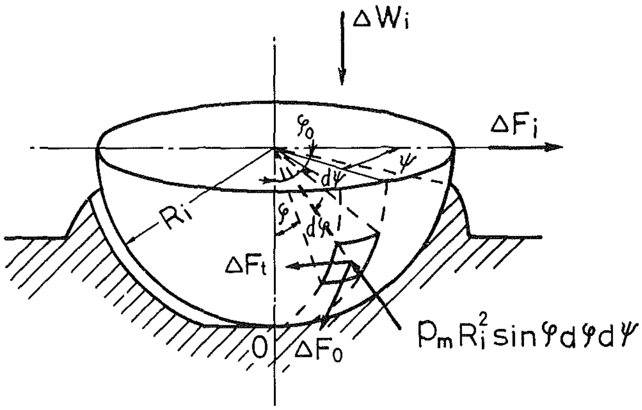

Fig. 17 Deformation of a soft flat surface by a hard spherical asperity

On the other hand, the normal load ΔW_i applied on the spherical asperity is given by

$$\Delta W_i = 2 \int_0^{\pi/2} \int_0^{\varphi_o} p_m R_i^2 \sin\varphi \cos\varphi d\varphi d\psi$$

$$- 2 \int_0^{\pi/2} \int_0^{\varphi_o} \mu p_m R_i^2 \sin^2 \varphi d\varphi d\psi$$

$$= \left\{ \sin^2 \varphi_o - \frac{2}{\pi} \mu \left(\varphi_o - \frac{1}{2} \sin 2\varphi_o \right) \right\} \frac{\pi}{2} p_m R_i^2 \qquad (12)$$

Hence, the coefficient of friction is expressed as follows

$$\mu_s = \frac{\Delta F_i}{\Delta W_i}$$

$$= \frac{\frac{2}{\pi}\left(\varphi_o - \frac{1}{2}\sin 2\varphi_o\right) + \mu\left(\frac{1}{2}\sin^2\varphi_o - \cos\varphi_o + 1\right)}{\sin^2\varphi_o - \frac{2}{\pi}\mu\left(\varphi_o - \frac{1}{2}\sin 2\varphi_o\right)} \qquad (13)$$

When the value of φ_o is very small, the equation (13) can be written as

$$\mu_s = \frac{\frac{4}{3\pi}\varphi_o + \mu}{1 - \frac{4}{3\pi}\mu\varphi_o} \qquad (14)$$

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DISCUSSION

A. W. J. de Gee[6]

It would be interesting to hear if the authors have considered the practical implications of their work. Personally, I believe that the μ_s versus H data for S25C on S25C (Fig. 13) are relevant to the technology of prestressed joints, which derive their strength from static friction between the contacting surfaces. As there is no statistical evidence for a decrease in μ_s with increasing P_n, it can be concluded that, for this combination of materials, the shear strength is proportional to the normal load on the joint. Also, the rather astonishing information is obtained that an increase in surface roughness does not appreciably influence the strength of the joint from 4–8 μ peak-to-valley height (this roughly corresponds to 20–40 μ in. rms, which is not much for a sandblasted surface).

It may be interesting to note that both observations completely concur with the results of experiments with prestressed joints, performed at our institute at Delft.

Upon discussing Fig. 10, the authors state that it is evident that, in spite of the fairly large fluctuations of experimental values, the general trend of these values agrees with equation (14). However, especially as each point is the average of five individual readings, it occurs to me that the systematic character of the "scatter" cannot be ignored. Actually, the data of Fig. 10(*b*) clearly suggest that the results, obtained with indenters of different curvature, fall on parallel curves with a slightly larger average slope than that of the single curve, shown by the authors.

This would mean that the value of μ_s is *not* exclusively determined by φ_0. Also, the value of $(\mu_s)_0$, by the authors found to be 0.12, splits up into four separate values, i.e., 0.05 (R = 0.20 mm), 0.09 (R = 0.32 mm), 0.11 (R = 0.78 mm) and 0.13 (R = 2.22 mm). See Fig. 18.

Do the authors agree that the experimental data point in this direction and can they suggest an explanation?

T. R. Thomas[7]

The authors are to be congratulated on an interesting paper. There are one or two points, however, which might repay clarification.

They define surface roughness in terms of a parameter H which they call the maximum asperity height. On referring to previous papers of theirs [9, 10][8] it appears that H is in fact the separation of the highest peak and the lowest valley in a profile of a certain

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[8] Numbers in brackets designate Additional References at end of discussion.

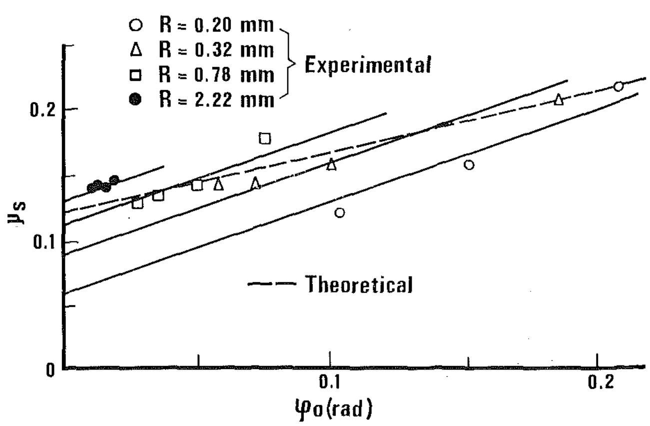

Fig 18 Coefficent of static function, μ_s versus Semiangle subtended by the track φ_0 for the spherical diamond slides on metals

length. It would be helpful if the authors could indicate the relationship between this and definitions of roughness with more immediate physical significance.

The reference cited for the expressions connecting H with the base angle θ of the conical asperities in fact quotes slightly different expressions and does not in any case explain their derivation. This is a pity as it would be interesting to learn the meaning which the authors have attached to θ as applied to a ground surface. Other workers have considered that a better asperity model for such a surface would be a pyramid with a diamond-shaped base [11, 12]. On this view, a ground surface would consist of an array of such pyramids aligned with their long diagonals parallel. Certainly photographic evidence [13, 14] seems to favor a ground surface model of long parallel asperities. However, it should be borne in mind that both models are only crude approximations; real surfaces do not look like an assembly of cones of equal base angle any more than they look like an array of pyramids.

The authors' micrographs of contact areas between a ground tool steel surface and a sandblasted copper surface are puzzling in that they seem to show that the distribution of contact spots is determined by the finish of the smoother surface. Assuming the standard deviations of the surfaces to be in the ratio of their maximum heights, the roughness of the ground surface is 0.01–0.02 times the standard deviation σ of the rougher surface, while the quoted values of nominal pressure and surface hardness correspond to a separation of 2σ [10]. Calculation shows that regions raised by 0.02σ on an ideally flat surface at a separation of 2σ would not increase the contact density under them by more than 5 per cent. Is it possible that what we are seeing is in fact the effect of undetected gross waviness in the ground specimens?

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Authors' Closure

The authors wish to express their appreciation to Messrs. A. W. J. de Gee and T. R. Thomas for their interest in this paper.

It can be concluded in Fig. 13 that the shear strength is proportional to the normal load as pointed out by Mr. de Gee. But we can also consider that the contact surfaces were highly work-hardened by grinding before those sandblasted, so that the normal load and the surface roughness would hardly have an effect on the shear strength of the joint.

It seems to be reasonable from our theory that if the normal load on the spherical diamonds becomes infinitesimal, the coefficient of friction owing to adhesion at interface for all of indenters of different curvature, μ must be ideally constant for the combination of materials. Thus we have doubt whether the experimental results fall on parallel curves with a slightly larger average slope than the broken line as shown in Fig. 18 Then it will need further experiments to confirm whether our experimental results in Fig. 10 clearly suggest the systematic character of the scatter as shown by Mr. de Gee.

We can consider that definition of roughness with more immediate physical significance than peak-to-valley height H provided in JIS (*Japanese Industrial Standard*) will be the root-mean square value σ, which is a standard deviation of the profile curve and for example, necessary to estimate the number and the average radius of contact points between two surfaces in contact [9, 10]. But the measurement of the root-mean square value is more difficult than the peak-to-valley height, so we can determine easily the values of σ from ones of H by using the experimental relationship $H \fallingdotseq 4\sigma$ for sandpaper-finished or sandblasted surfaces.

The slopes of facets over a profile curve obtained from a sandblasted and a ground surface were measured on specially recorded profile of larger magnification in horizontal direction than that in the previous paper [10], the magnification ratio of vertical to horizontal being $1 \sim 10$. The values of slopes of the facets were obtained from the results of $300 \sim 500$ times measurements in the direction of the median line. The relation between H and $\tan\theta$ in the previous paper [10] was obtained from the profile curves of the surfaces prepared by many kinds of finishes such as hand-lapping, sandpaper-finishing, and grinding.

It seems to be reasonable in the case of contact between ground surfaces without tangential microslip to consider that the asperity model would be a pyramid as pointed out by Mr. Thomas. But we could consider from photographic evidence that the asperity model would rather look like a cone than a pyramid as a result of change in shapes of contact asperities in process of tangential microship.

It will be possible that the undetected gross waviness in the ground specimens has an effect on the real contact area as suggested by Mr. Thomas. However, it will be impossible to conclude that the considerable part of the increase in real area of contact by the time gross sliding occurred, is owing to the waviness in the ground specimens.