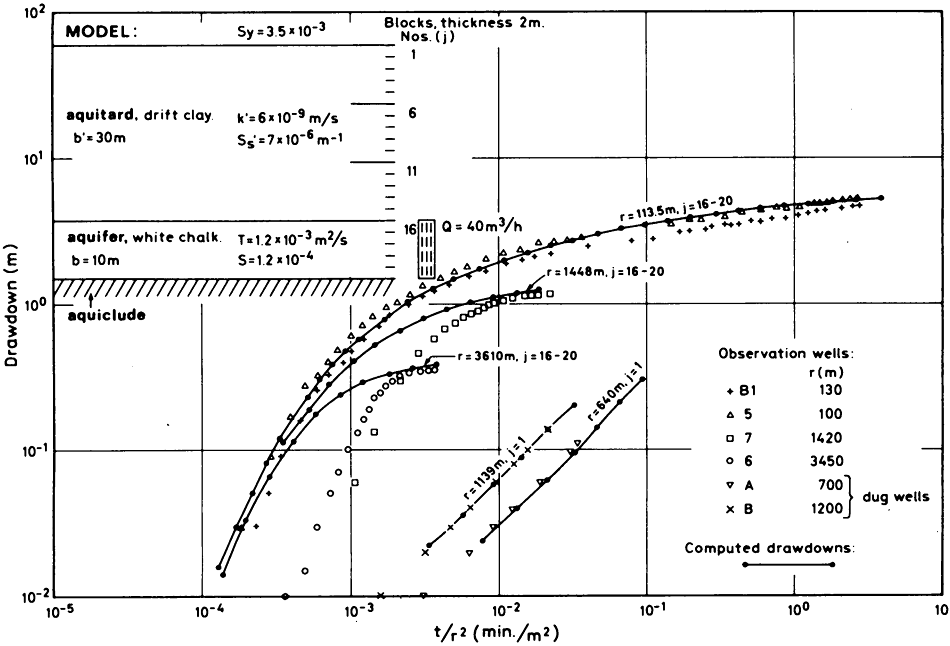

Fig. 6. Observed and modelled drawdowns versus t/r^2 during pumping test on well 3, Torkildstrup-Lillebrænde Waterworks, Falster, Denmark. Test reported by Kærgaard (1980).

2) Aquifer test on well 3, Torkildstrup-Lillebrænde Waterworks, Falster, Denmark. The test was reported by Kærgaard (1980). The system consists of an artesian white chalk aquifer (secondary permeability by glacial tectonics) overlain by a water table aquitard (drift clay). The bottom is solid chalk with very small permeability and storage.

The data from the nearest observation wells in the aquifer are simulated well by the model. The same goes for the water table wells *A* and *B* (dug wells). The more distant wells react somewhat later than the corresponding nodes in the model, but the fit is reasonable towards the leakage steady state. The late reactions are probably due to a combined effect of variations in aquitard storage and inhomogeneities in the aquifer.

Conclusions

The model has proved to be very useful for the study of drawdown evolutions during aquifer tests in stratified geohydrologic systems. Especially it has been able to reveal the limitations in the use of standard aquifer test formulas in such systems.

In interpretation work the use of the model is limited by the fact that horizontal inhomogeneities cannot be accounted for.

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