Comparative analysis of different probability distributions of random parameters in the assessment of water distribution system reliability

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ABSTRACT

During recent years, several methods based on the probabilistic approach have been proposed for the analysis of the performance of water distribution systems (WDSs). Uncertain elements are described by probabilistic laws chosen and parameterised on the basis of the network characteristics. However, the choice of the most suitable probabilistic distribution and of the statistical parameters can be difficult because of the lack of information about the WDSs. Among the stochastic parameters that affect the network performance, a fundamental role is played by the times to failure and repair of the system components. The impact of the chosen probability distributions of these fundamental variables on the evaluation of water distribution network reliability is analysed. The study is performed by using a technique capable of considering the mechanical failure of the network components, the spatial and temporal variations of the water demand and the uncertain distribution of the pipe roughness. This analysis allows quantification of the effect of any inaccuracy that may occur in the probabilistic characterisation of the random parameters.

Key words | failure and repair analysis, probabilistic model, reliability assessment, water systems

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INTRODUCTION

Techniques for the assessment of water supply network reliability evaluate the ability of the system to meet the required water demand under normal and abnormal operating conditions. This evaluation is concerned with many uncertain variables like the system topology, the failure of some network elements (valves, pumps, pipes, etc.), the spatial and temporal behaviour of the nodal demand and the pipe roughness.

A consolidated methodology to analyse the performance of water distribution systems (WDSs) is based on the probabilistic approach (Su *et al.* 1987; Gargano & Pianese 2000). By this method the hydraulic behaviour of the system is evaluated under different operating conditions, characterised by a proper probability of occurrence. The statistical analysis of the results provides an evaluation of the network performance in probabilistic terms. The uncertainties are assumed to be random variables whose probability distributions must be derived from the knowledge of the network characteristics. The availability of large data series is essential to ensure a suitable choice of the probability distributions and parameters. The lack of detailed information about the distribution networks and the shortage of historical measurement series can make it difficult to model the main random factors that affect the water supply system performance (Mailhot *et al.* 2000).

With regard to the most important variables concerning the water system reliability evaluation, times to pipe failure and repair must be considered. While few works deal with repair times, several failure models have been proposed in the literature. An exhaustive review about this topic is given by Kleiner & Rajani (2001). In their classification, one first, important, distinction is made between physically

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based and non-physically based models. The former category includes models taking into account the physical mechanisms that lead to pipe breakage. Since the parameterisation of such models requires the availability of data that can be obtained only at significant costs, such models are rarely applied in drinking WDSs.

For modelling pipe breakage in WDS, statistical models are normally used. These models use available historical data on past failures to formulate laws of the occurrence of breakages, assuming that such formulations remain valid in the future. Following Kleiner & Rajani's (2001) classification, these models can, in turn, be divided into deterministic and probabilistic. Both deterministic and probabilistic models may require different types of information, such as the year of installation of the pipes, the diameter, material, etc., and may or may not be parameterised in relation to a limited observation period.

Deterministic models predict breakage rate based on pipe age and breakage history and, in some cases, by taking into consideration various factors influencing the occurrence of breakage, such as the operating pressures and soil characteristics of the area overlaying the pipe (Clark *et al.* 1982). In such models, the age-dependent formulations are defined a priori. Among the first studies, Shamir & Howard (1979) suggested an exponential relationship between pipe breaks and age, while Kettler & Goulter (1985) adopted a linear age-dependent formulation.

In contrast, the probabilistic models use various probability functions to represent the interarrival times of pipe breakages in a WDS. Many authors have suggested that the survival time between different break orders needs to be described by different statistical distributions, since they exhibit different breakage behaviours (Andreou *et al.* 1987). In particular, the time to failure between successive breaks was observed to be very different than the time to failure from pipe installation to first break. The most commonly used statistical distributions for time to failure data associated with pipe breaks are the Weibull distribution for the first break and the exponential one for the successive break orders (Eisenbeis 1994; Mailhot *et al.* 2000). The parameterisation of such probability distributions may lead a model to reproduce situations in which the probability of breakage of a pipe increases, remains constant or decreases over time. An alternative approach is based on modelling failures in a water distribution network as a non-homogeneous Poisson process, using as input data failure times instead of interfailure times (e.g., Røstum 2000; Kleiner & Rajani 2012).

Besides different statistical approaches thoroughly described by Kleiner & Rajani (2001), recently data-driven modelling techniques have been used to predict pipe failures (Berardi *et al.* 2008; Tabesh *et al.* 2009).

The type and amount of information required for the parameterisation of each model and the availability of recorded data over a long time period represent discriminating factors for the model's application to real-life cases (Alvisi & Franchini 2010). Moreover, the natural variations that exist in all the factors that affect pipe deterioration and subsequent failure make the prediction of breaks in individual pipes even more difficult (Kleiner & Rajani 2012).

Almost all the previous studies are limited in assessing the ability of the different models to reproduce the number of breakages over the observation time and to predict the number in the years following the observation period. In this manner, the benefits derived from the planning of pipe rehabilitation in terms of reducing breakages can be quantified.

The impact of the changes of the probability distributions of times to failure and repair on reliability evaluation is analysed here in order to quantify the influence of any imprecision that can occur in the choice of probabilistic models and in its parameterisation. This paper represents an extension of a previous study (Darvini 2012), which analysed only the aspects related to the proper selection of the probability distribution functions of the time to repair of pipes. The present investigation is addressed through a technique based on a Monte Carlo numerical approach and a linear probabilistic hydraulic model (Darvini *et al.* 2008). The Monte Carlo procedure allows analysis of the aspects related to the mechanical failure of pipes and the time evolution of the demand, also considering the ageing of pipes and valves. From the knowledge of the probability density function (pdf) of failure and repair times of each network component by the Monte Carlo method a sample life cycle of the system is reproduced as a succession of normal and failure states of all the elements. The hydraulic analysis is performed by a numerical algorithm that integrates the simulator by Todini & Pilati (1988) and the iterative procedure by Todini (2003) to solving the cases of insufficient head that may occur in extended period simulations. From the statistical analysis of the results obtained for a large number of Monte Carlo simulations, the system nodal reliability is provided. The linear approach by Xu & Goulter (1998) is used to analyse the spatial variability of the water demand and the uncertainty in the pipe roughness. By assuming for these random variables a normal distribution, the probability distribution of the nodal heads for each network configuration is computed.

Several examples in the well-known case of the Anytown network are developed considering different models of probability distribution for the times to pipe failure and repair. Moreover, the effects of the changes of the statistical parameters of the probability distribution of the pipe roughness and demand spatial distribution on the reliability evaluation are analysed.

PROBABILITY DISTRIBUTIONS OF RANDOM PARAMETERS

The water system performance is affected by the temporary unavailability of one or more network elements related to mechanical failures. The time t of out-of-order is a random variable with pdf $f(t)$. A significant parameter of a supply system is the failure rate $\lambda(t)$. The latter gives the probability of failure in the unit of time, assuming the system to be normally operating at the initial time, and it is expressed as:

$$\lambda(t) = \lim_{\Delta t \to 0} \frac{\int_t^{t+\Delta t} f(t)\mathrm{d}t}{\Delta t} \tag{1}$$

From the knowledge of the time to failure pdf, the mean time to failure (*MTTF*) is given by:

$$MTTF = \int_0^{\infty} tf(t)\mathrm{d}t \tag{2}$$

By assuming an exponential distribution function for the time to failure pdf, it is $f(t) = \lambda \exp(-\lambda t)$, with rate λ constant in time, hence $MTTF = 1/\lambda$.

If the component is reparable, because of the complexity of the repair operations, the time t for the system to return to operation can be taken to be a random variable. The mean time to repair *MTTR* is:

$$MTTF = \int_0^{\infty} tg(t)\mathrm{d}t \tag{3}$$

If for the pdf $g(t)$ of the time to repair an exponential function $g(t) = \mu \exp(-\mu t)$, with repair rate μ constant in time is assumed, then $MTTR = 1/\mu$. The rate μ gives the probability that the system is again in normal operating state in the unit of time, assuming that at initial time all the system components are operating normally.

The exponential function is mostly used for describing the failure and repair times, but it is not the only available model. To our knowledge, there are few studies dealing with repair times. Bizzarri *et al.* (2002) analysed the recorded data of repairs in a WDS located in the Emilia Romagna region (Italy) during 6 years of observation between 1995 and 2000. They observed that most of the repairs required 2 working days and used for the repair time a uniform distribution between 5 and 55 hours. Instead, Salandin (2003), by analysing observed data of the water distribution network of Marghera, a dry land area of the city of Venezia (Italy), concluded that times to repair can be described by a log-normal distribution.

In contrast, many methods can be found in the scientific literature for modelling the pipe failure process.

As mentioned in the Introduction, the probability distributions most frequently used to represent inter-break times are the Weibull and exponential distributions (e.g., Eisenbeis 1994; Mailhot *et al.* 2000), although some authors have proposed different models. For example, Gustafson & Clancy (1999) proposed the gamma distribution to model the time to first break, while subsequent inter-break times were modelled as exponentials with diminishing *MTTF*.

In Table 1, the probability distributions most commonly used in reliability analyses and the relationships for the parameter estimation are reported. One can observe that for $k = 1$ the Weibull distribution reduces to the exponential one and for $k = 1$ and $\theta = 1/\lambda$ the gamma distribution gives the exponential with rate parameter λ.

Besides the times to failure and repair, the definition of the nodal demand and the evaluation of the associated uncertainty are the main factors in the reliability analysis of a WDS. The time evolution of the demand is

Table 1 | Probability distributions for reliability analyses

Probability distribution	pdf	Mean	Variance
Exponential	$\lambda e^{(-\lambda t)}$ $\lambda > 0, t \geq 0$	$\frac{1}{\lambda}$	$\frac{1}{\lambda^2}$
Uniform	$\frac{1}{b-a}$ $a < t < b$	$\frac{a+b}{2}$	$\frac{1}{12}(b-a)^2$
Log-normal	$\frac{1}{t\sigma\sqrt{2\pi}} e^{-\frac{(\ln t-\mu)^2}{2\sigma^2}}$ $t > 0$	$e^{\mu+\frac{\sigma^2}{2}}$	$\left(e^{\sigma^2}-1\right)e^{2\mu+\sigma^2}$
Weibull	$\frac{k}{\lambda}\left(\frac{t}{\lambda}\right)^{k-1} e^{\left(-\frac{t}{\lambda}\right)^k}$ $k, \lambda > 0, t \geq 0$	$\lambda\Gamma\left(1+\frac{1}{k}\right)$	$\lambda^2\left[\Gamma\left(1+\frac{2}{k}\right)-\left(\Gamma\left(1+\frac{1}{k}\right)\right)^2\right]$
Gamma	$\frac{1}{\theta^k}\frac{1}{\Gamma(k)} t^{k-1} e^{\left(-\frac{t}{\theta}\right)}$ $k, \theta > 0, t \geq 0$	$k\theta$	$k\theta^2$

characterised by a long-term trend depending on the development of users, by a seasonal cycle (due to climatic aspects or factors like tourism) and by a daily cycle related to the habits of users. The consumption rate is coupled with a significant random component related to an on-demand service. Generally, we consider only the daily cycle defined by mean values, and sometimes the random component is also taken into account with the introduction of a stochastic force with an assigned probability distribution (e.g., Bertola & Nicolini 2004). However, the generation of a large number of demand patterns requires a relevant computational effort.

The pipe roughness is also an uncertain parameter, but its mean value can be estimated with calibration procedures (e.g., Greco & Del Giudice 1999; Mallick *et al.* 2002). Xu & Goulter (1998), by following the assumption of Lansey *et al.* (1989), assume a normal distribution for the demand flow, the reservoir level and the pipe roughness.

UNCERTAINTY ANALYSIS IN WDSS

In water distribution networks, if the unknown nodal heads are defined as system state variables, for a junction that connects two or more pipes, the conservation of mass is written as:

$$\sum_{i\in\Omega_j} f_{ij}(H_i - H_j) - Q_j = 0 \quad (j = 1, \ldots, N) \tag{4}$$

where Q_j is the flow supplied at node j, H_j is the head at the node j and H_i is the head at all the nodes i connected directly to node j, N is the number of nodes with unknown heads, Ω_j is the set of nodes connected directly to node j and $f_{ij}(\ldots)$ is a nonlinear function relating the hydraulic loss and the flow rate in the pipe connecting nodes i and j.

Probabilistic hydraulic models for water distribution networks take uncertain parameters as random variables and attempt to derive the probabilistic characterisations of the nodal heads using probability theory. Solutions of such problems are, however, very complex, due to the nonlinearity of the hydraulic equations that relate the head loss and flow in a pipe. A solution is given by Monte Carlo simulation (MCS) which consists of successive evaluations of the equations describing the system for alternative realisations of the system parameters. MCS theoretically enables us to give account of all uncertainties; however, the number of MCS that ensures the convergence of the statistics increases with the number of different uncertainty causes, and the computing effort grows rapidly. To overcome this complexity, linear probabilistic hydraulic models for the analysis of the performance of water distribution networks are developed (Xu & Goulter 1998). In the present work, the uncertainty analysis is developed through a technique that blends a Monte Carlo method with the linear probabilistic model proposed by Xu & Goulter (1998). MCS gives proper account of pipe failures and the temporal variation of the demand, while the spatial variability of the nodal demand and the uncertainty in the pipe roughness are handled by the linear approach.

The linearised probabilistic model

The probabilistic model proposed by Xu & Goulter (1998) is briefly explained here. The model allows the uncertainty related to the demands, reservoir/tank levels and pipe roughness to be taken into account and is based on two assumptions: (1) demands, reservoir/tank levels and pipe roughness are normally distributed variables and (2) a linearised model of the system, that makes use of the expected values of nodal demands and pipe coefficients, is used in the vicinity of the nodal heads derived from a nonlinear model. The equations of the system linearised around the expected values of nodal demands, reservoir levels and pipe coefficients are developed using a first-order Taylor series expansion at the expected values of the nodal demands, reservoir/tank levels and pipe roughness.

Having defined as $\mathbf{H}_\mathrm{f}$ the vector of the fixed nodal heads, $\mathbf{H}_\mathrm{v}$ the vector of the piezometric heads of the remaining nodes, $\mathbf{e}$ the pipe roughness vector and $\mathbf{Q}$ the nodal demand vector, Equation (4) can be written as:

$$\mathbf{G}(\mathbf{H_v}, \mathbf{H_f}, \mathbf{e}, \mathbf{Q}) = \mathbf{0}. \tag{5}$$

By expanding in Taylor series Equation (5) around the mean positions $\overline{\mathbf{H}}_\mathbf{v}$, $\overline{\mathbf{H}}_\mathbf{f}$, $\overline{\mathbf{e}}$, $\overline{\mathbf{Q}}$, and by limiting the expansion to the first-order, one obtains:

$$\mathbf{G}(\overline{\mathbf{H}_\mathrm{v}}, \overline{\mathbf{H}_\mathrm{f}}, \overline{\mathbf{e}}, \overline{\mathbf{Q}}) + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{v}}\right|_{\overline{\mathbf{p}}} \mathbf{H}'_\mathbf{v} + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{f}}\right|_{\overline{\mathbf{p}}} \mathbf{H}'_\mathbf{f} + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{e}}\right|_{\overline{\mathbf{p}}} \mathbf{e}' + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{Q}}\right|_{\overline{\mathbf{p}}} \mathbf{Q}' + \mathbf{o}(\mathbf{1}) = \mathbf{0} \tag{6}$$

where the terms with superscripts represent the fluctuations around the expected value and $\overline{\mathbf{p}}$ refers to the set of the expected values $\overline{\mathbf{H}}_\mathrm{v}$, $\overline{\mathbf{H}}_\mathrm{f}$, $\overline{\mathbf{e}}$, $\overline{\mathbf{Q}}$ where the derivatives are computed. The last term on the left-hand side of Equation (6) represents the remainder of the series of the degree smaller than 1.

By separating the terms at different orders, one obtains:

$$\mathbf{G}(\overline{\mathbf{H}}_\mathrm{v}, \overline{\mathbf{H}}_\mathrm{f}, \overline{\mathbf{e}}, \overline{\mathbf{Q}}) = \mathbf{0} \tag{7}$$

$$\left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{v}}\right|_{\overline{\mathbf{p}}} \mathbf{H}'_\mathbf{v} + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{f}}\right|_{\overline{\mathbf{p}}} \mathbf{H}'_\mathbf{f} + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{e}}\right|_{\overline{\mathbf{p}}} \mathbf{e}' + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{Q}}\right|_{\overline{\mathbf{p}}} \mathbf{Q}' = \mathbf{0} \tag{8}$$

By multiplying Equation (8) with each of the fluctuations $\mathbf{H}'_\mathrm{v}, \mathbf{H}'_\mathrm{f}, \mathbf{e}', \mathbf{Q}'$, and averaging, the following system of four equations is obtained:

$$\left.\frac{\partial \mathbf{G}}{\partial \mathbf{H_v}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{H}'_\mathrm{v}\mathbf{H}'_\mathrm{v}\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{f}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{H}'_\mathrm{f}\mathbf{H}'_\mathrm{v}\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{e}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{e}'\mathbf{H}'_\mathrm{v}\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{Q}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{Q}'\mathbf{H}'_\mathrm{v}\rangle = 0 \tag{9}$$

$$\left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{v}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{H}'_\mathrm{v}\mathbf{e}'\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{f}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{H}'_\mathrm{f}\mathbf{e}'\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{e}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{e}'\mathbf{e}'\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{Q}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{Q}'\mathbf{e}'\rangle = 0 \tag{10}$$

$$\left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{v}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{H}'_\mathrm{v}\mathbf{Q}'\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{f}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{H}'_\mathrm{f}\mathbf{Q}'\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{e}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{e}'\mathbf{Q}'\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{Q}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{Q}'\mathbf{Q}'\rangle = 0 \tag{11}$$

$$\left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{v}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{H}'_\mathrm{v}\mathbf{H}'_\mathrm{f}\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{H}_\mathrm{f}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{H}'_\mathrm{f}\mathbf{H}'_\mathrm{f}\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{e}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{e}'\mathbf{H}'_\mathrm{f}\rangle + \left.\frac{\partial \mathbf{G}}{\partial \mathbf{Q}}\right|_{\overline{\mathbf{p}}} \langle \mathbf{Q}'\mathbf{H}'_\mathrm{f}\rangle = 0 \tag{12}$$

From the knowledge of the covariance matrices of pipe roughness, flow demands and reservoir levels, one can calculate the covariance matrix of the piezometric heads. Then, assuming that roughness, nodal demands and reservoir levels are uncorrelated, $\langle \mathbf{H}'_\mathrm{f}\mathbf{e}'\rangle = \langle \mathbf{e}'\mathbf{Q}'\rangle = \langle \mathbf{Q}'\mathbf{H}'_\mathrm{f}\rangle = 0$, the system (10)–(13) has an equal number of equations and unknowns. Once the covariance matrices $\langle \mathbf{H}'_\mathrm{v}\mathbf{e}'\rangle$, $\langle \mathbf{H}'_\mathrm{v}\mathbf{Q}'\rangle$ and $\langle \mathbf{H}'_\mathrm{v}\mathbf{H}'_\mathrm{f}\rangle$ from Equations (10)–(12), respectively, are obtained, the covariance matrix of the piezometric heads at the supply nodes can be found by using Equation (9):

$$\langle \mathbf{H}'_v \mathbf{H}'_v \rangle = -\frac{\partial \mathbf{G}}{\partial \mathbf{H}_v}\bigg|_{\bar{\mathbf{p}}}^{-1} \frac{\partial \mathbf{G}}{\partial \mathbf{H}_f}\bigg|_{\bar{\mathbf{p}}} \langle \mathbf{H}'_f \mathbf{H}'_v \rangle - \frac{\partial \mathbf{G}}{\partial \mathbf{H}_v}\bigg|_{\bar{\mathbf{p}}}^{-1} \frac{\partial \mathbf{G}}{\partial \mathbf{e}}\bigg|_{\bar{\mathbf{p}}} \langle \mathbf{e}' \mathbf{H}'_v \rangle - \frac{\partial \mathbf{G}}{\partial \mathbf{H}_v}\bigg|_{\bar{\mathbf{p}}}^{-1} \frac{\partial \mathbf{G}}{\partial \mathbf{Q}}\bigg|_{\bar{\mathbf{p}}} \langle \mathbf{Q}' \mathbf{H}'_v \rangle \tag{13}$$

The distribution function for the nodal heads can always be obtained, by mathematical convolution techniques, from knowledge of the joint distribution of nodal demands, reservoir levels and pipe roughness. In practice, such a task is often very complicated. However, if the nodal demands, reservoir levels and pipe roughness are normally distributed, the linearised nodal heads also follow a normal distribution. In this situation, only the mean and variance need to be estimated, by Equations (7) and (13), respectively, to derive the complete probability distribution of the head at each demand node.

The assumption of normal distributions may be relaxed without affecting the results of the model for large networks. By virtue of the central limit theory, the random nodal heads, which are a sum of linear functions of the random nodal demands and random pipe coefficients, are approximately normally distributed no matter what distribution the nodal demands and pipe roughness follow (Xu & Goulter 1998). Since in the present work the attention is focused on the comparison among different pdfs of times to failure and repair, the assumption of normality in deriving the probability distribution for nodal heads is maintained. However, the assumption of independency among random variables may not be valid for the nodal demands. Actual demands in the same areas may be correlated since the demands may rise and fall for the same causes as weather conditions. It is, however, relatively easy to include the dependence on the demand if the correlation coefficients for the nodal demands are known. Furthermore, demands may be temporally and spatially cross-correlated with reservoir levels. This limitation could influence the results and the development of a more realistic system reliability model to fully account for these operating factors is certainly needed.

Monte Carlo simulation

The solution of Equation (7) was obtained using the technique proposed by Darvini & Salandin (2004) here summarised. Since WDSs are repairable systems, the lifetime of each reparable component (pipe, pump, valve, etc.) is constituted by the succession of intervals of time of random duration T_r and T_f where the element is in either normal or abnormal operating conditions, respectively.

Starting from time $t = 0$, where all the system components are normally operating, at time t_k the system is in its state k, characterised by a specific combination of elements either normally operating or not. The probability that at time t, measured from time t_k, the system is still in the state k is given by the probability that time t is smaller than $T_{k,i}$, $T_{k,i}$ being the transition time that the i-th component takes to switch between a failure operating condition and a repair state or vice versa. The time T_k required to switch from the state k to the state $k+1$ is the minimum transition time computed among all the system components.

Any ageing process (increase of the pipe roughness or variation of the failure/repair rate) and/or the consequences from the rehabilitation or replacement of some elements (Darvini & Salandin 2004) can be taken into account in the story of the system.

For each state, the system of Equation (7) has to be solved to compute the nodal head and flow for all the nodes.

In the present work, the hydraulic analysis is performed according to a pressure-driven approach. For the pressure-driven model, a numerical algorithm that integrates the simulator proposed by Todini & Pilati (1988) with the iterative technique suggested by Todini (2003) for the simulation of the hydraulic behaviour of the system was developed. The pressure-driven analysis allows performance of a realistically extended simulation period by evaluating the actual nodal flow, also in the cases of insufficient nodal head derived from pipe failures.

From the statistics of the results obtained for each Monte Carlo simulation the reliability of the system can be estimated.

Reliability assessment

The definition of a failure of WDS is here based on inadequate delivery flow, and a failure is deemed to occur if at node i the supplied flow $Q_{e,i}$ is lower than the nodal demand $Q_{r,i}$.

For a single MCS, the probability that the supplied flow $Q_{e,i}$ is lower than $Q_{r,i}$ conditioned to the demand $Q_{r,i}$, is given by:

$$P\{Q_{e,i} < Q_{r,i} | Q_{r,i}\} \cong \sum_k \frac{T^*_{k,i}}{T_{\text{tot}}} \tag{14}$$

where $T_{\text{tot}} = \Sigma T_k$ is the total duration of simulation and $T^*_{k,i}$ is the duration, in the state k and for node i, where is $Q_{e,i} < Q_{r,i}$.

Due to the uncertainty of the flow demand, pipe roughness and reservoir levels, the head at the node i varies in comparison with the expected value $\overline{H}_i$ obtained by Equation (7) using the expected values of demands, reservoir/tank levels and pipe roughness. The covariance matrix $\langle H'_i H'_i \rangle$ can be computed by Equation (13). Therefore, the probability $P_{k,i}(Q_{e,i} < \overline{Q}_{r,i}) = P_{k,i}(H_i < H_{s,i})$ can be associated with the duration $T^*_{k,i}$ where $Q_{e,i}$, is smaller than the mean nodal demand $\overline{Q}_{r,i}$.

The probability that at node i the supplied flow $Q_{e,i}$ is lower than the mean nodal demand $\overline{Q}_{r,i}$ is given by:

$$\begin{aligned} P\{Q_{e,i} < \overline{Q}_{r,i} | \overline{Q}_{r,i}\} &\cong \sum_k \frac{T^*_{k,i}}{T_{\text{tot}}} P_{k,i}\{Q_{e,i} < \overline{Q}_{r,i}\} \\ &= \sum_k \frac{T^*_{k,i}}{T_{\text{tot}}} P_{k,i}\{H_i < H_{s,i}\} \end{aligned} \tag{15}$$

Equation (15) states that in WDS hydraulic failures can occur also in the absence of any mechanical failure, although the network works correctly for mean values of uncertain parameters.

The method can be applied for any value of the magnitude of the pressure shortfall, by setting a coefficient $\alpha = Q_{e,i}/Q_{r,i}$ in the range between 0 and 1.

METHODOLOGY

The proposed approach for the reliability assessment is based on the combined use of the MCS approach and the linearised model described in previous sections.

Each MCS consists of four steps. (1) For each pipe the time to failure is generated based upon the assumed distribution. The pipe with minimum time to failure is taken to be failed. (2) The hydraulic simulation is run and the computed hydraulic quantities necessary for the reliability evaluation are stored. (3) The head covariance is computed by Equation (13) to derive the complete head probability distribution. (4) For the failed pipe, a time to repair is generated; whereas, for the other pipes, a new value of time to failure is generated. Such a new generation of times is needed when the failure rate increases with time, by considering the deterioration process of pipes. If the time to repair of failed pipe is longer than the smaller time to failure of the pipes, the number of the pipes unavailable increases by one unit. Otherwise, all the components will result as normally operating. Steps 1–4 are repeated until T_{tot}. At the end of the lifetime of the system, the probability given by Equation (15) is computed. When the number of realisations reaches the given maximum value the output statistics are computed to provide the system reliability estimate.

Different pdfs of times to failure and repair are here compared to evaluate the influence of the choice of the probability distributions on the reliability assessment. The distributions of the time to repair are deduced from the analysis of recorded data in the water distribution network of Marghera, Italy (Salandin 2003). From the reading of the service orders (schematic description of the maintenance operations) collected during 12 years, a total of 531 data of times to repair was derived. The time to repair distribution can be described by a log-normal probability distribution as illustrated in Figure 1. Labelled as $T = \ln t$

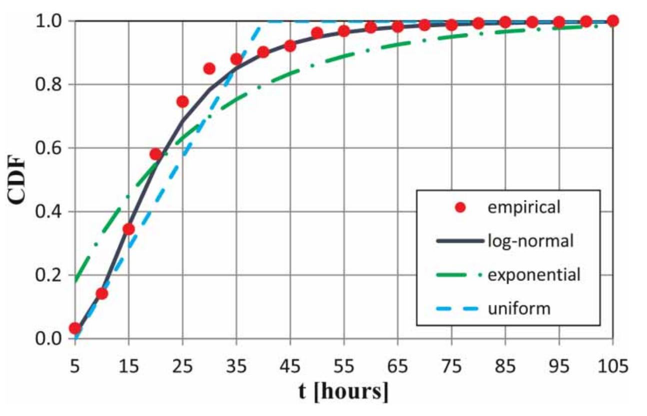

Figure 1 | Cumulative distribution function (CDF) of times to repair.

the logarithm of the time to repair, mean and variance are 2.93 and 0.362, respectively. In the absence of detailed information about times to repair for different classes of pipes, these statistical parameters are attributed to all the pipes.

Figure 1 also shows the cumulative distribution functions (CDFs) for the exponential and uniform probability distributions adapted to the sample. For the uniform distribution times to repair vary between 5 and 40 hours, and the mean value is 22.5 hours. Differences among various CDFs can be evaluated in terms of distance or dissimilarity coefficient (Gower 1971). The latter is 0.03 between log-normal and uniform distributions, while the value increases until 0.06 between log-normal and exponential ones.

Besides the analysis of the probability distribution concerning times to repair already developed in Darvini (2012), the investigation is here extended to different pdfs that can describe the distribution of times to failure of pipes.

On the basis of the available information about the pipe breaks occurring at the Marghera network, it was possible to deduce a relationship existing between the mean time to failure and the pipe diameter, being the latter one of the pipe characteristics that has the largest impact on the breakage rate. For the mean time to failure, the following expression is obtained:

$$MTTF_i = [0.2688\exp(-0.0023D_i)]^{-1} \quad (16)$$

where D_i is the diameter of the i pipe, given in millimetres.

Equation (16) confirms the evidence that pipe failure rates are much higher for smaller diameters (e.g., Su *et al.* 1987; Mays 1989; Pelletier *et al.* 2003); probably because of the thinner pipe walls and smaller moment of inertia (e.g., Kettler & Goulter 1985) or due to lesser care being adopted when laying in comparison with larger diameter pipes. Because of the scarcity of data and the fact that they are incomplete (inaccurate locations and approximate date of occurrence), it was not possible to find a different grouping criterion such as the pipe age or material.

In the absence of more information for the time to failure, we were unable to deduce the CDF by fitting experimental data and, thus, the most common CDF used in the literature is taken into consideration for the comparative analysis. In addition to the exponential distribution, fully defined by the failure rate $\lambda = 1/MTTF$, the Weibull and gamma pdfs are taken into account. As an example, the CDF of the exponential distribution for the mean value computed over all the pipes $\lambda = 0.133$ is reported in Figure 2.

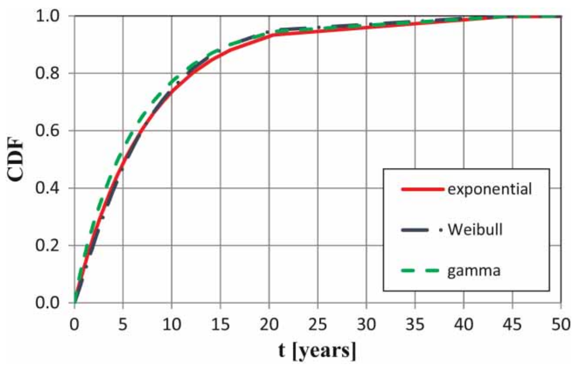

Figure 2 | Cumulative distribution function (CDF) of times to failure.

In order to retain a similar mean value of times to failure in all the cases, for the Weibull distribution, whose CDF is illustrated in Figure 2, the parameter k was set to 1.1. This value was chosen according to results reported in the scientific literature confirming that the parameter k is, generally, larger than 1, the value corresponding to the exponential distribution. This means that smaller times to failure have a smaller cumulative probability than that obtained assuming the exponential distribution and the behaviour is opposite for large times. For the gamma distribution, the parameter k was set to 0.90 and the CDF is shown in Figure 2. This value leads to times to failure smaller, on average, than times generated by using the exponential distribution. The dissimilarity coefficient among different CDFs are smaller than values computed for times to repair. The coefficient is 0.02 between exponential and log-normal distribution and 0.03 between exponential and gamma distribution.

CASE STUDY

The analysis was applied to the synthetic water distribution network of Anytown. The chosen network is a well-known bench-test conceived to compare results obtained by different design and optimisation methods (e.g., Walski *et al.* 1987). In the examples the network scheme is simplified as

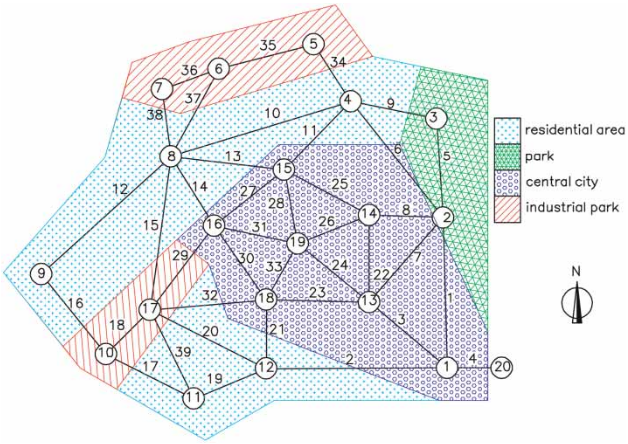

Figure 3 | Scheme of the water distribution network Anytown.

shown in Figure 3. The node elevations, the mean nodal demands, the pipe data and the mean roughness are reported in Tables 2 and 3. As explained in detail in the previous section, the *MTTR* value is set constant for all the pipes, while the *MTTF* is a function of the pipe diameter on the basis of Equation (16).

The covariance matrix of the pipe roughness has been assumed, for the sake of simplicity, as diagonal, i.e., the roughness values of any couple of different pipes are taken to be uncorrelated. Alternatively, it is possible to fix the matrix coefficients with reference to the year of pipe laying, assuming the roughness values of the pipes laid in the same period of time to be correlated.

The spatial fluctuations around each instantaneous value of the demand depend on the covariance matrix. In the absence of any information about the correlation coefficients ρ for the network, these have been taken on the basis of the different areas where the nodes are located: central city, industrial park, residential area and park. Thus, $\rho = 0.8$ was set for the nodes that belong to the same area and $\rho = 0.5$ for the nodes in different areas, while the correlation between the residential area and the central city demands was set to $\rho = 0.7$. The coefficient of variation ($CV = \sigma/\mu$) varies within the limits of validity of the linear theory, $CV(Q_r) \leq 0.4$ and $CV(e) \leq 1.0$ for the nodal demand and the roughness, respectively (Xu & Goulter 1998).

The illustrative example has been developed by considering the fulfilment of the flow service value with reference to the service piezometric head $H_{s,i} = 25$ m on each node. The period of simulation was set equal to 50 years and the number of MCSs was 500, to ensure the convergence of the required statistics. Since within such a long period all pipes deteriorate, the *MTTF* is increased in time by assuming an exponential law $\lambda(t) = \lambda(t_o)\exp[A(t - t_o)]$, where $\lambda(t_o)$ is the failure rate at time $t = t_o$ and the coefficient A is set to 0.1 years^{-1}. Moreover, the relationship $e(t) = e_o – \exp(\beta t)(e_o - e_n)$ was used for the pipe roughness at time t, where the β coefficient is taken to be of 0.15 years^{-1}. The initial values of e_n are reported in Table 3, while the final values e_o are obtained, doubling the initial ones.

Table 2 | Node elevation and nodal demand data for the illustrative network

Node	Elevation (m a.s.l.)	Demand (l/s)	Node	Elevation (m a.s.l.)	Demand (l/s)
1	6	31.5	11	36	25.2
2	15	12.6	12	15	31.5
3	15	12.6	13	15	31.5
4	15	37.9	14	15	31.5
5	24	37.9	15	15	31.5
6	24	37.9	16	36	25.2
7	24	37.9	17	36	63.1
8	24	25.2	18	15	31.5
9	36	25.2	19	15	63.1
10	36	25.2	20	6	0.0

DISCUSSION OF RESULTS

With reference to the Anytown network, several simulations have been run to evaluate the variation of the probability of supply failure as function of the probabilistic model that describes both times to failure and repair.

Effect of changes of CV(Q_r) and CV(e) on the system reliability

First, the influence of the value of the coefficient of variation of the nodal demand CV(Q_r) and roughness CV(e) on the system reliability is analysed. In Figure 4, the results obtained at nodes 6, 7 and 17 – situated in the industrial park – by assuming for the distribution of the time to failure

Table 3 | Pipe data for the illustrative network. For each pipe are given the initial node N1, the final node N2, the diameter D, the length L, the roughness e and the MTTF

Pipe	N1	N2	L (m)	D (mm)	e (mm)	MTTF (yr)	Pipe	N1	N2	L (m)	D (mm)	e (mm)	MTTF (yr)
1	1	2	3,657.6	304.7	1.5	7.50	21	12	18	1,828.8	355.6	2.0	8.43
2	1	12	3,657.6	762.9	2.0	21.51	22	13	14	1,828.8	762.2	2.0	21.47
3	1	13	3,657.6	699.6	2.0	18.59	23	13	18	1,828.8	304.7	2.0	7.50
4	1	20	30.5	457.2	1.0	10.65	24	13	19	1,828.8	152.4	2.0	5.28
5	2	3	1,828.8	253.9	1.5	6.67	25	14	15	1,828.8	598.0	2.0	14.72
6	2	4	2,743.2	203.9	1.5	5.95	26	14	19	1,828.8	253.9	2.0	6.67
7	2	13	2,743.2	304.7	2.0	7.50	27	15	16	1,828.8	253.9	2.0	6.67
8	2	14	1,828.8	253.9	1.5	6.67	28	15	19	1,828.8	253.9	2.0	6.67
9	3	4	1,828.8	253.9	1.5	6.67	29	16	17	1,828.8	203.1	1.5	5.94
10	4	8	3,657.6	203.1	1.5	5.94	30	16	18	1,828.8	203.1	2.0	5.94
11	4	15	1,828.8	253.9	1.5	6.67	31	16	19	1,828.8	253.9	2.0	6.67
12	8	9	3,657.6	203.1	1.5	5.94	32	17	18	1,828.8	203.1	1.5	5.94
13	8	15	1,828.8	253.9	1.5	6.67	33	18	19	1,828.8	253.9	2.0	6.67
14	8	16	1,828.8	203.1	1.5	5.94	34	4	5	1,828.8	355.6	1.0	8.43
15	8	17	1,828.8	203.1	1.5	5.94	35	5	6	1,828.8	406.4	1.0	9.47
16	9	10	1,828.8	304.9	1.5	7.50	36	6	7	1,828.8	152.4	1.0	5.28
17	10	11	1,828.8	394.6	1.5	9.22	37	6	8	1,828.8	203.9	1.0	5.95
18	10	17	1,828.8	355.6	1.5	8.43	38	7	8	1,828.8	609.6	1.0	15.12
19	11	12	1,828.8	203.1	1.5	5.94	39	11	17	2,743.2	406.4	1.0	9.47
20	12	17	1,828.8	606.9	1.5	15.02							

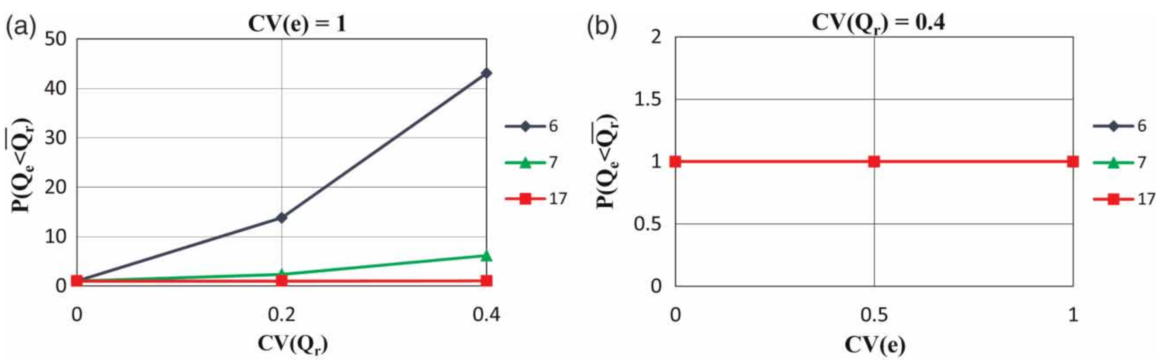

Figure 4 | Supply failure probability at nodes 6, 7 and 17 as function of CV(Q_r) for CV(e) = 1 (a); as function of CV(e) for CV(Q_r) = 0.4 (b). Case with exponential distribution of times to failure and log-normal distribution of times to repair.

an exponential function fully defined by the *MTTF* value are shown. While for times to repair a log-normal probability distribution with the statistical parameters obtained in the previous section was used. Figure 4 shows the supply failure probability as the function of CV(Q_r), by setting CV(e) = 1 (Figure 4(a)) and as function of CV(e), by setting CV(Q_r) = 0.4 (Figure 4(b)). The results are normalised by the value obtained with a coefficient of variation equal to 0.

Figure 4(a) clarifies that the supply failure probability is rather sensitive to the variations of CV(Q_r) and increases as the latter increases. The magnitude of such dependence varies from node to node. Node 6 is the most vulnerable to CV(Q_r) variations, while for node 17, situated closer to the source node, the variation is less pronounced. Figure 4(b) shows that the supply failure probability can be taken as insensitive to the variation of CV(e) for all the considered nodes. These findings agree with results reported in the work of Xu & Goulter (1998), where the authors state that nodal and system reliabilities for their example networks are moderately sensitive to variation in nodal demands and reservoir levels and relatively insensitive to variations in pipe roughness.

Effect of changes in the repair time pdf on the system reliability

The variation of the results obtained at the same nodes of Figure 4 by retaining the exponential distribution of the time to failure and by assuming for the time to repair different probability distributions is shown in Figure 5. The figure shows the results as function of CV(Q_r), by setting CV(e) = 1 (Figures 5(a), 5(c) and 5(e)) and as function of CV(e), by setting CV(Q_r) = 0.4 (Figures 5(b), 5(d) and 5(f)).

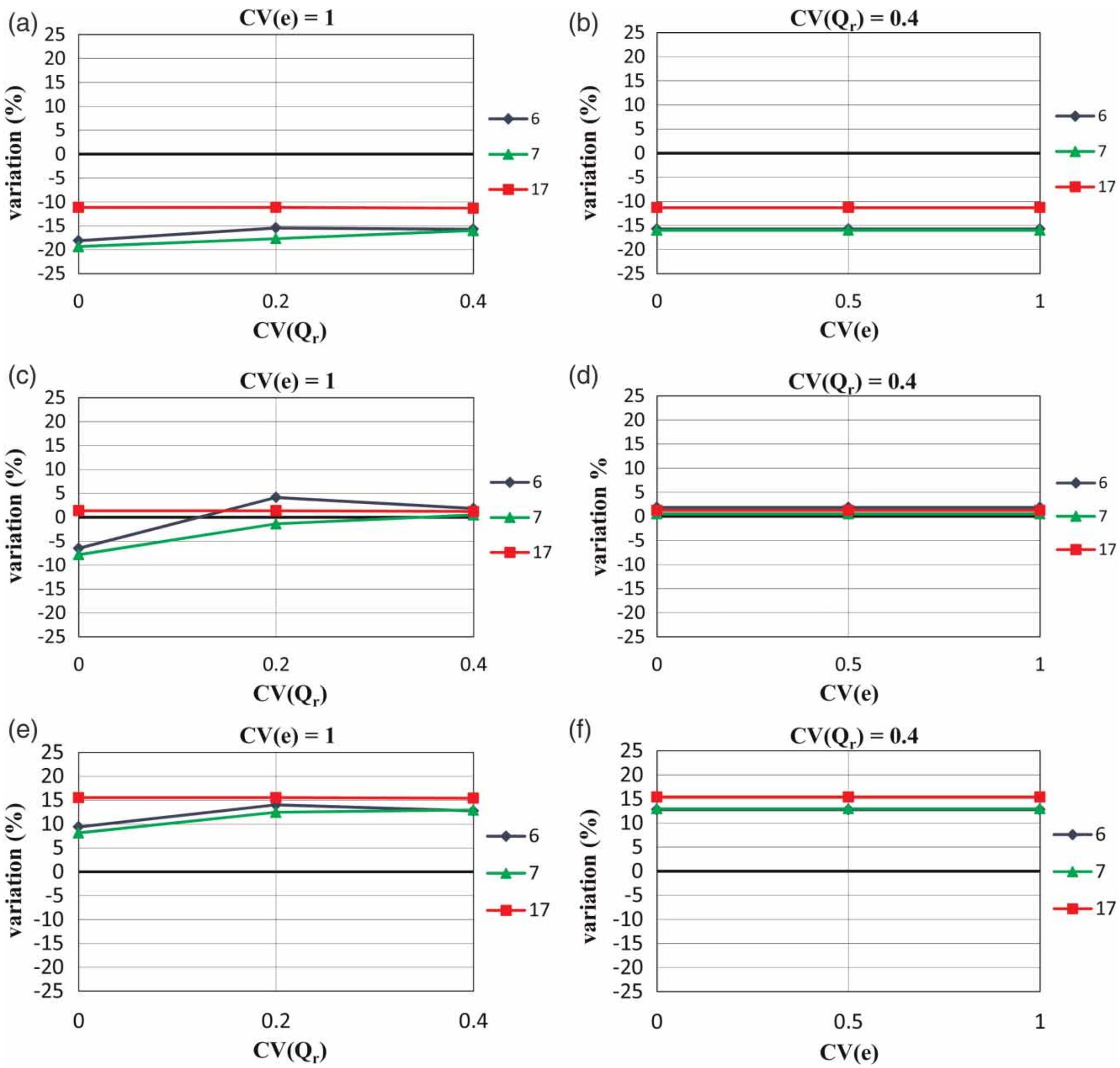

Figure 5 | Variation of the supply failure probability at nodes 6, 7 and 17 in comparison with the results of Figure 4 as function of CV(Q_r), for CV(e) = 1 (a), (c) and (e); as function of CV(e), for CV(Q_r) = 0.4 (b), (d) and (f). Case with exponential distribution of times to failure and various distributions of times to repair: exponential distribution (a) and (b); uniform distribution of times between 5 and 40 hours (c) and (d); uniform distribution of times between 5 and 45 hours (e) and (f).

Results of Figures 5(a) and 5(b) are obtained by replacing the log-normal distribution of times to repair with an exponential distribution fully defined by the *MTTR* value. The supply failure probability reduces by about 15%. The reduction can be explained as a consequence of the generation by the Monte Carlo procedure of times to repair smaller, on average, than times derived by a log-normal distribution, as can be observed from Figure 1. Because the pipes remain out of order over shorter times, a minor probability of supply failure is obtained. The variation is less pronounced at node 17 which is closer to the source node, while it is more evident for the other nodes. The variation does not depend on CV(e) for all the nodes considered (Figure 5(b)) and on CV(Q_r) for node 17 (Figure 5(a)).

The variation of the results obtained by assuming a uniform probability distribution for the time to repair with times between 5 and 40 hours is shown in Figures 5(c) and 5(d). If the log-normal distribution of times to repair is replaced with a uniform one, variations less pronounced than the previous case are found. Moreover, the supply failure probability changes in different ways for different nodes. Figure 5(c) demonstrates that at nodes 6 and 7 the supply failure probability reduces by 7% for CV(Q_r) = 0 and CV(e) = 1, but the variation tends to vanish as CV(Q_r) increases. The supply failure probability increases by 1.4% at node 17 for CV(Q_r) = 0 and CV(e) = 1 and the reduction is not very sensitive to the CV(Q_r) value. The variation is independent of CV(e) for all the nodes considered (Figure 5(d)).

Finally, the influence of the distribution parameters was analysed in the case of the time to repair uniform distributed. As an example, the duration of times to repair was varied by setting such times between 5 and 45 hours. The results of the simulations are illustrated in Figures 5(e) and 5(f). Since the pipes are unavailable for a time longer than the previous case, the supply probability failure increases with respect to the results of Figure 4 for all the control nodes. Such a variation does not depend on CV(e) and it is not very sensitive to CV(Q_r). The increase is smaller at nodes 6 and 7 and is larger at node 17, where it reaches 15%.

Furthermore, the comparison between the pairs of Figures 5(c) and 5(d) and Figures 5(e) and 5(f) shows that, for the uniform pdf, an increase of about 11% of the *MTTR* can lead to an increase of the supply failure probability of over 15% at the nodes considered here.

Effect of changes in the failure time pdf on the system reliability

Figure 6 shows the influence on the reliability evaluation of changes in the probability distributions of times to failure. Variations are computed with reference to results of Figure 4, obtained with a time to failure exponentially distributed.

Dealing with times to failure, one can observe that variations of results due to changes in pdfs are comparable with those obtained by varying the distributions of times to repair, although the dissimilarity indices among different pdfs are much smaller.

The effect on the results obtained at the control nodes if one assumes a Weibull distribution of times to failure instead of an exponential one, is shown in Figures 6(a) and 6(b). The probability of supply failure tends to decrease for all the nodes. This result is due to the fact that the times to failure obtained by assuming a Weibull distribution are larger, on average, than those generated by an exponential pdf, as can be deduced from Figure 2. Figure 6(a) shows that the reduction of the probability of supply failure is maximum for CV(Q_r) = 0 and CV(e) = 1 at node 7, where it reaches 18%. As CV(Q_r) increases the variation tends to reduce. At node 6 the probability of supply failure reduces by about 6%, while at node 17 the reduction is 3% and the variation seems to be independent of the coefficients of variation of the nodal demand and roughness. Figure 6(b) confirms that the variation is independent of CV(e) for all the nodes.

Figures 6(c) and 6(d) show the results in terms of the variation of the probability of supply failure obtained by replacing the exponential probability distribution of times to failure with a gamma distribution. As can be deduced from Figure 2, times to failure are smaller, on average, than those generated from an exponential pdf. Therefore, the probability of supply failure increases for all the nodes considered. Unlike the case of Figures 6(a) and 6(b), the variation is larger at node 17 for CV(Q_r) = 0 and CV(e) = 1, where it is 16% and insensitive to both CV(Q_r) and CV(e).

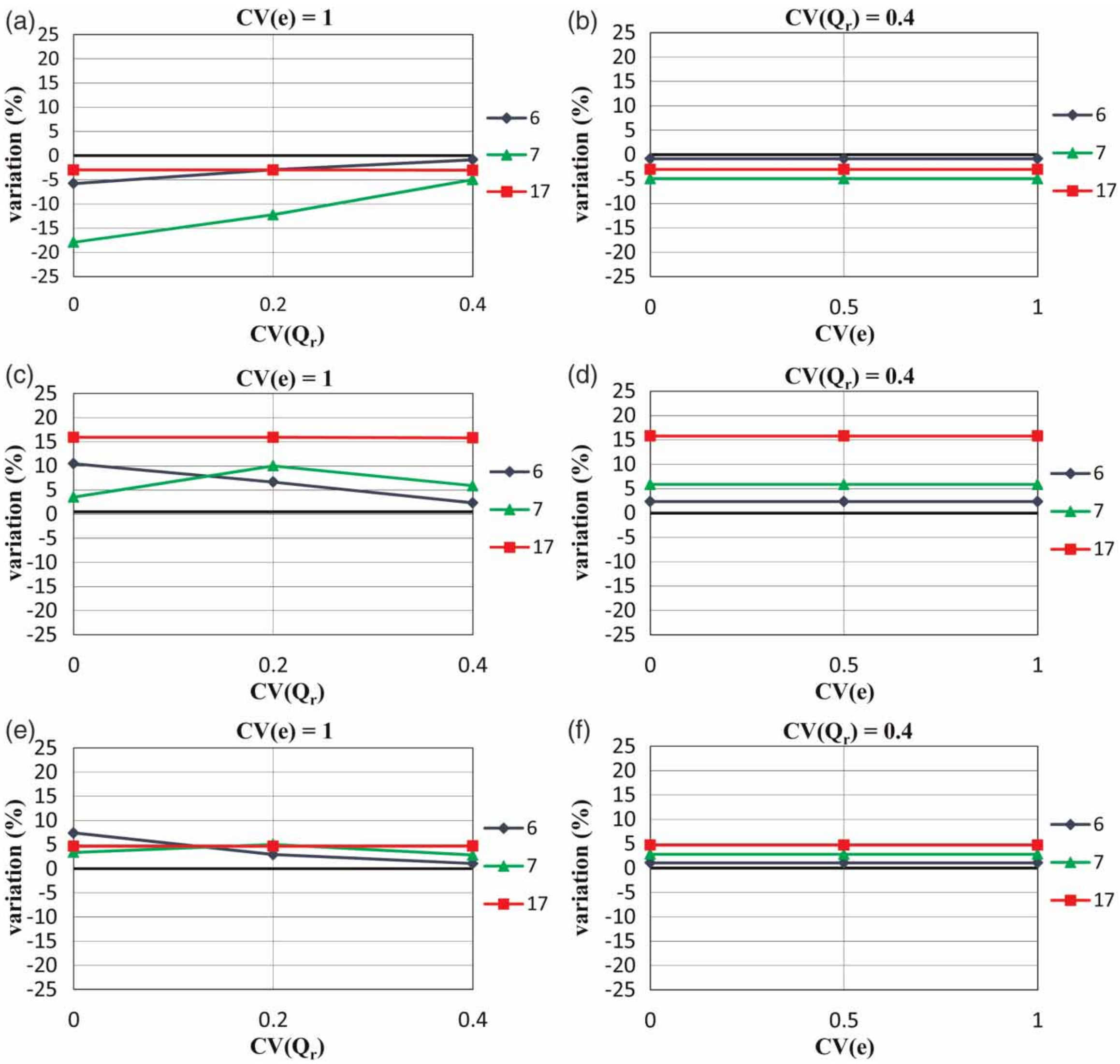

Figure 6 | Variation of the supply failure probability at nodes 6, 7 and 17 in comparison with the results of Figure 4 as function of CV(Q_r), for CV(e) = 1 (a), (c) and (e); as function of CV(e), for CV(Q_r) = 0.4 (b), (d) and (f). Case with log-normal distribution of times to repair and various distributions of times to failure: Weibull distribution (a) and (b); gamma distribution with $k = 0.9$ (c) and (d); gamma distribution with $k = 0.95$ (e) and (f).

A further simulation has been run by varying the value of the parameter k of the gamma distribution from 0.9 to 0.95 to verify the effect of a small variation of the statistical parameters in the evaluation of the probability supply failure at each node of the network.

The variations obtained in comparison with the results of Figure 4 are reported in Figures 6(e) and 6(f). Figure 6(e) shows that a slight modification of the parameter k, corresponding to an increase of *MTTF* value of 5%, can lead to a significant difference in the results, mainly for smaller values of CV(Q_r). For CV(Q_r) = 0, the variation of the probability of supply failure reduces in comparison with that obtained in Figure 6(c) at all nodes; at node 17, the reduction is maximum and reaches 10%. Being k closer to 1, the value corresponding to the exponential distribution, this result was expected. For CV(Q_r) = 0.4, the supply failure probability is very close to the results of Figure 4 and variations tend to vanish. As in all the previous cases, the variations of the results are substantially independent of the CV of the pipe roughness for all the nodes examined.

CONCLUSIONS

A comparative analysis of different probability distributions of times to failure and repair has been made to analyse their impact on the reliability assessment of WDSs. This analysis allows quantification of the influence on the evaluation of the probability of supply failure of inaccuracies that can occur in the choice of probability laws and their parameterisation, on the basis of available data of the network. The variability of the nodal demand and pipe roughness are also taken into consideration.

Results of simulations run for the case of the Anytown network demonstrate that, among the different probability distributions here analysed to describe times to repair, use of the exponential distribution leads to the least precautionary results. In fact, the change of the log-normal distribution with the exponential one gives a reduction of the failure supply probability down to 20% at the control nodes. The substitution of the log-normal distribution with the uniform one, with the same *MTTR*, also gives a reduction of the supply failure probability at two nodes considered, but the differences are less pronounced than in the previous case. As expected, by increasing the *MTTR* for the uniform distribution, the supply failure probability increases at all control nodes.

Dealing with times to failure, variations of the results due to changes in pdfs are comparable with those obtained by varying the distributions of times to repair, although dissimilarity coefficients among different CDFs are smaller. Among the different pdfs taken into consideration, the Weibull distribution leads to the least precautionary results, by reducing the supply failure probability down to 20%. In contrast, replacing the exponential distribution with the gamma pdf and assuming a value of the parameter k slightly smaller than 1, to retain a similar *MTTF* in both cases, leads to more precautionary results and the nodal supply failure probability increases up to 15%.

Although the proposed method was applied to the specific case of the Anytown network, some outcomes are relevant for practical applications. Results show that errors done in the estimate of the probability distribution of times to failure, although smaller than those made in the statistical analysis of times to repair, affect in the same proportion the nodal supply failure probability. This means that the probability distributions of the time to failure must be chosen more carefully than that of the time to repair of pipes. Moreover, the usual practice of adopting the exponential distribution, that is fully defined by the mean value, seems to be reasonable for describing the failure times. In contrast, the adoption of the same pdf for times to repair should be made with care. Indeed, the use of the exponential pdf gives probabilities of occurrence larger than those derived by uniform or log-normal distributions with the same *MTTR* for shorter times to repair and smaller for longer times. Hence, the reliability of the system may be overestimated.

Moreover, from the sensitivity analysis concerning the effect of the coefficient of variation of the nodal demand $CV(Q_r)$ and the pipe roughness $CV(e)$ on the system reliability estimation, we conclude that at the nodes placed far from the source, variations in the results due to the change of the pdf of times to failure and repair are greater for smaller values of $CV(Q_r)$, and discrepancies decrease for increasing $CV(Q_r)$. In contrast, at the nodes closer to the source, variations seem to be not very sensitive to the value of $CV(Q_r)$. Results do not depend on the value of the coefficient of variation of the pipe roughness for all the nodes analysed. Therefore, while the variability of the pipe roughness can be neglected without making significant errors in the system reliability assessment, the value of the coefficient of variation of the nodal demand has a strong impact on the results. In particular, for larger values of $CV(Q_r)$ the influence of the choice of the probability distribution of both the times here considered on the nodal supply failure may become insignificant.

A final aspect that is worth highlighting is the importance of considering the availability of a system of gate valves in existing networks and its influence on the reliability (e.g., Giustolisi *et al.* 2008). Although in the case study analysed in this work, this aspect has been neglected, during simulations the actual valve locations must be taken into account to isolate the portion of the network unavailable throughout the duration of the repair. By ignoring the existing system of gate valves, the results of the simulations, in terms of nodal supply failure probability, may be underestimated in comparison with real-life conditions.

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