$$J_4 = \frac{J_1}{\sqrt{J_1^2 + J_2^2}}$$

$$J_5 = \frac{J_2}{\sqrt{J_1^2 + J_2^2}} \qquad (24)$$

In Fig. 6(*b*) the resulting J_4 and J_5 are shown as a function of J_1/J_2, the longitudinal slip to side slip ratio. There is no effect of spin pole offset on the influence of side slip on traction.

8 Traction Results at Constant J_4

Many forms of the traction drives employ a mechanism to alter the normal load according to the magnitude of the torque transmitted. These mechanism are designed to operate the drive at a constant fraction of the maximum available traction, i.e. at a constant fraction of J_4 most commonly about 75 percent. Taking J_4 as a constant (=0.75), it is now possible to replot the results of all the traction calculations on two graphs: (i) The influence of spin on longitudinal slip (Fig. 7) and (ii) the influence of side slip on longitudinal slip (Fig. 7(*b*). These curves may be divided into three regions.

Region (i). Spin (J_3) and side slip (J_2) have a negligible effect on longitudinal slip (J_1). In this region the curves are asymptotic to the result given by equation (22) (Fig. 4)— shown by the elastic line to the left of Figs. 7—7(*b*).

Region (ii). A transition region in which a complete elastic-plastic analysis is required. The curves in this region are derived from the results given in Figs. 5 and 6.

Region (iii). A region in which elastic deformation is negligible so that the curves are asymptotic to the rigid-plastic analysis which gives a linear relationships between J_1 and J_3 and between J_1 and J_2. This is the region to which the analysis of Wernitz and Magi apply.

9 Losses in Traction Drives Under Spin

The total friction losses in the contact of a traction drive under spin consists of two components, the slip loss and the spin loss. Thus the nondimensional loss is defined by:

$$J_7 = J_1 \, x \, J_4 + J_3 \, x \, J_6 = \frac{\overline{G}}{\pi\sqrt{ab}Uh\overline{\tau}c^2}(\Delta U \cdot F_x + \omega \cdot T)$$

The component in the brackets is the actual power loss in the contact. However the efficiency of the drives is better expressed by a loss factor proportional to the ratio of the power lost to the power transmitted

$$LF = \frac{\overline{G}\sqrt{ab}}{h\overline{\tau}_c}\left(\frac{\text{Power lost}}{\text{Power input}}\right) = \frac{\overline{G}\sqrt{ab}}{h\overline{\tau}_c}\left(\frac{\Delta U \cdot F_x + \omega \cdot T}{U \cdot F_x}\right) = J_7/J_4$$

The loss factor thus calculated as a function of slip traction J_4 is shown in Fig. 8 for an aspect ratio of $k = 1$ for various values of spin. It may be observed that the loss factor increases with increasing spin. When the spin is high the loss factor is a minimum at $J_4 \simeq .75$.

The loss factor may also be plotted for a constant traction value J_4. This is shown in Fig. 9 for the various aspect ratios and values of spin. At low values of spin the loss factor is constant and given by the slip J_1 corresponding to that aspect ratio and traction J_4 fraction. When the spin increases the loss factor increases until again it becomes asymptotic to the loss factor for the rigid/plastic analysis.

Conclusion

The work of Johnson and Tevaarwerk on the rheology of fluids in highly loaded EHD contacts suggests that typical traction fluids can be conveniently modelled for traction drive analysis as elastic-perfectly plastic solids. In this model the fluid is characterized by two independent parameters: its shear modulus G and its limiting or critical shear stress τ_c. The mean effective value of these parameters are best found by a traction test on a simple 2 disk machine at the appropriate conditions of speed, pressure and temperature. The traction transmitted by a rolling contact and the power dissipated in viscous heating are governed by not only the slip in the rolling direction (longitudinal slip) but also by the spin which is inevitable in variable speed drives and also by side slip when it arises. The elastic-plastic model has been used to derive traction and loss curves under various combinations of slip, spin and side spin. Three regions of behavior have been identified. If the spin or side slip is sufficiently small, as indicated in Figs. 7 and 7(*b*), the traction is independent of spin and side slip and depends upon the longitudinal slip only as given in Fig. 9. If the spin or side slip are sufficiently large, elastic effects in the fluid and also in the rollers can be neglected and the traditional method of analysis based on the Coulomb friction law (rigid-plastic model) is satisfactory. For moderate spin or side slip, elastic effects are not negligible as the results in this paper show.

For maximum efficiency i.e. for minimum loss factor the drive should transmit about 75 percent of its limiting traction.

Acknowledgment

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DISCUSSION

S. H. Loewenthal[1]

The authors have made a timely and most welcome contribution to the understanding of lubricant behavior in power transmitting, traction contacts. Traction drives are currently the subject of renewed interest due to improvements in their power capacity through better lubricants and cleaner steels and their potential of reducing automotive fuel consumption. The solutions presented in this paper in graphical form to the authors' elastic-plastic model of traction will be of practical value in the design and optimization of contact geometry for these types of transmissions.

In the interest of independently verifying the accuracy of the proposed method, the discusser, for comparison purposes, has arbitrarily selected four sets of traction test data generated with a simple twin disk machine from the report of Gaggermeier [12]. In Gaggermeier's experiments, the effect of side slip under zero spin and the effect of spin under zero side slip on the traction-versus-slip curve was deter-

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[1] NASA Lewis Research Center, Cleveland, Ohio.

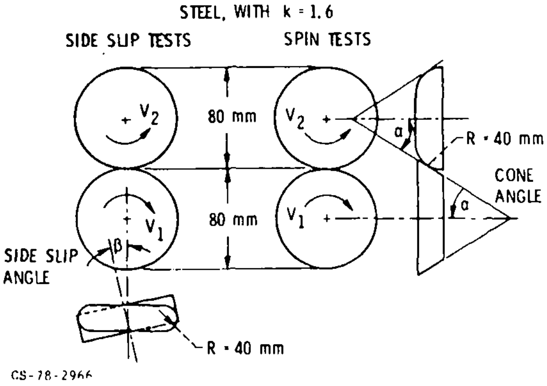

Fig. 9 Test disks for Gaggermeier's traction experiments [12]

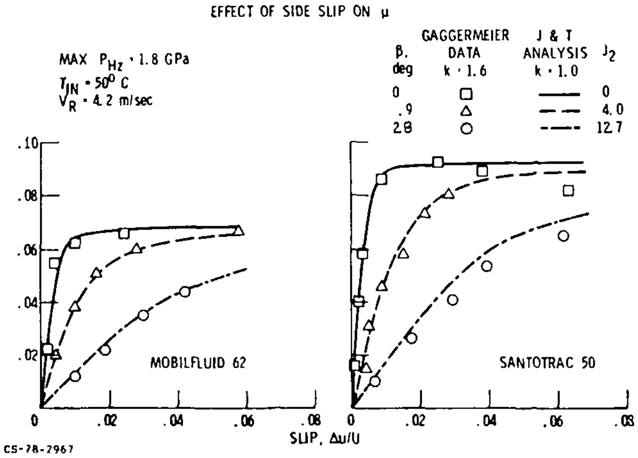

Fig. 10 Comparison of Johnson and Tevaarwerk analysis with test data

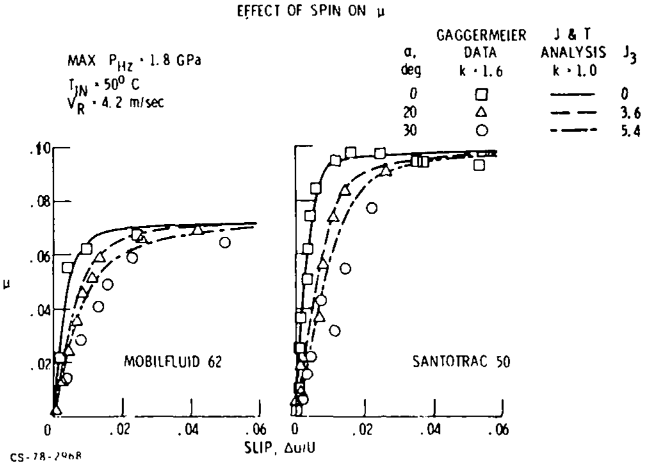

Fig. 11 Comparison of Johnson and Tevaarwerk analysis with test data

mined with the test disk geometry shown in Fig. 9. The data for these tests under one test operating condition with two lubricants appear in Figs. 10 and 11. Also shown on these figures are the predicted traction curves from the authors' analysis. The theoretical curves were generated by the discusser using the graphical solutions shown in Figs. 5 and 6. The values for $\overline{G}$ and $\overline{\tau}_c$ were found from equations (18) and (20) by measuring the initial slope m and peak traction coefficient μp from zero-spin/zero-side-slip-traction curve fitted through Gaggermeier's data.

As shown in Figs. 10 and 11, the predicted values of μ from the Johnson and Tevaarwerk analysis are in rather good agreement with the test data. This is despite the fact that the predictions were derived from Figs. 5 and 6 where the contact ellipticity ratio, k = 1.0, whereas the test disks had k = 1.6. As expected, the isothermal Johnson and Tevaarwerk analysis has a slight tendency to overpredict μ for the higher values of slip, side slip and spin as thermal effects become more pronounced. Thermal corrections to the theory would be helpful but are not essential from a design standpoint. This is because most traction drive contacts are designed to operate on the linear, isothermal portion of the traction curve and at the lowest possible values of side slip and spin.

It is apparent from the side slip data displayed in Fig. 10 that even a relatively small misalignment angle of 2.8 degrees causes a substantial reduction in μ, underscoring the need to maintain accurate alignment of roller components in traction drives. In contrast to this, the traction coefficient data with spin in Fig. 11 under the same operating conditions show surprisingly little adverse effect to spin even for disks with a relatively large cone angle of 30 degrees. Is there a physical explanation which the authors can give for the above observation?

Author's Closure

The authors would like to thank Dr. S. Loewenthal for his discussion and the comparison that he made between the results as published here and the experimental work by Gaggermeir [12].

In answer to the question at the end of the discussion it may be commented that slip angles and spin angles cannot be directly compared for their degree of influence on the traction. What is of importance is the degree of slip that results from these angles. In the case of the side slip experiment, the amount of side slip is given by $U\tan\beta$ while the average slip due to spin on the contact is given by $U\sqrt{\overline{ab}}\sin\alpha/r$, ($\beta$ is the sideslip angle and α is the spin angle). The reason for the smaller influence of the spin can be seen directly in that the term $\sqrt{\overline{ab}}/r$ is of order 10^{-1}, making the degree of slip due to spin quite a bit smaller than for the side slip experiment.