effect the integration indicated in Equation [14]. However, a useful lower bound can be obtained from the following inequality, reference (7)

$$\frac{1}{A}\int_A \log(1-\alpha\hat{p})\,dA \leqslant \log\left[\frac{1}{A}\int(1-\alpha\hat{p})\,dA\right]\quad[15]$$

This may be written

$$\frac{1}{A}\int_A \log(1-\alpha\hat{p})\,dA \leqslant \log\left[\frac{1}{A}(A-\alpha\hat{W})\right]\quad[16]$$

where $\hat{W} = \int \hat{p}\,dA$ and corresponds to the total load for the case of constant viscosity. Thus

$$W \geqslant -\frac{A}{\alpha}\ln\left(1-\alpha\frac{\hat{W}}{A}\right)\quad[17]$$

The inequality of the type used in Equation [15] is true for the general class of convex functions rather than merely the logarithm function so that any slider-bearing problem having a convex function for $p(\hat{p})$ may be subjected to this treatment. As we have indicated, this is always the case for physically plausible variations of viscosity with pressure.

Other Design Factors

The other design factors such as center of pressure, coefficient of friction, and oil flow may be set up quite easily in terms of the true pressure. The evaluation of the integrals involved, however, is best done by any of the standard numerical methods.

Although the details will not be presented here, the procedure will be exemplified by the calculation for the center of pressure $\bar{x}$. This is given by

$$\bar{x} = \frac{\int\int xp\,dx\,dy}{W}\quad[18]$$

If one substitutes for p its value in terms of $\hat{p}$ in the numerator, it is easy to see how a numerical evaluation may be effected. The denominator which is the total load has been discussed already.

An interesting qualitative result may be obtained regarding the center of pressure. In Fig. 5, there is represented schematically

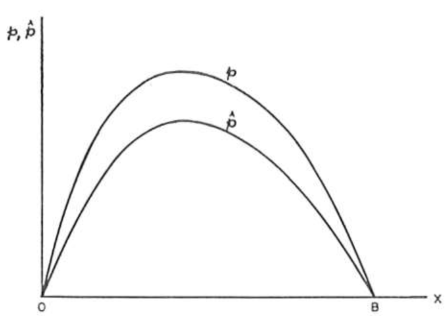

Fig. 5

a section of the slider bearing in the x, z-plane. The distributions of p and $\hat{p}$ are shown. In accordance with Equation [12] $p \geqslant \hat{p}$ and as pointed out herein, the maxima occur at the same value of x. It has been shown by the authors that the effect of increasing the ordinate of the $\hat{p}$-curve by a factor which increases with an increase of $\hat{p}$ is to displace the centroid of the curve toward the position of the maximum ordinate. The proof of this as well as other properties of these curves will appear elsewhere.

Numerical Example

The following example is given by Shaw and Macks (8) for a slider bearing:

B = 2 in.
L = 2 in.
Max film thickness = 0.00059 in.
h_0 = min film thickness = 0.0003 in.
U = 320 ips
$\mu_0 = 5 \times 10^{-6}$ reyn (SAE 20 oil)
Mean bearing temperature = 120 F

Considering viscosity and temperature constant, they find the pressure at the center to be about 8000 psi and a total load of 22,500 lb. This is based on the assumption of no side leakage, and constant viscosity. Considering the slider as finite and the viscosity constant, the Michell solution (4) gives 4730 psi for the center pressure and 9810 lb for the total load. The solution of Charnes and Saibel (4) gives 5050 psi for the pressure and 9650 lb for the total load. According to the work of Hersey and Shore (5) and Dow (9) a medium oil having a viscosity of 5 microreyns at 120 F has a value of α of about 15×10^{-5} in²/lb. Using this value together with the foregoing data, the pressure at the center is calculated from Equation [10] using the value of $\hat{p}$ from reference (4)

$$p = \frac{1}{\alpha}\ln\frac{1}{1-0.142\,\frac{\alpha\mu_0 UB}{h_0^2}}$$

and is equal to 9450 psi when the variation of viscosity is taken into account.

The total load calculated from Equation [17] gives

$$W \geqslant 12{,}000 \text{ lb}$$

BIBLIOGRAPHY

1 "Studies in Lubrication IX," by M. Muskat and H. H. Evinger, *Journal of Applied Physics*, vol. 11, 1940, pp. 739–748.

2 "Theory of Lubrication," by M. D. Hersey, John Wiley & Sons, Inc., New York, N. Y., 1938.

3 "The Theory of Rate Processes," by S. Glasstone, K. Laidler, and H. Eyring, McGraw-Hill Book Company, Inc., New York, N. Y., 1941.

4 "On the Solution of the Reynolds Equation for Slider-Bearing Lubrication," by A. Charnes and E. Saibel, ASME Preprint No. 51—A-43, presented at the Annual Meeting, Atlantic City, N. J., November 25–30, 1951, of The American Society of Mechanical Engineers.

5 "Viscosity of Lubricants Under Pressure," by M. D. Hersey and H. Shore, *Mechanical Engineering*, vol. 50, 1928, pp. 221–232.

6 "Influence of Pressure on Film Viscosity in Heavily Loaded Bearings," by S. J. Needs, Trans. ASME, vol. 60, 1938, p. 347.

7 "Inequalities," by G. H. Hardy, J. E. Littlewood, and G. Polya, Cambridge University Press, London, England, 1934.

8 "Analysis and Lubrication of Bearings," by M. C. Shaw and E. F. Macks, McGraw-Hill Book Company, Inc., New York, N. Y., 1949.

9 "The Effects of Pressure and Temperature on the Viscosity of Lubricating Oils," by R. B. Dow, *Journal of Applied Physics*, vol. 8, 1937, pp. 367–372.

Discussion

By H. Blok.[5] The present paper is thought to be very useful for obtaining a coherent view into the field concerned; this is particularly true if this paper is read in conjunction with the papers to be mentioned in the footnotes.[6,7,8,9]

[5] Professor of Mechanical Engineering, Technical University, Delft, Holland.

The basic transformation indicated by Equation [2] seems to date back to 1925, when it was used by Prof. W. Weibull.[6] It would appear, that, because it is in Swedish, this paper is less known than it deserves; this may explain why repeatedly the transformation has later been arrived at independently by others.

A short abstract of, and some extensions and physical limitations to, the paper of Weibull was recently given in a paper by the writer,[7] which was read before the 1951 Third World Petroleum Congress.

Finally, still more information can be obtained from a paper,[8] which was read by Prof. E. M'Ewen before the recent Gear Lubrication Symposium in London. In this paper the viscosity-pressure function is represented by the power relationship suggested in a paper by Hersey and Lowdenslager,[9] instead of by Weibull's and the authors' exponential relationship. In the discussion to this paper it was pointed out that the two sets of results deduced from the two relationships bear a close similarity to one another if they are expressed in the unifying manner indicated in that discussion.

H. J. Henry.[10] For the design of journal bearings, design factors such as $\mu n/p$, pU together with p are considered. Certain critical values of these design factors for a number of bearing materials have been compiled from values obtained from existing journal bearings that function properly. Bearings designed on the safe side of these design factors on occasion fail. The reason for this failure may be difficult to ascertain. There are perhaps dozens of other design factors which might have been considered, any one of which might have caused the failure.

In the case of slider bearings, we can hope for a more exact analysis to give us more reliable design factors. The usefulness of this analysis, if more reliable design factors are compiled, should be apparent.

The writer would like to ask the authors whether there has been, or is contemplated, any correlation of this work with values of load-carrying capacity obtained experimentally. Certainly the usefulness of this work would be extended.

One inconsistency in the paper is noted. In deriving the relationship for pressure, viscosity at a given temperature is taken as function of pressure. The viscosity certainly varies with temperature as well as pressure. Is it sufficiently accurate to consider the temperature of the oil film as being constant?

Authors' Closure

The authors wish to thank Professor Blok for his interesting and helpful discussion. The transformation used in Equation [1] to relate the problem to that of constant viscosity is a standard one which has been used from earliest times in works on differential equations. It was not the intent of the authors to attribute it to any one in particular in the lubrication field. It is interesting to know that it has been found useful by several authors in this field. As Professor Blok points out, the particular form of the viscosity-pressure relationship does not affect the general conclusions arrived at in the present paper.

In later papers the authors and F. Osterle consider viscosity as a function of both pressure and temperature. This constitutes an adiabatic treatment; the present one is isothermal.

Between the two lies the true situation. Thus in answer to the question of Dr. Henry, there is no inconsistency. The authors contemplate some experimental work in the future and indeed would like very much to see work of this sort carried on in other places too.

[6] "Glidlager-Teori med Variabel Viskositet" ("Theory of Slider Bearings With Variable Viscosity"), by W. Weibull, *Teknisk Tidskrift Mekanik*, vol. 55, 1925, pp. 164–167.

[7] "Viscosity—Temperature—Pressure Relationship," by H. Blok, Proceedings of the Third Works Petroleum Congress, E. J. Brill, Leyden, Holland, 1951, section vii, pp. 304–319.

[8] "Effect of Variation of Viscosity With Pressure on the Load-Carrying Capacity of the Oil Film Between Gear Teeth," by E. M'Ewen, Gear Lubrication Symposium, London, 1952; to be published in the Proceedings of the British Institute of Petroleum.

[9] "Film Thickness Between Gear Teeth—A Graphical Solution of Hallson's Problem," by M. E. Hersey and D. B. Lowdenslager, Trans. ASME, vol. 72, 1950, pp. 1035–1042.

[10] Director of Engineering, Chicago Pneumatic Tool Company, Franklin, Pa.