Nordic Hydrology, 27 (4), 1996, 275-294
No part may be reproduced by any process without complete reference

Predicting Sedimentgraphs for a Small Agricultural Catchment

K. Banasik
Warsaw Agricultural University – SGGW, Poland

D. E. Walling
University of Exeter, EX4 4RJ, U.K.

The key components of a sedimentgraph prediction procedure for small agricultural catchments are outlined in the paper. An instantaneous unit sedimentgraph (IUSG) based on the IUH and on a dimensionless sediment concentration distribution is developed, and used for transforming the sediment produced during a specified rainfall duration into a sedimentgraph. Rainfall-runoff-suspended sediment transport data from the River Dart basin, in Devon, UK, are used to evaluate several relationships for sediment yield estimation. The relationship between the lag time of the direct runoff hydrograph and the sedimentgraph is analysed, and the use of these lag times for estimating an IUSG (sediment routing) parameter is examined. The effectiveness of the proposed sedimentgraph prediction procedure is demonstrated by the successful regeneration of a measured sedimentgraph.

Introduction

The type of sediment data required for a drainage basin depends on the nature of the problem to be addressed. Information concerning the form of the sedimentgraph (graph of suspended sediment flux *versus* time) associated with a storm runoff hydrograph is, for example, essential for sediment yield assessment, for providing input data for prediction models of sediment deposition in reservoirs, for designing efficient sediment control structures, and for water quality prediction. In these situations, and particularly for investigating non-point pollution in which sediment must be viewed as both a pollutant in itself and a carrier for other pollutants, it is important to obtain accurate estimates of suspended sediment transport during individual storms. In the absence of intensive sediment monitoring programmes in many drainage basins, use is frequently made of prediction or modelling procedures to estimate storm-period sediment response. However, many of the recently developed erosion and sediment yield models are applicable only to small areas or single fields, and at the same time require large amounts of input data, which are frequently unavailable. The development of simple sedimentgraph prediction procedures could therefore be seen as providing a useful advance by narrowing the gap between the state-of-the-art in research tools and practical needs. The large amount of rainfall-runoff-sediment transport data collected in small catchments by various research organisations and hydrological agencies could provide a basis for estimating the required parameters for such sedimentgraph models. Because the prediction of a sedimentgraph is based on the instantaneous unit sedimentgraph, which is in turn derived from the IUH, and a sediment routing parameter, the approach also has potential for application to ungauged areas. This contribution attempts to develop an improved procedure for predicting storm-period sedimentgraphs and demonstrate the main components of the procedure by using data collected from a small agricultural catchment in Devon, UK.

The Sedimentgraph Approach

One of the first studies of the relationship between the ordinates of the streamflow hydrograph and the sedimentgraph for a small catchment was undertaken by Johnson (1943). He developed a distribution graph of suspended sediment concentration which is analogous to the hydrograph. His results, which demonstrated that there was commonly a rapid initial rise of sediment concentration with increasing discharge and that the suspended sediment concentration reached a peak before the discharge peak, have been widely applied as a general behavioural model of suspended sediment response to heavy rainfall. Rendon-Herrero (1974; 1978) developed the approach further in his studies of the 39 km^2 Bixler Run watershed in Pennsylvania. He proposed use of the unit sedimentgraph (USG) which was defined as the distribution graph of suspended sediment flux resulting from one unit of sediment yield produced in the watershed by rainfall of a given duration. The USG ordinates are obtained by dividing the storm sediment discharge, expressed in mass per time units (M/T) by the total sediment yield for the event, expressed in mass per unit area (M/L^2). The ordinates of the USG have the dimensions of area and time (L^2/T). Different USGs were established for rainfall events of different duration. A relationship between the log-transformed values of effective rainfall depth and sediment yield per unit area for individual events was also established for the watershed for prediction purposes. Separate relationships were established for summer and winter events.

Another technique for estimating the sedimentgraph, based on the instantaneous unit sedimentgraph (IUSG) and the modified Universal Soil Loss Equation (MUSLE) was proposed by Williams (1978). According to his definition the IUSG represents the distribution of the suspended sediment load generated by an instantaneous burst of rainfall producing one unit of runoff. The IUSG therefore represents the product of the instantaneous unit hydrograph (IUH) and the suspended sediment concentration distribution (SCD), *ie.*

$$s_t = u_t \, c_t \tag{1}$$

where s_t are the IUSG ordinates, u_t are the IUH ordinates and c_t are the SCD ordinates computed using the formula

$$c_t = c_0 \exp(-\beta \, t \sqrt{d}) \tag{2}$$

where c_0 is initial (source) sediment concentration, t is time, β is a routing parameter and d is the particle diameter of suspended sediment. Source sediment concentration is assumed to be proportional to the effective rainfall depth, and the source sediment yield is therefore proportional to the square of effective rainfall depth. However, since the MUSLE estimates the total sediment yield for the whole event, the sediment discharge response is computed by convoluting the IUSG with the time distribution of effective rainfall squared. Use of the MUSLE to estimate the event sediment yield makes the method easy to transfer to other regions (although some workers have shown that the MUSLE may produce overestimates in some locations (*e.g.* Madeyski and Banasik 1989; Finney *et al.* (1993). Furthermore, the assumption that source sediment production is proportional to the square of effective rainfall can also be questioned. Williams' own tests of the method based on 50 events from five watersheds located in Texas showed good agreement between the predicted and measured sedimentgraphs. However, he concluded that further tests were needed at other locations to determine whether the model was more generally applicable.

A modified version of the IUSG as proposed by Williams (1978) was used in a study by Singh *et al.* (1981). To estimate the SCD they replaced the sediment routing parameter and the square root of particle diameter by a single parameter B defined as

$$B = \beta \sqrt{d} \tag{3}$$

producing the formula

$$c_t = c_0 \exp(-B \, t) \tag{4}$$

Using rainfall-runoff-sediment yield data for 13 events on a small (4 km^2) watershed near Oxford, Mississippi, the IUH as specified by the Nash (1957) model and an optimization technique for estimating the IUH and IUSG parameters, they concluded that the IUSG is essentially identical to the IUH for a specified event, which in turn suggests that the routing parameter B in Eq. (4) is close to zero (the mean value was 0.08 h^{-1}). In a parallel investigation, Singh and Chen (1981), demonstrated that a linear relationship between log-transformed values of sediment yield and effective rainfall existed for 21 watersheds, ranging in area from 45 to 2,200 km^2, located in ten states of the U.S.A.

Further work on developing a synthetic unit sedimentgraph was undertaken by Chen and Kuo (1986). Because Rendon-Herrero (1974; 1978) produced unit sedimentgraphs for various durations, they focussed on the *one-hour unit sedimentgraph.* This is obtained from measured data by dividing the ordinates of the sedimentgraph, produced by rainfall of one-hour duration, by the total sediment yield for the event. The one-hour unit sedimentgraph (with ordinate units of 1/T) can be defined as the distribution of suspended sediment associated with (or in other words the watershed response to) a one hour storm producing one unit of sediment. The method is very convenient, but requires a means of estimating the sediment produced during each time interval of effective rainfall. The limitation of the procedure is that the authors did not provide a mathematical procedure for deriving the one-hour unit sedimentgraph from measured data.

In the study of Chen and Kuo (1986), the amount of sediment produced during each one-hour time increment of effective rainfall (which is termed the »effective sediment erosion intensity«) is computed from values of the effective rainfall intensity using a relationship which is assumed to be the same as the relationship between the ordinates of the sedimentgraph (»effective sediment yield rate«) and the ordinates of the hydrograph (»effective runoff rate«). Using about 80 recorded events from seven small watersheds (0.2-232 km^2) in the eastern USA (Maryland and Virginia) they derived a regional formula for estimating the USG parameters (q_{sp} – peak sediment discharge, t_{st} – time to peak, T_s – time base) as a function of five watershed characteristics (*i.e.* area, main channel length, main channel slope, mean basin elevation, and a soil erodibility factor).

Another method for estimating the IUSG, based on the time-area diagram concept, is presented by Das and Agarwal (1990). In developing their procedure, they employed a time-area diagram of mobilized sediment (equivalent to effective rainfall in rainfall-runoff analysis), and a sediment storage constant. The amount of sediment mobilized during the event is estimated using a log-log linear relationship with effective rainfall. Using 4-5 events collected each year during the period 1977-80 from a 1,025 km^2 mountainous watershed in India, the sediment storage constant was estimated and shown to increase over the period. This increase was accounted for in terms of the extensive soil conservation measures which had been adopted on the watershed.

The preceding brief review demonstrates that the main components associated with estimating of sedimentgraphs include:

- a means of estimating the sediment produced during the rainfall-runoff event
- a method for distributing the sediment moblised during the event through the duration of the storm

- a unit sedimentgraph technique for transformation of the record of sediment mobilisation into the sedimentgraph for the basin outlet.

In this paper the following aspects of the sedimentgraph approach are considered within a view to developing an improved procedure:

a) development of a new IUSG procedure based on the IUH derived using the Nash model and on a dimensionless sediment concentration distribution (DSCD). The IUSG is then used for estimating the one-hour USG, which in turn is employed for transforming the sediment produced by a given rainfall duration into a sedimentgraph,
b) use of rainfall-runoff-suspended sediment transport data available for the River Dart basin in Devon, UK to evaluate a sediment yield prediction procedure. The form of commonly used relationships (*i.e.* MUSLE, or sediment yield versus effective rainfall amount), is compared with other relationships using rainfall characteristics,
c) establishment of relationships for estimating the distribution of sediment generation (analogous to effective rainfall) during rainfall events. The distribution of sediment produced during the rainfall duration is needed to provide the values to be transformed (using the above one-hour USG) into the sedimentgraph,
d) examination of the relationship between the lag time of the direct runoff hydrograph and sedimentgraph for recorded events. These lag times are needed for the estimation of IUSG parameters,
e) demonstration of the effectiveness of the proposed procedure in regenerating a measured sedimentgraph.

The IUSG Procedure

As applied in this paper, the Instantaneous Unit Sedimentgraph (IUSG) is defined as the time distribution of suspended sediment flux associated with an instantaneous burst of rainfall producing one unit of *sediment*. The definition is similar to that employed by Williams (1978), except that in his definition the IUSG is the response to one unit of effective rainfall. It is also similar to that of Chen and Kuo (1986), but their USG was derived for hourly time intervals, and the instantaneous graph was not developed.

The IUSG presented here is based on the IUH derived by Nash (1957) *i.e.*

$$u_t = \frac{1}{k\,\Gamma(N)} \left(\frac{t}{k}\right)^{N-1} \exp\left(\frac{t}{k}\right) \tag{5}$$

and on the first-order kinetic equation (similar to Eq. (4), *cf.* Hammer and Mac Kichan 1981 p. 313) written in dimensionless form and termed the dimensionless sediment concentration distribution (DSCD)

$$c_t = \exp(-B\,t) \qquad (6)$$

where u_t are the ordinates of the IUH (h^{-1}), N and k are the Nash model parameters; N is the number of reservoirs, k is the retention time of the reservoirs (h), c_t are the ordinates of the DSCD, B is the sediment routing parameter (h^{-1}), and t is time (h). Insertion of Eqs. (5) and (6) into Eq. (1), and dividing the right hand side of the resulting equation by

$$g \equiv \int_0^\infty u_t\, c_t\, dt \qquad (7)$$

produces the following formula (Banasik 1994)

$$s_t = \frac{B\,k+1}{k\,\Gamma(N)} \left(t\left(B+\frac{1}{k}\right)\right)^{N-1} \exp\left(-t\left(B+\frac{1}{k}\right)\right) \quad \text{for} \quad B > -\frac{1}{k} \qquad (8)$$

where s_t are the IUSG ordinates (h^{-1}). The IUSG has two parameters, N and k which are also IUH parameters and a third, the sediment routing parameter B. It is clear that when B equals zero the right hand side of Eq. (8) assumes the form of the Nash IUH (Eq. (5)), which means that the shape of the IUSG and IUH will be the same. This was indicated in the discussion of the study of Singh *et al.* (1981), introduced above.

One of the characteristic values of the Nash model is the retention of the system or lag time, which is defined as the time elapsed between the centroids of effective rainfall and the direct runoff hydrograph.

For the IUH the lag time is estimated using the formula

$$\text{LAG} \equiv N\,k \qquad (9)$$

For the IUSG, the lag time (LAGs) can be calculated using the equation

$$\text{LAGs} \equiv \frac{N\,k}{1+B\,k} \qquad (10)$$

Making use of Eq. (10), the routing parameter B can be computed using the formula

$$B \equiv \frac{N}{\text{LAGs}} - \frac{1}{k} \qquad (11)$$

or

$$B \equiv \frac{\text{LAG/LAGs} - 1}{k} \qquad (11a)$$

Since the IUH parameters (N and k) can be estimated from rainfall-runoff analysis, and the lag time for the sedimentgraph, LAGs, can be estimated from measured data, the third parameter of the IUSG, *i.e.* the routing parameter B, can be estimated using Eq. (11).

The distribution of sediment produced during the storm event is needed to provide the values to be transformed by the IUSG into the sedimentgraph and for finding the centroid of sediment production to compute the sediment lag time.

Analysis of Measured Rainfall-Runoff-Suspended Sediment Transport Data

The Study Catchment and Data Availability

The River Dart is a west-bank tributary of the River Exe in South-West England, with a catchment area of 46 km^2. The catchment has an absolute relief of 228 m. Permanent pasture dominates the land use. The runoff and suspended sediment data have been collected by the Department of Geography at the University of Exeter for the gauging station at Bickleigh, which is also equipped with a continuous recording turbidity meter. Estimation of suspended sediment concentration was based on well defined relationships between turbidity at the measurement point and discharge-weighted mean suspended sediment concentration in the river cross-section. The relationships were established by field calibration. Rainfall data were available for Way Farm, located within the catchment, where two autographic rain recorders were installed. The rainfall data were not corrected for aerodynamic errors. A more detailed description of the watershed, the soil types and the location of the rainfall and river gauging stations is provided by Webb and Walling (1985).

For the period 1982-84, all those storm runoff events in which the peak discharge was greater than three times the discharge at the time at which the direct runoff began, and for which rainfall data existed, have been selected for analysis. This provided a total of 39 events, of which 6 evidenced multipeaked storm hydrographs. The rainfall amounts associated with the events varied from 8.9 mm to 36.2 mm with a mean value of 16.1 mm. The peak discharges of the events varied from 1.52 m^3s^{-1} to 26.6 m^3s^{-1}, with a mean value of 7.9 m^3s^{-1}. The maximum values of suspended sediment concentration for the individual runoff events varied between 99 mgl^{-1} and 1,475 mgl^{-1}, with a mean value of 560 mgl^{-1}. The variation of suspended sediment concentration measured in the River Dart at Bickleigh during a sequence of storm hydrographs is discussed in detail by Walling and Webb (1987). Rainfall, discharge and suspended sediment concentration data recorded at one hour time interval were used in the study.

Direct runoff hydrographs and sedimentgraphs were obtained from available discharge and suspended sediment concentration records, after subtracting base flow and base sediment transport.

Runoff-Sediment Yield and Rainfall-Sediment Yield Relationships

The aim of this part of the analysis was to establish a relationship between storm-period sediment yield, Yd (in Mg), as the dependent value, and various measures of rainfall and runoff as independent variables. Such relationships are needed for estimating the sediment produced by a rainfall-runoff event, and can also provide a basis for establishing the distribution of sediment production during a rainfall event. The relationships evaluated in the paper are as follows:

Relationship-I)

$$Yd = a(V_d Q_p)^b \tag{12}$$

where Yd is direct sediment yield (Mg), V_d is direct runoff (m^3) and Q_p is the peak discharge of the direct runoff hydrograph (m^{3}s^{-1}),

Relationship-II)

$$Yd = a\,H^b \tag{13}$$

where H is effective rainfall (mm)

Relationship-III)

$$Yd \equiv a\left(\sum_{j=1}^{n} \Delta H_j^2\right)^b \tag{14}$$

where ΔH_j is the effective rainfall in the j^{th} one hour time interval (mm), n is the number of time intervals

Relationship-IV)

$$Yd = a\left(\sum_{j=1}^{n} \Delta H_j\, \Delta P_j^{b1}\right)^b \tag{15}$$

where ΔP_j is the measured rainfall in the j^{th} one hour time interval (mm). The effective rainfall distribution, *i.e.* the ΔH_j-values used in relationships III and IV, and also in further computations was established using the SCS-CN (Curve number) method (USDA-SCS 1972). After the retention parameter S of the SCS-CN method was computed from the measured rainfall and the hydrograph of direct runoff for each event, the cumulative effective rainfall for the event was estimated using the formula

$$H_t = \begin{cases} 0 & , \text{ for } P_t - 0.2\,S \leq 0 \\[2ex] \dfrac{(P_t - 0.2\,S)^2}{P_t + 0.8\,S} & , \text{ for } P_t - 0.2\,S > 0 \end{cases} \tag{16}$$

where: H_t is the cumulative effective rainfall (mm) in time t, P_t is the cumulative rainfall (mm), and S is the watershed retention parameter (mm) related to total measured rainfall P (mm) and direct runoff H (mm) as

$$S \equiv 5\left(P + 2\,H - (4\,H^2 + 5\,H\,P)^{0.5}\right) \tag{17}$$

The amount of effective rainfall during each time intervals was computed as

$$\Delta H_j \equiv H_{(t-\Delta t,\, t)} = H_t = H_{(t-\Delta t)} \quad \text{for} \quad t = j\,\Delta t\ , \quad j = 1, 2, \ldots\ n \tag{18}$$

The runoff factor from the MUSLE (Williams 1975), *i.e.* $V_d\ Q_p$, was used in relationship I as the independent variable, because the equation has been widely applied for sediment yield estimation. The power function form of relationship II was selected since it has been used in several similar investigations (*e.g.* Singh and Chen 1981; Chen and Kuo 1986; Das and Agarwal 1990). The forms of relationships III and IV were selected with a view to also using the relationship for estimating the time distribution of sediment production during a storm.

The estimates for parameters a, b and $b1$ in relationships I-IV and the coefficients of determination and standard error of the estimate associated with the log-transformed relationships between observed and predicted sediment yield values are shown in Fig. 1 and listed in Table 1. It is evident from Fig. 1 and Table 1 that a strong linear relationship exists between the log transformed values of observed sediment yield for the 39 events and those estimated using relationships I-IV. Coefficients of determination for the relationships range between 0.810 and 0.865 (*i.e.* the amount of variance explained by the relationships varies from 81.0% to 86.5%). The coefficient of determination obtained using effective rainfall as the independent variable (R-II), is lower than obtained using the MUSLE runoff factor (R-I). However, slightly better relationships in comparison with R-I, were obtained using the rainfall parameters *i.e.* R-III and R-IV.

In the case of relationship I (Eq. (12)), which employs the MUSLE runoff parameter ($V_d\ Q_p$), the value of the exponent $b = 0.552$ is very close to that used in the MUSLE ($b \equiv 0.56$). Incorporating the other USLE factors for the Dart catchment, the relationship takes the form

$$Yd = 1.18 (V_d Q_p)^{0.552}\ K\ L\ S\ C\ P \tag{19}$$

where K is the soil erodibility factor (Mg ha h/(MJ ha cm)), L, S, C and P are the dimensionless USLE factors for slope length, slope steepness, cover-management and control practice, respectively. In the MUSLE (using the same units for all the factors) the value of the constant increases from 1.18 to 8.96 indicating that the MUSLE would overpredict single event sediment yields in the Dart watershed by about 7.5 times (*i.e.* 8.96/1.18) for values of $V_d\ Q_p$ equal to one unit. The overprediction would only increase slightly with increases in the value of $V_p\ Q_p$. The form of Eq. (19) is convenient for prediction, and particularly for transferring the relationship to other watersheds with similar hydrological conditions, but no information is provided regarding the variation of sediment production during a storm.

The value of parameter b in relationship II ($b = 0.982$ *i.e.* very close to 1) suggests that the production of sediment by different events is nearly proportional to the amount of effective rainfall. This relationship could also be applied to sediment production during an event, if it assumed that the same amount of effective rainfall at the beginning of an event and at the end of an event would produce approximately

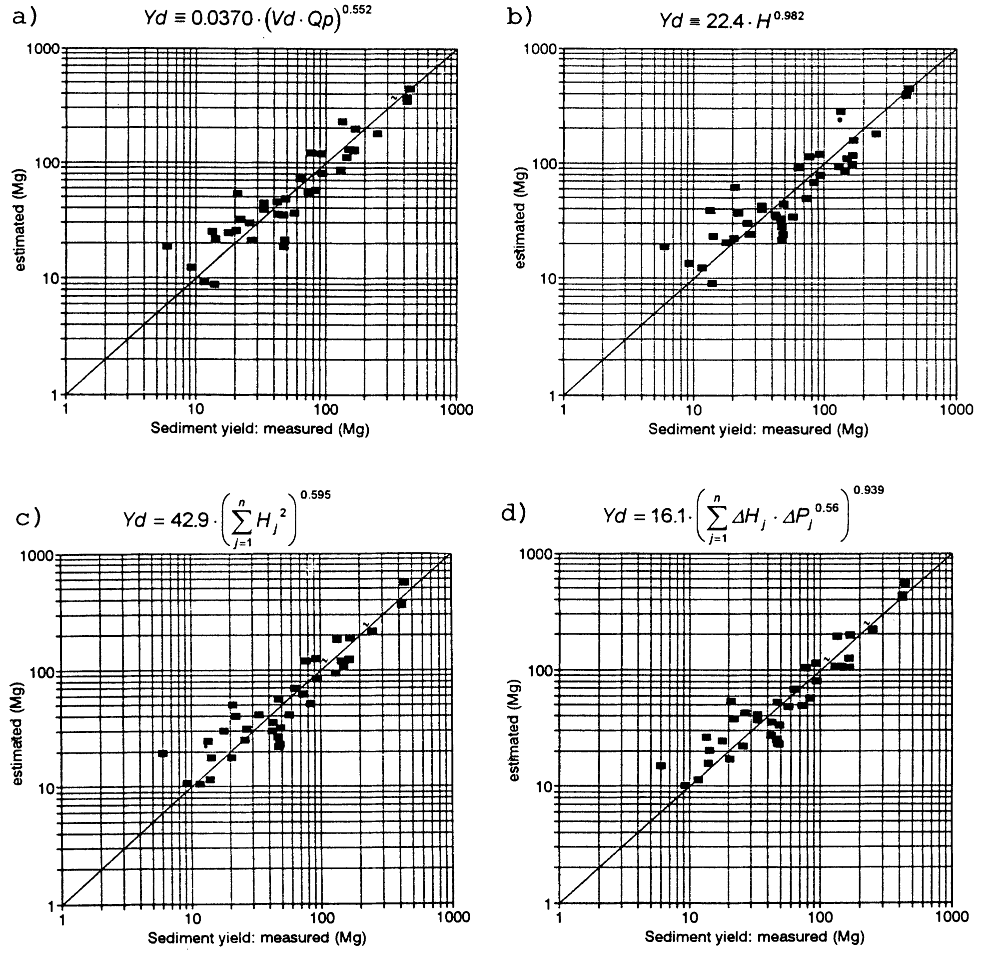


Fig. 1. A comparision of measured sediment yield and values estimated using relationships: R-I (a), R-II (b), R-III (c) and R-IV (d).

Table 1 – Characteristics of relationships I-IV

Relationship	Parameters		Coefficient of determination-r^2	Standard error of estimate-SEE
	a	*b*		
R - I	0.0370	0.552	0.847	0.187
R - II	22.7	0.982	0.810	0.209
R - III	42.9	0.595	0.855	0.182
R - IV*	16.1	0.939	0.865	0.176

*$b1 = 0.56$

the same amount of sediment. However the value of exponent *b* has been found to be greater than 1 in many investigations (Singh and Chen 1981; Chen and Kuo 1986) suggesting that this assumption is unlikely to be correct.

The exponent of R-III (*b*=0.595 *i.e.* less than 1) suggests that for different events, sediment production is not a linear function of the square of effective rainfall. Assuming the same relationship for sediment production during an event, the amount of sediment produced by one unit of effective rainfall at the beginning of the event would be higher than at the end of the event. A similar conclusion could be drawn from the parameters of R-IV, although this effect would be less marked since the value of *b* is closer to 1.0.

Sediment Production and Sediment Yield Estimation

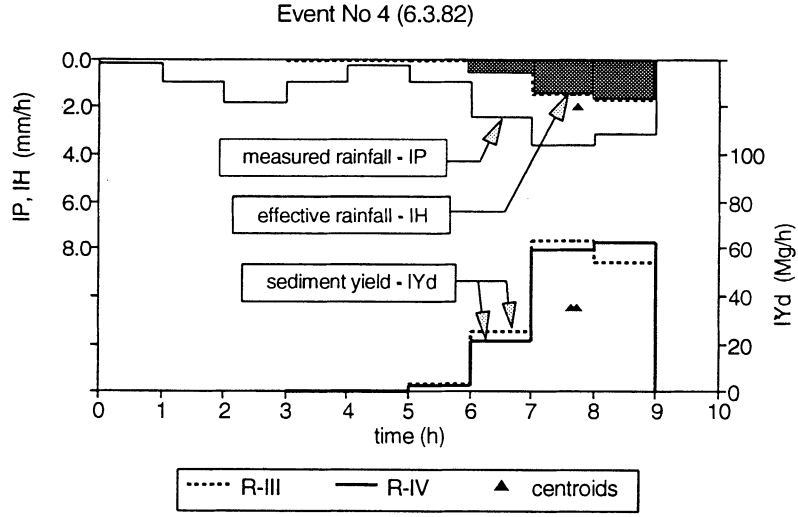
Relationships R-III and R-IV appear to conform to generally-accepted concepts concerning sediment production during a rainfall event (*cf.* Johnson 1943; Walling and Webb 1987), *i.e.* that more sediment is produced at the beginning of an event than later in the event. Considering two (or one two-peaked) hypothetical events with similar direct runoff hydrographs, occurring one after the other, with the first following a dry period, sediment production during the first event will commonly be greater than during the second event. R-IV should therefore produce a better prediction of sediment yield and of sediment distribution during the two events (and also during the two-peaked event), because a lower value of total rainfall is needed to produce a second runoff hydrograph of the same size as the first (*i.e.* involving the same effective rainfall), because of the increased catchment moisture status. Eq. (15) indicates that the lower value of total rainfall associated with the second peak will result in a reduced amount of sediment. However for a hypothetical rainfall event of long duration with a constant intensity, when the effective rainfall intensity will increase with time, R-III may give a better approximation of the distribution of sediment production during the event. In different situations, different assumptions concerning sediment mobilization by rainfall may be required. It seems that the first hypothetical example is more likely than the latter. Because R-IV also produces a higher coefficient of determination, this relationship has been used in preference in subsequent analysis. Sediment production for each time interval of effective rainfall can be estimated, by analogy with Eq. (18), using the following formula

$$\Delta Yd_j = Yd_j - Yd_{j-1} \quad \text{for} \quad j = 1, 2, \ldots n \tag{20}$$

were Yd_j is the cumulative sediment production computed from R-III or R-IV *i.e.*

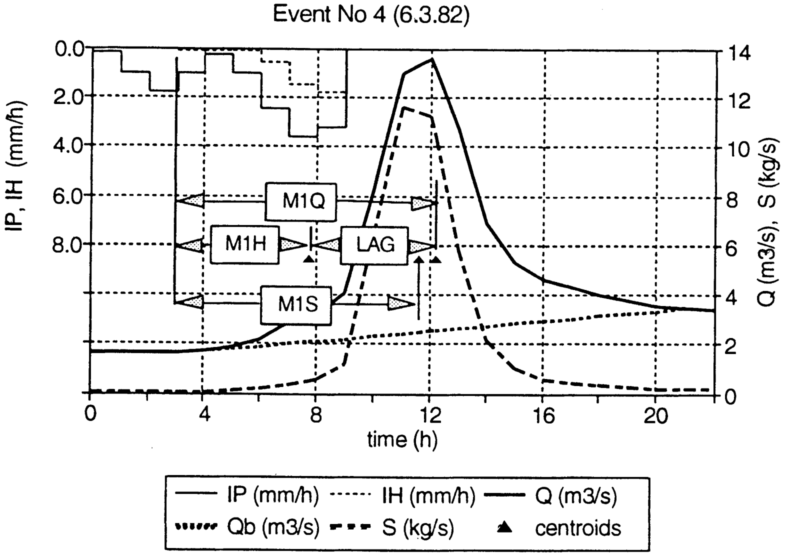
$$Yd_j = a\,\Big(\sum_{i=1}^{j} \Delta H_i\, \Delta P_i^{b1}\Big)^{b} \quad \text{for} \quad j = 1, 2, \ldots n \tag{21}$$

An example of the distribution of sediment production during a rainfall event, estimated using R-III and R-IV is shown in Fig. 2. The distribution of sediment produc-



Event No 4 (6.3.82)

Fig. 2. The distribution of effective rainfall and sediment production estimated using R-III and R-IV, during an event (6.3.82). The centroids of the hyetograph of effective rainfall and the sediment production graphs are also shown (*IP* is intensity of measured rainfall ($\Delta P/\Delta t$), IH is intensity of effective rainfall ($\Delta H/\Delta t$) and *IYd* is intensity of sediment production ($\Delta Yd/\Delta t$)).



Event No 4 (6.3.82)

Fig. 3. The interrelationship between the centroids of effective rainfall, the hydrograph of direct runoff and the sedimentgraph for the event of 6.3.82; *M*1*H*, *M*1*Q* and *M*1*S* represent the first (statistical) moments of effective rainfall, the direct runoff hydrogaph and the sedimentgraph respectively; LAG = lag time for the hydrograph.

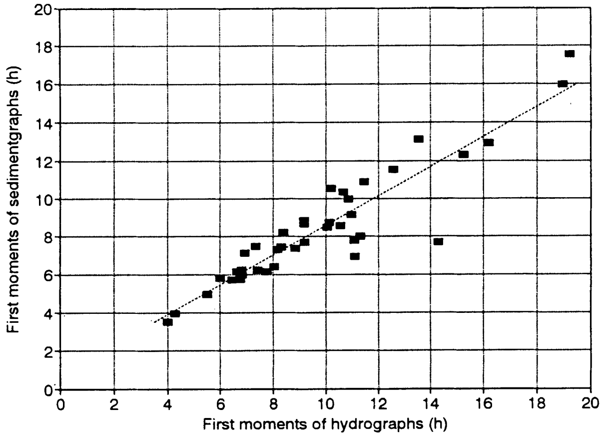


Fig. 4. The relationship between the first statistical moments of the sedimentgraphs and hydrographs for the River Dart basin.

tion during a rainfall event also determines the location of its centroid (Fig. 2), which in turn influences the value of LAGs, defined for the IUSG by Eq. (10), and for a measured event as the elapsed time between the occurrence of the centroids of sediment production and of the direct sedimentgraph. An example of the interrelationship between the centroids of the effective rainfall, the hydrograph of direct runoff and the sedimentgraph are shown in Fig. 3. The precision with which the centroids of the direct runoff hydrograph and the sedimentgraph can be estimated depends primarily on the precision of the measured data (because the base flow and the base sediment which are separated, represent only a small part of the measured hydrograph and sedimentgraph), but the precision of the effective rainfall and particularly the sediment production estimates depends also on assumptions about their distributions. Comparison of the first moments of the direct runoff hydrographs and sedimentgraphs provides an indication of the difference in centroid location and permits some suggestions regarding the differences between LAGs and LAG. The relationship between the first statistical moments of the sedimentgraphs and hydrographs for the 39 analyzed events is presented in Fig. 4. The regression relationship between the moments, shown as a dashed line in Fig. 4, was established as

$$M1S = 0.776 + 0.781\ M1Q \qquad (22)$$

where $M1S$ and $M1Q$ are the first statistical moments of the sedimentgraph and direct runoff hydrograph (h), respectively.

The relationship between the lag time of the sedimentgraphs computed using the two relationships for estimating sediment production (R-III and R-IV) and the lag time of the hydrographs are shown in Fig. 5. The following relationships were computed

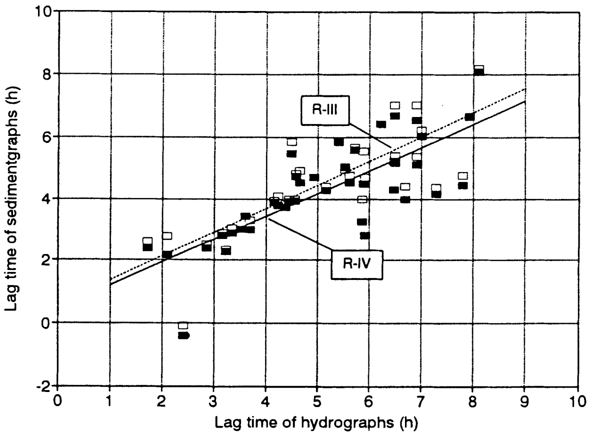


Fig. 5. The relationship between the lag time of the sedimentgraphs computed using the two relationships for estimating sediment production (R-III – open squares and dotted line, and R-IV – filled squares and solid line) and lag time of the hydrographs.

$$LAG_{s(III)} \cong 0.557 + 0.775\ LAG \qquad (23)$$

and

$$LAG_{s(IV)} = 0.447 + 0.745\ LAG \qquad (24)$$

where $LAG_{s(III)}$, $LAG_{s(IV)}$ are the lag times of sedimentgraphs generated using relationships R-III, and R-IV (h), and LAG is the lag time of the hydrograph (h). It is evident from Eqs. (22)-(24) and Figs. 4 and 5 that there is only a very small difference between the moments and lag times of the hydrograph and sedimentgraph for small values of $M1Q$ ($\cong$4) and LAG ($\cong$2). The difference increases for greater values of hydrograph moment and LAG, *i.e.* the sedimentgraph values increases more slowly than those of the hydrographs. This means that for events of short LAG (about 2 h) in the Dart catchment, the shape of the IUSG and IUH will be similar (B about 0), and that for events with a greater LAG the routing parameter will be higher, causing the IUSG to be more slender than the IUH.

Having estimated LAGs for the sedimentgraph, and the N and k parameters of the IUH from rainfall-runoff analysis, the third parameter of the IUSG – the sediment routing parameter B, can be computed from Eq. (11). The values of N, k and B so determined for the Dart river basin were found to vary from event to event, which variation had been also reported in several previous investigations (*e.g.* Sarma *et al.* 1973; Banasik 1994). These parameters, estimated for each event by using the measured data, were applied for regeneration of storm hydrograph and sedimentgraph.

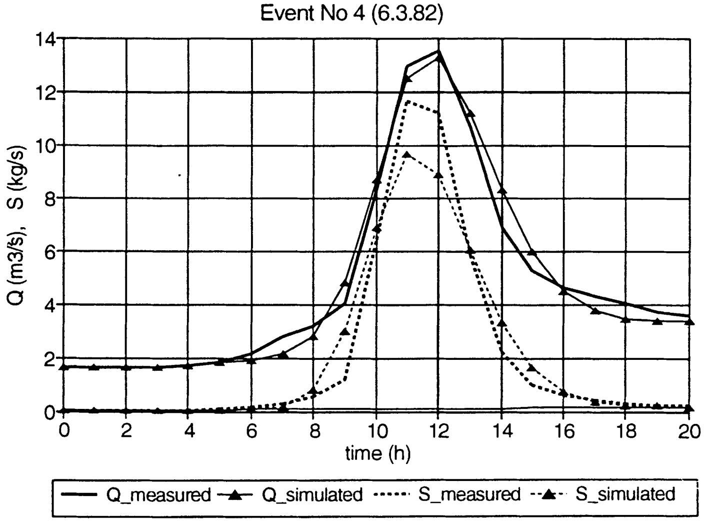


Fig. 6. An example of measured and simulated hydrographs and sedimentgraphs (sediment production was estimated using Eqs. (20) and (21)).

Sedimentgraph Estimation

The sedimentgraph may be computed by convolution of the USG with the graph of sediment production. The unit sediment graph USG, which is derived from the IUSG, and expresses the temporal distribution of sediment transport at the outlet of the watershed associated with 1 Mg of sediment produced by effective rainfall falling in time interval Δt is

$$us_k \equiv \frac{1}{3.6\,\Delta t}\int_{t-\Delta t}^{t} s_\tau \, d\tau \quad \text{for} \quad t=\Delta t\, k\,, \quad k \equiv 1,2,\ \ldots\ m \qquad (25)$$

where us_k are the ordinates of the USG (kg s^{-1} Mg^{-1}), Δt is time step used in computation (h), s_τ are the ordinates of IUSG computed from Eq. (8) (h^{-1}).

The sedimentgraph may be computed by the following convolution

$$\mathrm{SED}_i = \sum_{j=1}^{\min(i,n)} \Delta Yd_j \, us_k \quad \text{for} \quad k=i-j+1\,, \quad i=1,2,\ \ldots\ m+n-1 \qquad (26)$$

where SED_i are the ordinates of sedimentgraph (kg s^{-1}), m is the number of IUSG ordinates, n is the number of time increments in the sediment production graph.

The viability of the above sedimentgraph and hydrograph estimation procedure, has been tested by assessing their success in regenerating measured hydrographs and sedimentgraphs. In these tests the total amounts of effective rainfall and sediment produced for a given event were derived directly from the measured runoff hydrograph and sedimentgraph. The time distribution of these values were, however, computed according to Eqs. (16)-(18) (hyetograph of effective rainfall) and Eqs. (20)-(21) (sediment production graph) after the parameters *CN* and *a* had been established.

The parameters of the IUH and IUSG, estimated for each event from the data (hyetograph of effective rainfall and direct runoff hydrograph as well as from the sediment production graph and sedimentgraph) were used in the regeneration of the hydrograph and sedimentgraph. The regeneration thus undertaken to demonstrate the potential application of the procedure for hydrograph and sedimentgraph simulation for cases in which the IUH and IUSG parameters as well as the effective rainfall and sediment production graph can be estimated. An example of measured and simulated hydrographs and sedimentgraphs for the event of 6.3.82 is given in Fig. 6. Qualitative comparison of the observed and simulated data based on visual match, peak reproduction, *etc.*, as well as various statistical measures, as applied by Sarma *et al.* (1973) and including the special correlation coefficient and the integral square error, confirm the effectiveness of the simulation procedure (Banasik 1990; 1994).

Potential Applications

The IUSG procedure presented above affords a simple technique for predicting storm period sedimentgraphs for watersheds for which ~~the primary~~ components of the procedure, *i.e.* the total sediment yield and sediment production graph and the parameters of the IUSG, can be estimated. The procedure may thus be applied in situations where rainfall-runoff-suspended sediment flux data (gauged catchments) are available or in ungauged catchments where existing formulae can be used to assemble the required estimates.

The results of the analysis of the river Dart data could thus be used in the following applications:

i) Relationships I-IV (Eqs. (13)-(15)) and the associated parameters given in Table 1 could be used for computing the sediment yield for events without sediment measurements. Relationships I and II could be used if the direct runoff hydrograph is estimated (*H* in R-II represents effective rainfall which is equal to direct runoff). Relationships III and IV require only the rainfall and effective rainfall hyetograph data, so they can be applied in cases where runoff data is unavailable, although parameter *CN* must be known. Because permanent pasture dominates the land use of the river Dart catchment, and no clearly marked influence of different seasons on the sediment yield was identified, no land cover parameter was used in the relationships I-IV. This, and also the lack of inclusion of soil type and topographic variables, limits the applications of these relationships to other catchments, even with similar climatic conditions.

ii) Eq. (19), which takes the form of a recalibrated MUSLE, could be used for predicting the storm period sediment yield from small ungauged catchments experiencing similar climatic conditions.

iii) R-III and R-IV appear to afford an effective basis for distributing sediment production during storm event for catchments for which only sediment yield estimates exists.

The main limitation of the applications of the prediction procedure to ungauged catchments would appear to be the specificity of most sediment yield estimation formulae (Hadley *et al.* 1985), the high variability of the IUH parameters (*e.g.* this study, Sarma *et al.* 1973; Banasik 1994) and limited information on the values of the IUSG (sediment routing) parameter.

Conclusions

The following conclusions can be drawn from this study:

1) A range of different procedures for using the USG and IUSG for sedimentgraph prediction have been presented in the literature. However, common features include: i) a procedure for sediment yield estimation, ii) assumptions about sediment production during an event, iii) use of a function or relationship to transform the graph of sediment production into the sedimentgraph.
2) The procedure presented in this paper comprises: i) use of a relationship for estimating sediment production and yield for an event based on the associated rainfall characteristics, ii) development of an instantaneous unit sedimentgraph as a function for transforming the sediment production graph into the sedimentgraph. The IUSG defined here has three parameters, two of which are common with the IUH, which are easy to establish using measured data.
3) The analysis of rainfall-runoff-sediment transport data from the River Dart catchment showed that: i) a strong linear relationship exists between log-transformed values of event sediment yield and the runoff parameters of the MUSLE, but the original MUSLE would overestimate the sediment yield from that watershed significantly (× 7.5 or more), ii) similar strong relationships existed with rainfall parameters. The latter relationships can also be used for estimating the distribution of sediment production during a rainfall event.
4) Analysis of measured hydrographs and sedimentgraphs has also shown that, with the exception of events with small values of LAG, the lag times of the sedimentgraph are shorter than the lag times of the hydrographs.

5) The study has demonstrated usefulness of the approach in regenerating sedimentgraphs for the study catchment. The sedimentgraph procedure presented could provide an effective basis for predicting storm event sedimentgraphs from ugauged agricultural catchments with similar climatic conditions. However, further analysis using data from other catchments is required to verify the relationship between rainfall characteristics and sediment production, as well as to identify the factors controlling the value of the sediment routing parameter – *B*.

Acknowledgment

The study described in this paper was initiated within the framework of the British Council Academic Link Programme (WAW-992/313) and continued with the support of the CEC's Action for Cooperation in Sciences and Technology with Central and Eastern European Countries (CIPA3510PL922573 and TEMPUS-IMG-94-PL-2219). The support provided by these organisations is gratefully acknowledged.

References

Banasik, K. (1990) Sedimentgraph model for a small watershed, Proc. Int. Workshop on Application of Mathematical Models for the Assessment of Changes in Water Quality. Atelier International UNESCO-IAHS-ENIT, Tunis, pp. 302-311.

Banasik, K. (1994) Model sedymentogramu wezbrania opadowego w małej zlewni rolniczej (Sedimentgraph model of rainfall event in a small agricultural watershed). Treatises and Monographs, Publications of Warsaw Agricultural University – SGGW (in Polish), 120 pp.

Chen, V. J., and Kuo, C.Y. (1986) A study on synthetic sedimentgraphs for ungaged water sheds, *J. Hydrology*, Vol. 86, pp. 35-54.

Das, G., and Agarwal, A. (1990) Development of conceptual sediment graph model, *Trans. in Agriculture, ASAE, Vol. 33*(1), pp. 100-104.

Finney, V. L., Banasik, K., Needham, S., and Young, R. A. (1993) Comparision of DR-USLE, SEGMO and AGNPS with two rainfall events, Proc. Int. Symp. Runoff and Sediment Yield Modelling, SGGW-Warsaw, pp. 195-200.

Johnson, J. W. (1943) Distribution graphs of suspended-matter concentration, *Trans. ASCE, Vol. 108*, pp. 941-964.

Hadley, R. F., Lal, R., Onstad, C. A., Walling, D. E., and Yair, A. (1985) Recent developments in erosion and sediment yield studies, Technical Documents in Hydrology, IHP, UNESCO, Paris, 127 pp.

Hammer, M. J., and Mac Kichan, K. A. (1981) *Hydrology and Quality of Water Resources*, Wiley & Sons, New York, 486 pp.

Madeyski, M., and Banasik, K. (1989) Applicability of the modified universal soil loss equation in small Carpathian watersheds. *Catena Supplement, Vol. 14*, pp. 75-80, Cremlingen.

Nash, J. E. (1957) The form of the instantaneous unit hydrograph, International Association of Scientific Hydrology, Publication No 42, pp. 114-118.

Rendon-Herrero, O. (1974) Estimation of washload produced on certain small watersheds. *J. Hydraulics Division, Proc. ASCE, Vol.100, No. HY7*, pp. 835-848.

Rendon-Herrero, O. (1978) Unit sediment graph, *Water Resour. Res., Vol. 14*, No. 5, pp. 889-901.

Sarma, P. B. S., Delleur, J. W., and Rao, A. R. (1973) Comparision of rainfall-runoff models for urban areas, *J. Hydrol., Vol. 18*, pp. 329-347.

Singh, V. P., Baniukiewicz, A., and Chen, V. J. (1981) An instantaneous unit sedimentgraph study for upland watersheds, Proc. Int. Symp. on Rainfall-Runoff Modeling, – Modeling Components of Hydrologic Cycle, Mississippi State University, Mississipi, USA, p. 539-554.

Singh, V. P., and Chen, V. J. (1981) On the relation between sediment yield and runoff volume, Proc. Int. Symp. on Rainfall-Runoff Modeling, – Modeling Components of Hydrologic Cycle, Mississippi State University, Mississipi, USA, p. 555-570.

USDA – Soil Conservation Service (1972) *National Engineering Handbook*, Section 4, Hydrology, Washington D. C.

Walling, D. E., and Webb, B. W. (1987) Suspended load in gravel-bed rivers: UK experience. In: *Suspended Sediment in Gravel-bed Rivers*, eds. C. R. Thorne, J. C. Bathurst and R. D. Hey, John Wiley & Sons Ltd, London.

Webb, B. W., and Walling, D. E. (1985) Nitrate behaviour in streamflow from a grassland catchment in Devon, *U. K. Water Res. Vol. 19*, No. 8, pp. 1005-1016.

Williams, J. R. (1975) Sediment yield prediction with universal equation using runoff energy factor. In: Present and Prospective Technology for Predicting Sediment Yield and Sources, Proc. Sediment-Yield Workshop, ARS-40, pp. 244-252, USDA – Sedimentation Laboratory, Oxford, Miss.

Williams, J. R. (1978) A sediment graph model based on an instantaneous unit sedimentgraph, *Water Resourc. Res., Vol. 14, No. 4*, pp. 659-664.

First received: 14 April, 1995
Revised version received: 22 December, 1995
Accepted: 15 January, 1996

Address:
K. Banasik,
Warsaw Agricultural University - SGGW,
Department of Hydraulic Structures,
ul. Nowoursynowska 166,
02-787 Warsaw,
Poland.

D. E. Walling,
University of Exeter,
Department of Geography,
Amory Building, Rennes Drive,
Exeter, Devon,
EX4 4RJ, U. K.