Capacity optimization of hydropower storage projects using particle swarm optimization algorithm

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ABSTRACT

A mixed integer optimization model is formulated for capacity optimization of a hydropower storage project with control on reliability of meeting the project’s firm energy yield. Particle swarm optimization (PSO) is used as the optimization algorithm, in which the method of sequential streamflow routing is called for objective function evaluations. Two types of problems are studied. The first one is an optimal design problem, in which the reservoir’s normal and minimum operating levels as well as the powerplant’s production capacity are optimized while reservoir releases are determined using a predefined operating policy. Two models are presented for this problem. In the first one (model A1), the normal and minimum operating levels are considered as the PSO decision variables, whereas the production capacity is iteratively adjusted in the simulation model. In the second model (model A2), the production capacity is also searched for by the PSO and the reliability constraint on meeting the system’s energy yield is satisfied using a penalty approach. In the second problem, reservoir releases as operational variables are also optimized by considering either the unknown parameters of linear release rules (model B1) or reservoir releases (model B2) as the PSO decision variables. The proposed models are employed for optimal design and operation of the Bakhtiari Hydropower Dam project in Iran. Results indicate that the PSO algorithm is capable of finding good solutions for the models explained while LINGO software employing gradient-based optimization techniques fails to solve the problems. Moreover, the system’s performance is much more affected by optimizing the design variables than the operational ones, unless greater penalties are assigned to severe energy deficits.

Key words | capacity optimization, hydropower systems, particle swarm optimization

NOTATION

The following symbols are used in this paper:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$x_i$</td>
<td>PSO $i$th particle</td>
</tr>
<tr>
<td>$v_i$</td>
<td>rate of $i$th particle’s position change</td>
</tr>
<tr>
<td>$p_i$</td>
<td>the best position of the $i$th particle found so far</td>
</tr>
<tr>
<td>$p_g$</td>
<td>the global best position identified in the entire population</td>
</tr>
<tr>
<td>$\chi$</td>
<td>PSO constriction factor</td>
</tr>
<tr>
<td>$\omega$</td>
<td>PSO inertia weight</td>
</tr>
<tr>
<td>$c_1$, $c_2$</td>
<td>PSO cognitive and social parameters</td>
</tr>
<tr>
<td>$C$</td>
<td>total cost</td>
</tr>
<tr>
<td>$CRF$</td>
<td>capital recovery factor</td>
</tr>
<tr>
<td>$r$</td>
<td>interest rate</td>
</tr>
<tr>
<td>$DC$</td>
<td>cost of dam construction</td>
</tr>
<tr>
<td>$PC$</td>
<td>cost of power-plant construction</td>
</tr>
<tr>
<td>$PeC$</td>
<td>cost of turbine construction</td>
</tr>
<tr>
<td>$E(t)$</td>
<td>generated energy in month $t$</td>
</tr>
<tr>
<td>$E_{\text{max}}$</td>
<td>maximum energy that can be produced</td>
</tr>
<tr>
<td>$e_p(t)$</td>
<td>power-plant efficiency</td>
</tr>
<tr>
<td>$FE$</td>
<td>target monthly firm energy yield</td>
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### INTRODUCTION

Limitations of fuels, pollutions caused by fossil fuels and benefits of hydroelectric power plants have led to the increased development of hydropower projects in recent decades. In order to design and construct hydroelectric systems including dams and reservoirs and power plants, their operation should be analyzed over a representative hydrologic period to account for streamflow variability.

Mathematical programming models including simulation and optimization models are extensively used in hydropower reservoir systems analysis. Simulation models, as they examine the system's performance under known design or operational conditions, are more computationally efficient; thus they are suitable for analyzing the underlying system with more detail. However, they require that the model builder specifies an operating policy, by which it is decided how much water should be released from the reservoir system in each time period.

Use of rule curves (Loucks & Sigvaldason 1982), heuristic rules, hedging rules (Tu et al. 2003) and optimization during simulation or re-optimization (Ginn & Houck 1989) are common methods for specifying operating policies required in simulation models. Lund & Guzman (1999) discussed operating rules for parallel and series hydropower systems. Ford (1990) developed a reservoir operation simulator called ResQ with the objective of meeting energy demands and water supply. Afzali et al. (2008) developed a reliability-based simulation model with one-period optimization sub-models for a multi-reservoir hydropower system operation.

In addition to simulation, optimization models have also been used in hydropower systems operation (for example, Gablinder & Loucks 1970; Kim & Palmer 1997; Barros et al. 2003). Successive linear programming (SLP) (Yeh et al. 1979; Grygier & Stedinger 1985) and sequential quadratic programming (SQP) (Powell 1983; Marino & Loaiciga 1985; Diaz & Fontane 1989) are among the optimization algorithms used in hydroelectric systems analysis. Pereira & Pinto (1985) developed a stochastic dual DP for determining optimal operation of a combined hydro-thermal system. Paudyal et al. (1990) developed a multi-reservoir optimization model for determining optimal capacities of the reservoirs. Mousavi et al. (2004) employed interior-point methods for optimizing a multi-reservoir system operation with the objectives of power generation and water supply. Mousavi et al. (2004a) compared the results of a DP-based optimization model with those of the HEC5 simulation model in a multi-reservoir system functioning for water supply and hydropower generation.

Minimizing the cost of design and operation of a hydropower dam reservoir system may be formulated as a nonlinear optimization problem. In this problem, hydraulic, hydrologic and physical equations simulate the proper performance of the system under different design and/or operational conditions. Coupling a search-based optimization algorithm with a reservoir operation model in a hydropower design problem is an example of a broader class of design optimization methods which could be used in many hydraulic and water resources engineering applications (Bakhtyar et al. 2007). Meta-heuristic optimization techniques and simulation–optimization methods linking a simulation model to a heuristic or population-based random search evolutionary algorithm are becoming more...
attractive in solving optimization models. Bozorg Haddad et al. (2008) employed honey-bee mating optimization, as a meta-heuristic algorithm, in deriving optimal operation rules for reservoir systems. The advantage of heuristic methods lies in their ability to locate solutions to combinatorial optimization problems with greater efficiency than implicit enumeration techniques. They are also advantageous because they more easily accommodate the discontinuities and nonlinearities of real functions than do gradient-based algorithms. On the other hand, the realistic incorporation of reliability into the optimization of reservoir system design and operation remains a particularly difficult task after decades of research. While most of this research has worked with methods based on linear or dynamic programming, little has been done to find out how well the problem could be handled by a simulation model linked to an optimization model (Ndiritu 2005). Water supply and energy systems have to satisfy different demands that each require various levels of reliability and this needs to be incorporated in analyses for efficient system design and operation (Ndiritu 2005). Dandy et al. (1997) compared the methods for yield assessment of multiple reservoir systems.

This study presents an approach for determining the capacity of a dam reservoir and its power-plant system as well as the reservoir operating rules maximizing the system’s energy yield subject to a reliability constraint on energy supply. Linking the PSO algorithm, as a random search evolutionary optimization algorithm, to the method of sequential streamflow routing (SSR), by which the reservoir operation is simulated, is followed to deal with the problem of optimal design and operation of a hydropower reservoir system. There are few cases in which the PSO algorithm has been used in optimization of reservoir systems operation (Baltar & Fontane 2006; Kumar & Reddy 2007). In the following paragraphs, the global variant is explained (the local variant can be easily derived through minor changes).

In the PSO algorithm, each particle is a candidate solution equivalent to a point in a $D$-dimensional space, so the $i$th particle can be represented as $x_i = (x_{i1}, x_{i2}, \ldots, x_{iD})$. Each particle flies through the search space, depending on two important positions, $p_i = (p_{i1}, p_{i2}, \ldots, p_{iD})$, the best position the current particle has found so far (pbest); and $p_g = (p_{g1}, p_{g2}, \ldots, p_{gD})$, the global best position identified in the entire population (gbest). The rate of the $i$th particle’s position change is given by its velocity $v_i = (v_{i1}, v_{i2}, \ldots, v_{iD})$. Equation (1) updates the velocity for each particle in the next iteration step, whereas Equation (2) updates each particle’s position in the search space:

$$v_{id}^{n+1} = \chi (\omega v_{id}^n + c_1 r_1(p_{id}^n - x_{id}^n) + c_2 r_2(p_{gd}^n - x_{id}^n))$$  

$$x_{id}^{n+1} = x_{id}^n + v_{id}^{n+1}$$

where $d = 1, 2, \ldots, D; i = 1, 2, \ldots, N$, and $N$ is the size of the swarm; $\chi$ is called the constriction factor which is used in constrained optimization problems in order to control the magnitude of the velocity (in unconstrained optimization problems it is usually set equal to 1.0); $\omega$ is called inertia weight; $c_1$, $c_2$ are two positive constants, called cognitive
and social parameters, respectively; \( r_1, r_2 \) are random
numbers uniformly distributed in \([0,1]\); and \( n = 1, 2, \ldots \),
\( n_{\text{max}} \) represents the iteration number.

Experimental results indicate that it is better to initially
set the inertia to a large value, in order to promote global
exploration of the search space, and gradually decrease it to
get more refined solutions (Shi & Eberhart 1998a,b). Thus,
an initial value around 1.2 and a gradual decline toward
zero can be considered as a good choice for \( \omega \). Therefore,
PSO updates the inertia weight in each iteration using the
following equation:

\[
\omega^n = \omega_{\text{max}} - \frac{\omega_{\text{max}} - \omega_{\text{min}}}{n_{\text{max}}} \times n
\]  

(3)

where \( \omega^n \) is the inertia weight in iteration \( n \), and \( \omega_{\text{max}} \) and
\( \omega_{\text{min}} \) are respectively the maximum and minimum inertia
weights. Recent works report that it might be better to choose a larger cognitive parameter, \( c_1 \), than a social
parameter, \( c_2 \), with \( c_1 + c_2 \leq 4 \) (Carlisle & Dozier 2001).
The PSO algorithm starts with a set of randomly generated
solutions (particles). The algorithm then updates the swarm
using Equations (1) and (2) in each iteration. This process is
continued until it satisfies stopping criteria.

Some previous studies have discussed the trajectory of
particles and the convergence of the standard PSO
algorithm. It has been shown that the trajectories of the
particles oscillate in different sinusoidal waves and converge quickly, sometimes prematurely. In each iteration,
particles are attracted toward the pbest and gbest positions.
They eventually loose their exploration capability in future
iterations. This situation may occur in early stages of the
search. In fact, this does not even guarantee that the
algorithm has converged to a local minimum and it merely
means that all the particles have converged to the best
position discovered so far by the swarm.

One of the main reasons for the problem of premature
convergence is the stagnation of particles, making them
unable to explore any new regions of the search space. In
order to ensure escaping from premature convergence, a
strategy may be employed to drive those lazy particles and
let them explore better solutions. If a particle’s velocity
decreases to a threshold \( v_c \), a new velocity is assigned using
Equation (4). Thus, a turbulent PSO (TPSO) (Liu &
Abraham 2001) has been used in this study in which a new
velocity update equation is employed as follows:

\[
v'^n_{id} = \begin{cases} 
v^n_{id} & \text{if } |v^n_{id}| \leq v_c \\
\rho(u(1,1)v_{\text{max}}) & \text{if } |v^n_{id}| \geq v_c 
\end{cases}
\]  

(4)

where \( u(1,1) \) is a random number uniformly distributed in
the interval \([-1,1]\] and \( \rho \) is a scaling factor which controls
the domain of the particles oscillation according to \( v_{\text{max}} \). \( v_c \)
is the minimum velocity threshold, a tunable threshold
parameter to limit the particle’s minimum velocity. The
performance of the algorithm is directly related to two
parameter values, \( v_c \) and \( \rho \). A large \( v_c \) shortens the
oscillation period, providing a great probability for particles
to leap over local minima using the same number of
iterations. However, a small \( v_c \) compels particles into a
quick “flying” state, which leads them not to search the
solution space and forcing them not to refine the search. In
other words, a large \( v_c \) facilitates global search while a
smaller value facilitates local search. One can adjust the
search ability by changing \( v_c \) dynamically. The value of
\( \rho \) changes the particle oscillation domain directly. It is
possible for particles not to jump over the local minima if
there would be a large number of local minima in the search
space. However, the particle trajectory would more prone
to oscillate because of a smaller value of \( \rho \). For a desired
exploration–exploitation trade-off, it is better to divide the
search into three stages. In the first stage the values for \( v_c \)
and \( \rho \) are set to a large and a small value, respectively. In
the second stage, \( v_c \) and \( \rho \) are set to medium values and in the
last stage, \( v_c \) is set to a small value and \( \rho \) is set to a large
value. This enables particles to take very large steps to
explore solutions in the early stages, by scanning the whole
solution space for good local minima and then in the final
stages particles perform a fine grain search.

CAPACITY OPTIMIZATION OF HYDROPOWER
STORAGE PROJECTS

In hydropower reservoir systems, selection of the reservoir’s
normal and minimum operating levels as well as the power-
plant’s production capacity which yields the best economic
performance may be of great importance. On the other
hand, the net benefit and energy potential of these systems
for any possible combinations of those design variables are under hydrologic variability of inflows to the reservoir. This requires analysis of hydropower systems’ performance over a long representative hydrologic period in order to estimate the expected costs and benefits resulted from the project for those combinations as candidate solutions. Since there are many alternatives for the values of the design variables to choose from, the best values resulting in the highest expected net benefit can be determined by an optimization model. On the other hand, as the model solution is under hydrologic uncertainty, the energy yield of the system will be a random variable, each value of which will be realized at a certain level of reliability. In this case, designers may like to formulate an optimization model in which the reliability of meeting energy yield is under the control of the designer. The general formulation of the optimization model for this purpose may be written as follows:

\[
\text{Min } \text{Cost}_{\text{total}}^1 = \text{CRF} \times (\text{DC} + \text{PC} + \text{PeC}) - \text{fvalue} \times \text{nyear} \times \sum_{t=1}^{12} \text{FE} - \text{value} \times \sum_{t=1}^{T} \text{SE}(t) + \sum_{t=1}^{T} \text{Spill}(t) \tag{5}
\]

S.t:

\[
\text{DC} = d_1 \times S_{\text{max}}^2 + d_2 \times S_{\text{max}} + d_3 \times z_1 \tag{6}
\]

\[
\text{DC} \leq \text{BigM} \times z_1 \tag{7}
\]

\[
\text{PC} = p_1 \times \text{Pcap}^2 + p_2 \times \text{Pcap} + p_3 \times z_2 \tag{8}
\]

\[
\text{PC} \leq \text{BigM} \times z_2 \tag{9}
\]

\[
S(t + 1) = S(t) + Q(t) - R(t) - \text{Spill}(t) - L(t) \quad \forall t \tag{10}
\]

\[
E(t) = 2.75 \times R(t) \times (0.5 \times (h(t) + h(t + 1)) - h_{\text{tail}}(t) - h_{\text{f}}(t)) \times e_p(t) \quad \forall t \tag{11}
\]

\[
h(t) = k_1 \times S(t)^3 + k_2 \times S(t)^2 + k_3 \times S(t) + k_4 \quad \forall t \tag{12}
\]

\[
h_{\text{tail}}(t) = k_5 \times R(t)^2 + k_6 \times R(t) + k_7 \quad \forall t \tag{13}
\]

\[
\text{FE} = \text{Pcap} \times pf \times n\text{hours} \quad \forall t \tag{14}
\]

\[
E(t) \geq \text{FE} \times z(t) \quad \forall t \tag{15}
\]

\[
\sum_{t=1}^{T} z(t) \geq \text{TarREL} \quad \forall t \tag{16}
\]

\[
\text{SE}(t) = (E(t) - \text{FE}) \times z(t) \quad \forall t \tag{17}
\]

\[
E(t) \leq E_{\text{max}} = \text{Pcap} \times n\text{hours} \quad \forall t \tag{18}
\]

\[
S_{\text{min}} \leq S(t) \leq S_{\text{max}} \quad \forall t \tag{19}
\]

\[
R_{\text{min}} \leq R(t) \leq R_{\text{max}} \quad \forall t \tag{20}
\]

The model’s objective function defined in Equation (5) is to minimize the total cost, \( \text{Cost}_{\text{total}}^1 \), (or to maximize the total net benefit), which includes the capital and variable construction costs of dam, DC, power plant, PC, and tunnel, PeC, subtracted by the benefits resulting from firm and secondary energy sales. Dam construction cost, DC, is a function of reservoir capacity (dam height) as defined in Equation (6). The power-plant construction cost, PC, is a function of the powerplant capacity (Pcap) as defined in Equation (8). In Equations (6) and (8), \( d_1 \) to \( d_3 \) and \( p_1 \) to \( p_3 \) are constants determined by fitting the best quadratic curves to the cost functions of dam and power-plant constructions, respectively. Inequalities (7) and (9) guarantee that DC and PC values become zero in case they are not constructed, as \( z_1 \) and \( z_2 \) are respectively binary variables accounting for fixed costs of dam and power-plant construction and BigM is a positive large number. PeC is the cost of the penstock which does not affect the solution as it is assumed to be a constant value herein. CRF = \( r(1 + r)^{\text{nyear}} / (1 + r)^{\text{year}} - 1 \) is the capital recovery factor, converting the present value costs to their equivalent uniform annual costs and \( r \) is the annual discount rate of money. The last term in the objective function is defined to ensure that spillage may occur only if necessary and when \( S(t) = S_{\text{max}} \).
Equation (10) is the balance equation wherein \( S(t) \) is the beginning-of-month reservoir storage, \( Q(t) \) is the inflow to reservoir, \( R(t) \) is the turbine release, \( Spill(t) \) is the spilled release and \( L(t) \) is the net evaporation and seepage losses, all in period \( t \). Equation (11) is the power equation in which the energy generated in each month, \( E(t) \), is estimated depending upon the net head on the turbine, turbine discharge and power-plant efficiency, \( e_p(t) \). The net head is calculated as the difference between reservoir level, \( h(t) \), and tailwater level, \( h_{tail}(t) \), from which minor and frictional losses in conveyance structures, \( h_{fr}(t) \), are subtracted. The topographic relations including elevation-storage and tailwater-discharge equations are respectively defined by constraints (12) and (13) in which \( k_1 \) to \( k_4 \) and \( k_5 \) to \( k_7 \) are constants of the equations.

The firm energy, \( FE \), is the monthly energy yield that can be produced in adverse hydrologic conditions at a certain level of reliability. The firm energy yield that can be produced is estimated using Equation (14) in which \( pf \) is the planned plant factor by which the number of hours per day that power plant operates at its maximum production rate, i.e. the power-plant’s production capacity, is defined. Although the firm energy can be a seasonal variable depending on each calendar month \( m \) in general, we define it as an amount of monthly energy that is produced reliably at least TarREL\% of total months \( T = m \times nyear \) over a planning horizon with \( nyear \) years. This is satisfied through constraints (15) and (16) in which \( z(t) \) are binary variables equal to zero if the energy generated is less than the target energy yield and to one, otherwise. \( SE(t) \) is the secondary energy in period (month) \( t \), which is the energy produced in excess of the firm energy as determined by Equation (17). In the objective function, \( fvalue \) and \( svalue \) are respectively the firm and secondary energy unit sales.

Finally, constraint (18) takes care of the upper bound on monthly energy generation in which \( nhours = 720 \) is the number of hours per month. Also, the upper and lower bounds on monthly reservoir storages and releases are satisfied through constraints (19) and (20), in which \( S_{min} \) and \( S_{max} \) are respectively the minimum and maximum reservoir storage volumes while \( R_{min} \) and \( R_{max} \) are the minimum and maximum turbine releases, respectively.

As we can see, the model is a non-convex mixed integer nonlinear programming (MINLP) model, which is an NP-hard (Nondeterministic Polynomial-time hard) problem. MINLP consists of finding solutions to problems combining the combinatorial nature of mixed integer programming (MIP) with nonlinear programs’ (NLPs) difficulties. These problems contain discrete and numerical variables as well as linear and nonlinear constraints. Since both MIP and NLP are NP-hard problems, solving such problems can be really complex (Bussieck & Pruessner 2003). That is why meta-heuristic and evolutionary-based global optimization algorithms like PSO might be of interest as promising optimization methods to deal with solving this type of models.

It is worth mentioning that constraints (15) and (16) may be viewed as the deterministic equivalent of the probabilistic constraint \( Pr[E(t) \geq FE(t)] \geq TarREL \). In other words, the historical inflow time series \( Q(t) \) realized in past years over a long-term period are taken as realizations of the associated streamflow stochastic process. In fact, given a large number of historical or synthetically generated samples of inflow series, we can convert an explicit optimization model with a chance (reliability) constraint on meeting the energy yield to an implicit stochastic optimization model by adding binary variables \( z_t \) to the model’s formulation. This will cause a stochastic LP/NLP model to become a deterministic MILP/MINLP model. It is noted that solving the explicit stochastic model requires some kind of simplifications on the relation between release and inflow/storage random variables. This is due to the fact that the only variable whose probability distribution is known, before we get the model solved, is the inflow variable. Some kind of simplifications may be found, for example in release rules used in LDR (linear decision rule) models or a more advanced formulation presented by Fletcher & Ponnambalam (1998) as an explicit stochastic optimization model for reservoir storage-yield analysis. Moreover, converting an explicit chance constraint on energy yield into its deterministic equivalent is not as easy as that on release yield, because of nonlinearity of the energy equation.

In practical hydropower storage systems, a reliability-based reservoir simulation (RBS) model is commonly employed for a limited number of design combinations rather than using an optimization scheme. In the following, the single-reservoir RBS model is presented first and then it is explained how it will be linked with the PSO
algorithm for solving the MINLP formulation defined by Equations (5)–(20).

**SINGLE-RESERVOIR RBS MODEL**

Assume a hydropower single-reservoir system with the given normal and minimum reservoir operating levels and a specified plant factor. Then a monthly RBS model may be used to determine the maximum firm energy yield that can be produced. The steps taken in a single-reservoir RBS model are as follows.

**Estimation of an initial production capacity**

An initial production capacity may be estimated by the following equation:

$$P_{\text{cap}} = \frac{2.75 \times Q_{\text{ave}} \times h_{\text{max}} \times \alpha}{\text{pf} \times \text{nhours}}$$

(21)

where $P_{\text{cap}}$ is the estimated production capacity, $Q_{\text{ave}}$ is the mean monthly inflow to the reservoir, $h_{\text{max}}$ is an initial estimation for maximum net head on the turbines as the difference between the normal and tailwater levels, $\alpha$ is a decreasing factor considering the effect of dry periods ($0.5 < \alpha < 1$) and pf and nhours are previously defined.

**Estimation of the system’s energy yield**

Annual energy demand and its monthly distribution may be obtained from the load and/or power market studies and considering whether the hydropower system is to operate as a base, intermediate or peaking system. In the absence of such studies and where the energy system is not a hydro-based one, it may be desirable to maximize the system’s energy yield that can be produced reliably. Given the estimated production capacity ($P_{\text{cap}}$) and the specified plant factor (pf), one may estimate the system’s monthly energy yield as defined in Equation (14).

**Simulation of reservoir operation**

In this step, the reservoir system operation is simulated over a representative hydrologic period using sequential streamflow routing (SSR) method to estimate the energy-yield reliability. To specify the operating (release) policy in the simulation model, one may set the energy generated $E(t)$ (Equation (11)) to be equal to the estimated energy yield $FE$ (Equation (14)) in each time period $t$ based on which the turbine release can be calculated as follows:

$$R(t) = \frac{P_{\text{cap}} \times \text{nhours} \times \text{pf}}{2.75 \times (0.5 \times (h(t) + h(t + 1)) - h_{\text{tail}}(t) - h_{\text{hf}}(t)) \times \epsilon_{\text{ep}}(t)}$$

(22)

In Equation (22), $h(t + 1)$, $h_{\text{tail}}(t)$ and $h_{\text{hf}}(t)$ depend on the turbine release making the equation implicit with respect to $R(t)$. Therefore, Equation (22) is solved iteratively in each time period $t$. To do so, an initial end-of-month reservoir storage is assumed, yielding an initial release, $R(t)$, from Equation (22). Then the new end-of-month reservoir storage is determined from the continuity equation, i.e. Equation (10). The new end-of-month storage is then compared with that initially assumed. If they are not the same, the next estimation of the turbine release will be calculated by Equation (22), based on the new end-of-month storage. The procedure is repeated until getting the end-of-month storage and release volumes converged to the same values in two successive iterations.

It should be noted that the end-of-month storage in any iteration is checked if it is within its acceptable range [$S_{\text{min}}$, $S_{\text{max}}$]. If it is above $S_{\text{max}}$, the turbine release and the energy generated will be increased so that the ending storage equals $S_{\text{max}}$. Of course the excess turbine release and the energy generated (secondary energy) are limited respectively by the hydraulic capacity ($R_{\text{max}}$) and the maximum energy that can be generated ($E_{\text{max}}$). If any of those limits are touched, the excess release, not contributing in energy generation, will spill. On the other hand, if reservoir storage falls below $S_{\text{min}}$, the end-of-month storage is set to $S_{\text{min}}$ and consequently the turbine release will decrease. In this situation, the energy generated will be less than the estimated monthly energy yield (FE), resulting in a failure ($\zeta(t) = 0$ due to energy deficit) in that month. This operating policy implies that the release in each time period is determined so that the energy generated equals the estimated energy yield, if possible. By repeating the above-mentioned procedure over the simulation horizon, one can

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estimate the energy-yield reliability as $\sum_{t=1}^{T} z(t)/T$. Then the production capacity is adjusted accordingly, if required, as will be explained hereafter.

**Adjustment of the estimated production capacity**

If the estimated reliability is within the desired range specified for target reliability ($\text{TarREL} - \delta \leq \text{REL} \leq \text{TarREL} + \delta$), the estimated production capacity and energy yield will be acceptable, otherwise they will be increased or decreased, accordingly. Then all of the steps explained above are repeated until the production capacity and energy yield values are converged and the reliability of meeting the energy yield arrives at its specified target value. The converged values are, in fact, the maximum production capacity and energy yield values that can be achieved at the specified level of reliability.

The RBS model may be performed for different normal and minimum operating levels. This trial-and-error-based approach may not arrive at an optimum solution. It is therefore desired to make use of the optimization model defined by Equations (5)–(20) in order to determine the optimal design configuration of the system.

**THE SIMULATION–OPTIMIZATION MODEL**

In this section, the models integrating the PSO algorithm and the RBS model are presented. Two problems, each with two formulations resulting in four optimization models, are solved. In the first type of problems with models A1 and A2, only the design variables are optimized while in the second type with models B1 and B2, the design and operational variables are optimized simultaneously. The type and number of decision variables in each of the models are summarized in Table 1.

The selection of the above-mentioned models is based on the fact that two types of design and operation problems may exist in reservoir management problems. Design problems are about determining capacities of the system’s components while operation problems are about how to operate an existing system so as to meet the objectives which the system has been designed for. However, in design problems the system’s performance under any combination of design variables depends on operational variables, by which the system’s operation is controlled. The operational variables may be determined either by using a predefined operating rule or by considering them as unknown decision variables. The problem under study is a design problem whose unknowns are normal and minimum operating levels as well as the power-plant’s production capacity; but the system’s performance under any set of those unknowns depends on reservoir releases, which are unknowns too. In the A-type models (A1 and A2), the PSO decision variables are the only three design variables mentioned and reservoir releases are determined using a predefined operating rule, while the B-type models (B1 and B2) take both the design and operational variables as the PSO decision variables. Therefore, comparison of the results of models A and B will show how well a simple predefined operating policy may perform compared to optimizing the operational variables. It also shows how well the PSO model performs when the search space of the model increases dramatically as the number of operational variables is usually much more than that of the design variables.

In model A1, given the normal and minimum operating levels generated by the PSO, the RBS model simulates the system operation iteratively in order to determine the maximum system’s energy yield or the power-plant’s production capacity at a specified reliability level.

<table>
<thead>
<tr>
<th>Model</th>
<th>PSO variables</th>
<th>No. of PSO variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>Normal water level, Minimum operating level</td>
<td>2</td>
</tr>
<tr>
<td>A2</td>
<td>Normal water level, Minimum operating level, Production capacity</td>
<td>3</td>
</tr>
<tr>
<td>B1</td>
<td>Normal water level, Minimum operating level, Production capacity, Parameters of linear release rules in Equation (24)</td>
<td>39</td>
</tr>
<tr>
<td>B2</td>
<td>Normal water level, Minimum operating level, Production capacity, Monthly reservoir releases</td>
<td>495</td>
</tr>
</tbody>
</table>
The annual firm and secondary energies are also calculated from the results of the RBS model. Subsequently, particles fitness values (objective function values) are evaluated using Equation (5) and pbest and gbest solutions are determined. After evaluating fitness values, the PSO algorithm updates particle velocities according to Equation (1). Having the updated velocities, each particle changes its position and the new values of design variables (normal and minimum operating levels) are determined by Equation (2). The procedure is continued until the minimum total cost (maximum total benefit) associated with the fitness of the best particle, gbest, does not change over a number of consecutive iterations. The flow diagram of model A1 is presented in Figure 1.

The difference between models A1 and A2 is about how to satisfy the reliability constraint of the optimization model on meeting the system’s energy yield. In model A1, the power-plant’s production capacity is determined by a direct search scheme explained in the RBS model while in model A2 the production capacity is searched for by the PSO algorithm itself, using a penalty term added to the objective function as follows:

\[
\text{Cost}^2_{\text{total}} = \text{Cost}^1_{\text{total}} + (|\text{REL} - \text{TarREL}|) \ast P
\]  

(23)

where \(\text{Cost}^2_{\text{total}}\) is the total cost in model A2, \(\text{Cost}^1_{\text{total}}\) is as defined previously, \(\text{REL}\) is the reliability level resulting from simulation of reservoir operation, \(\text{TarREL}\) is the target or desired reliability level and \(P\) is a penalty factor that should be fine-tuned by trial and error. The penalty term would guide particles to converge toward a production capacity satisfying the reliability constraint. Comparison of results of
models A1 and A2 may show how well a penalty approach can perform in satisfaction of an important constraint of the underlying optimization model, which is a reliability constraint. Moreover, since the optimization problem solved by each of the models is the same, their results must be almost the same and this could be a checkpoint for the model’s verification.

Regarding the B-type models, although both models B1 and B2 take operational variables as decision variables of the PSO algorithm, the form of the variables are not the same. Model B2 takes reservoir releases as decision variables while parametric release rules are used in model B1 as follows:

\[ R(t) = a(m) \times S(t) + b(m) \times Q(t) - c(m) \]

\[ t = 1, 2, \ldots, T, \quad m = 1, 2, \ldots, 12 \]

where \( a(m), b(m) \) and \( c(m) \) are unknown parameters. The PSO algorithm generates three design decision variables of model A2 plus \( 12 \times 3 = 36 \) coefficients used in Equation (24). Therefore, the formulation of model B1 is the one represented by Equations (5)–(20) plus additional constraints defined by Equation (24). Thus it is a more constrained model with a smaller feasible space.

The formulation of model B2 is exactly the same as the model defined by Equations (5)–(20) in which the reservoir releases, \( R(t) \), are considered as the PSO decision variables. Therefore, in our case study with 41 years of planning horizon and three design variables, the optimization problem will include 495 decision variables, of which 492 variables are operational ones. The flow diagram of models A2, B1 and B2 is presented in Figure 2. Model B2 has the biggest degree of freedom in searching for the best values of decision variables, whereas it is the most difficult optimization model to solve by the PSO in terms of the dimensionality of its search space. Moreover, model B2 can take advantage of an unreal assumption which is about having perfect foresight on the future inflows, while operating rules used in model B1 needs only the current-month hydrologic information. Therefore, comparison of their solutions may show to what extent the assumption of perfect foresight on future inflows can help the model obtain better solutions in terms of the optimality criterion. Although model B2 has more freedom to search for better optimality criteria, no operating rules are offered by this model. Thus, rule inference techniques, such as multiple regression or ANNs, are needed to fit optimal release time series, if one may be interested in finding release operating rules. Moreover, comparison of the solutions of models B1 and B2 can show how much the optimality criterion may be affected by imposing linear release rules on the optimization model formulation.

**CASE STUDY AND INPUT DATA**

The real case study of this paper is Bakhtiari Dam, to be built on the Bakhtiari River in the west of Iran. Tables 2 and 3 present input parameters to the models, i.e. basic characteristics of the system and the topographic data of Bakhtiari Dam. The desired reliability of energy yield is considered as 90%. The power-plant efficiency and plant factor are set to 92.12% and 0.25, respectively. Tail-water elevation and head losses are considered constants as given in Table 2. Cost values at different dam and power-plant capacities are presented in Tables 4 and 5, respectively. Firm and secondary energy unit sales, i.e. fvalue and svalue, are set to 160 Rials (Rial is the Iranian currency) per kWh.
Cost and benefit input data are obtained from the economic report of the project prepared by the consulting company in charge of the Bakhtiari Dam project, wherein an annual discount rate of $r = 0.07$ has been adopted (Mahab-Ghods 2002). Although unit sales of firm and secondary energies are considered to be the same as that used in the report, they might have different values in general. The life period of the system, lifeyear, is considered as 50 years.

The PSO parameters were chosen by trial and error as follows: the population size is considered as 10 in model A1, 20 in model A2, 40 in model B1 and 80 in model B2, the maximum and minimum values of inertia weight are considered as $w_{\text{max}} = 1.5$, $w_{\text{min}} = 0.1$ and acceleration constants as $c_1 = 2.3$ and $c_2 = 1.2$. The penalty factor was set to $P = 10^4$. The algorithm stopped if the objective function had no improvement over 50 succeeding iterations.

**RESULTS AND DISCUSSION**

With reference to the aforementioned data, the optimization models were executed. Optimal values of the objective function, design variables, average annual firm and secondary energies and the reliability level resulting from the models are presented in Table 6. Figure 3 shows the variation of the model’s objective function against the PSO iterations. The results show that models A1 and A2 perform almost the same in terms of their overall performance. This is expected as they solve one problem by two different approaches in meeting the reliability constraint. Model B2 outperforms the other ones in terms of its objective value. The difference, however, between the optimal objective value of model B2 and that of models A1 and A2 is not as much as expected. Although operational variables are optimized in model B2 and its search space (495-dimensional) is much larger than that of models A1 and A2 (3-dimensional), models A1 and A2 have arrived at objective values which are close to that of model B2. In other words, although a specific operating rule is

### Table 2 Basic characteristics of Bakhtiari Dam and its powerplant system (Mahab-Ghods 2002)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum normal water level (masl)</td>
<td>830</td>
</tr>
<tr>
<td>Dead water level (masl)</td>
<td>660</td>
</tr>
<tr>
<td>Tail-water elevation (masl)</td>
<td>533.5</td>
</tr>
<tr>
<td>Generator efficiency</td>
<td>92.12%</td>
</tr>
<tr>
<td>Head loss (meter)</td>
<td>3</td>
</tr>
<tr>
<td>Plant factor</td>
<td>0.25</td>
</tr>
<tr>
<td>Target reliability</td>
<td>90%</td>
</tr>
</tbody>
</table>

### Table 3 Elevation–capacity table of Bakhtiari reservoir system (Mahab-Ghods 2002)

<table>
<thead>
<tr>
<th>Elevation (masl)</th>
<th>Capacity (MCM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>532</td>
<td>0</td>
</tr>
<tr>
<td>550</td>
<td>0.01</td>
</tr>
<tr>
<td>575</td>
<td>0.02</td>
</tr>
<tr>
<td>592</td>
<td>0.03</td>
</tr>
<tr>
<td>593</td>
<td>1.014</td>
</tr>
<tr>
<td>600</td>
<td>4.32</td>
</tr>
<tr>
<td>625</td>
<td>23.62</td>
</tr>
<tr>
<td>650</td>
<td>97</td>
</tr>
<tr>
<td>675</td>
<td>241.28</td>
</tr>
<tr>
<td>700</td>
<td>481.52</td>
</tr>
<tr>
<td>725</td>
<td>847.73</td>
</tr>
<tr>
<td>750</td>
<td>1,371.57</td>
</tr>
<tr>
<td>775</td>
<td>2,084.12</td>
</tr>
<tr>
<td>800</td>
<td>3,031.3</td>
</tr>
<tr>
<td>825</td>
<td>4,269</td>
</tr>
<tr>
<td>830</td>
<td>4,582.37</td>
</tr>
</tbody>
</table>

### Table 4 Costs of construction of Bakhtiari Dam at various capacities (Mahab-Ghods 2002)

<table>
<thead>
<tr>
<th>Capacity (MCM)</th>
<th>Cost ($10^9$ Rials)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,941.61</td>
<td>781.44</td>
</tr>
<tr>
<td>2,652.43</td>
<td>897.52</td>
</tr>
<tr>
<td>3,526.38</td>
<td>1,016.07</td>
</tr>
<tr>
<td>4,587.37</td>
<td>1,139.20</td>
</tr>
<tr>
<td>4,590.00</td>
<td>5,500.00</td>
</tr>
</tbody>
</table>

### Table 5 Costs of construction of Bakhtiari powerplant at various production capacities

<table>
<thead>
<tr>
<th>Installed capacity (MW)</th>
<th>Cost ($10^9$ Rials)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>0</td>
</tr>
<tr>
<td>913</td>
<td>152.45</td>
</tr>
<tr>
<td>1,014</td>
<td>162.40</td>
</tr>
<tr>
<td>1,110</td>
<td>173.25</td>
</tr>
<tr>
<td>1,220</td>
<td>189.30</td>
</tr>
<tr>
<td>1,500</td>
<td>225.00</td>
</tr>
</tbody>
</table>
used in models A1 and A2, without optimizing reservoir releases, their objective functions are very close to that of model B2 with a much larger degree of freedom in searching for releases.

There might be two reasons for the above result. The first one is because of the fact that the PSO algorithm might not have found the global optimum solution of model B2 due to the very large multi-dimensional and non-convex

<table>
<thead>
<tr>
<th>Item</th>
<th>Model A1</th>
<th>Model A2</th>
<th>Model B1</th>
<th>Model B2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost (10⁹ Rials)</td>
<td>-24,090.8</td>
<td>-24,086.8</td>
<td>-22,905.0</td>
<td>-24,171.2</td>
</tr>
<tr>
<td>Reliability of meeting energy yield (%)</td>
<td>90.04</td>
<td>89.84</td>
<td>90.04</td>
<td>90.04</td>
</tr>
<tr>
<td>Normal water level (masl)</td>
<td>830</td>
<td>830</td>
<td>830</td>
<td>830</td>
</tr>
<tr>
<td>Min. operating level (masl)</td>
<td>814.5</td>
<td>814.5</td>
<td>742.0</td>
<td>761.0</td>
</tr>
<tr>
<td>Powerplant's production capacity (MW)</td>
<td>1,143.79</td>
<td>1,143.8446</td>
<td>802.8</td>
<td>1,116.1</td>
</tr>
<tr>
<td>Annual firm energy (MWh)</td>
<td>2.4166 x 10⁶</td>
<td>2.4159 x 10⁶</td>
<td>1.734 x 10⁶</td>
<td>2.4054 x 10⁶</td>
</tr>
<tr>
<td>Annual total energy (MWh)</td>
<td>3.1884 x 10⁶</td>
<td>3.18847 x 10⁶</td>
<td>3.0370 x 10⁶</td>
<td>3.2831 x 10⁶</td>
</tr>
</tbody>
</table>

Figure 3 | Variation of particles fitness (objective) values in different iterations of the models. Top left: model A1, top right: model A2, bottom left: model B1, bottom right: model B2.
search space of the model, although it exists. The second reason is that the model A1’s optimal solution may be the global optimum (or near-optimum) solution for model B2. In other words, because of tight interrelationships between releases of different periods in Model B2, i.e. continuity equations and other constraints, the model’s feasible space is a very small part of the very large search space of the model. Therefore, the global optimum of model B2 is very close to that of model A1. This means that in spite of making releases relaxed to vary in the PSO, they are highly constrained within a very narrow feasible region compared to the very large and multi-dimensional search space of model B2.

To verify the above hypotheses, more investigations were carried out. First, the optimal solution of model A1 was put among the initial solutions of model B2. Moreover, attempts were made to increase the model’s exploration capability by running the model with different PSO parameter values, enlarging the swarm size and also introducing turbulence into the PSO. However, it was found that the model cannot find a significantly better solution than the one put among its initial-population solutions. This result may reinforce the hypothesis of near-optimality of the model A1’s solution for model B2. The validity of this conclusion is further explored as explained in the following paragraph.

In reservoir systems functioning for water supply, standard operating policy (SOP), in which the reservoir release in each time period equals water demand if possible, may be optimal or near-optimal if water shortages are penalized linearly. In hydropower systems, the concerns in refill and drawdown periods are respectively to avoid unnecessary spills and energy deficits. Accordingly, as long as energy deficits are penalized linearly in drawdown periods and unnecessary spills are avoided in refill periods, one may expect that the solution obtained by a model employing SOP would be a good or a near-optimal one for an optimization model including the entire planning horizon. In the case of assigning severe penalties to more intense failures, the models optimizing the system operation over a long-term horizon are able to prevent severe deficits by performing hedging rules. In other words, the models optimizing the system operation are more efficient if more severe deficits are penalized more than linearly.

Otherwise, they may not show a significant advantage over a model which makes use of a myopic operating policy like SOP. This point of view was also supported by the results of a simulation–optimization model developed for multireservoir hydropower systems operation (Afzali et al. 2008). Now let us analyze the operating policy used in models A1 or A2 and the objective function of the models developed herein to find out how severe energy deficits are penalized.

The operating policy used in models A1 or A2 is something similar to the SOP, but for a reservoir system functioning for hydropower generation. The SOP is nothing other than making the release in each time period equal to that period’s water demand, if possible. What is followed in models A1 or A2 is to make reservoir releases as much as required for meeting an estimated energy yield, if possible. In the objective function, the energy deficit is not penalized explicitly. However, in months in which deficits are permitted, a potential benefit equal to the amount of the deficit of energy multiplied by the unit value of firm energy (160 Rials per kWh) will be lost (opportunity cost). This means that, in models A1 or A2, the penalties assigned to energy deficits are penalized linearly. The same linear penalty is imposed on the model’s objective function when releases spill. As a result, the operating policy employed in models A1 or A2 may be a good or near-optimal policy for a model optimizing the system operation over the entire planning horizon, subject to a constraint on meeting the desired energy production reliably.

A constructive examination for verifying the above-mentioned point is to make the models more sensitive to the extent of energy deficits. This may be done by adding an additional nonlinear penalty term to the model’s objective function. It will be responsible to take vulnerability of failures into account in the models which have already considered the reliability measure. Therefore, a penalty function was added to the objective function as follows:

$$P_{\text{val}} = C_{\text{val}} \times \sum_{t=1}^{T} [(1 - E(t)) \times (FE - E(t))^k]$$  \hspace{1cm} (25)$$

where \(P_{\text{val}}\) is the penalty due to the failure in meeting the firm energy yield \(FE\), estimated by Equation (14), \(E(t)\) is the energy generated determined by Equation (11), \(k\) is a
larger-than-unity power of the penalty function, \( C_{P_{\text{vul}}} \) is a scaling factor, making the order of magnitude of the \( P_{\text{vul}} \) term balanced compared to other terms in the objective function and \( z(t) \) are the binary variables defined before. Considering larger values for \( k \) results in larger penalties assigned to energy deficits and thus makes the models more sensitive to more severe energy deficits. Two values for parameter \( k \), i.e. 2 and 4, were tested just to make the differences between the models results more transparent.

Table 7 presents the results obtained by the modified models A1 and B2. As seen in the table and for \( k = 4 \), model A1 has resulted in a minimum cost equal to \(-24,000.5\) (10\(^9\) Rials), while model B2 has found a minimum cost equal to \(-24,013.9\). As expected, model B2, in this case, is able to take advantage of optimizing the operational variables, whereas model A1 suffers from the lack of utilizing a policy which does not have any foresight on future inflows, resulting in excess design and operational costs. On the other hand, it is seen in case \( k = 2 \) that the results of the modified models A1 and B2 do not show a significant difference and model A1 is still capable of finding a near-global optimum or good solution. It is worth mentioning that the perfect foresight on future inflows in model B2 may not be realistic in terms of uncertain future inflows. However, this is an issue not studied in this paper and we are just interested in finding out why model B2, with perfect foresight, does not perform better than model A1, with no foresight, in terms of optimization capability.

Another result presented in Table 6 is that model B1 finds a lower value for optimal production capacity (802 MW) than other models do (1,100–1,200 MW), although the optimal objective function (\(-22,905\)) of model B1 is not that far away from that of other models (for instance, \(-24,171.2\) in model B2). Imposing a linear structure on reservoir releases in model B1 makes the feasible space of the model smaller than that of model B2. As a result, arriving at a higher cost in this model, compared to that of model B2, is reasonable. To verify this, monthly linear regression equations were fitted to time series of optimum releases obtained by model B2 in different calendar months. Results revealed a relatively low coefficient of determination values (\(R^2\)). Moreover, the system operation was simulated using the fitted linear rules, while keeping the design variables in the simulation model the same as the ones obtained by model B2. It was found that the resulting simulated cost (\(-16,195\)) is greater than that of model B1 (\(-22,905\)). Such a result may be taken as a checkpoint for the results of model B1, showing that its optimum objective value is reasonable. However, the question of why the resulting production capacity in this model is smaller than that of other models is still unanswered.

It can be concluded, by conducting some examination, that model B1 has done its best to locate a good solution in terms of the objective value so that the linear release structures imposed on the model are fulfilled. That solution has been achieved by decreasing the power-plant construction cost through reducing the production capacity. This may show that the benefit resulting from decreasing the power-plant construction cost may dominate the cost resulting from the reduction of the production capacity (resulting in a reduced firm energy generation). This might be because of the fact that unit values of both firm and secondary energies are considered the same as 160

---

### Table 7

<table>
<thead>
<tr>
<th>Item</th>
<th>( k = 2 )</th>
<th>( k = 4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total cost (10(^9) Rials)</td>
<td>-24,000.5</td>
<td>-24,013.9</td>
</tr>
<tr>
<td>Reliability of meeting the energy yield (%)</td>
<td>90.04</td>
<td>90.04</td>
</tr>
<tr>
<td>Normal water level (masl)</td>
<td>830</td>
<td>830</td>
</tr>
<tr>
<td>Min. operating level (masl)</td>
<td>815</td>
<td>807.5</td>
</tr>
<tr>
<td>Powerplant's production capacity (MW)</td>
<td>1,130.9</td>
<td>1,118</td>
</tr>
<tr>
<td>Annual firm energy (MWh)</td>
<td>2.4325 \times 10^6</td>
<td>2.2431 \times 10^6</td>
</tr>
<tr>
<td>Annual secondary energy (MWh)</td>
<td>7.5571 \times 10^5</td>
<td>9.4037 \times 10^5</td>
</tr>
</tbody>
</table>
(Rials per kWh). To check this, another computer run was carried out in which the unit value of firm energy sale was increased and the unit value of secondary energy sale was decreased. It was found that the optimal production capacity increases (910 MW) but not as much as that of other models. We realized through evaluating the objective function of model A2 that there are some good solutions (in terms of the objective value) around 800 MW of production capacity even for model A2 (or A1), although the production capacity for these solutions is smaller than the optimal capacity of the model (see Figure 4).

As the final point, it is worth mentioning that although the optimization problems developed and solved in this paper are for a single-reservoir system, they are quite difficult optimization models to solve. The reason is that the models are non-convex models. Moreover, considering a reliability constraint in the models requires a large number of binary variables included in the model’s formulation. This means that the optimization models are MINLP programs with many local optima which are NP-hard and thus difficult-to-solve problems.

Regarding the above point, several attempts were made to solve model B2, the formulation of which is given by Equations (5)–(20), by LINGO 10 software (LINGO Systems Inc. 2006), in which a combination of gradient-based NLPs and the branch-and-bound algorithm is used. The LINGO’s nonlinear solver employs both successive linear programming and generalized reduced gradient algorithms. The model includes 3,949 decision variables, 494 and 3,448 of which are respectively binary and nonlinear variables. It also includes 5,428 constraints, of which 1,971 constraints are nonlinear. After several model executions, each with more than $4 \times 10^5$ iterations and 1 hour of CPU time taken, the solver failed to arrive at a feasible solution, although the multistart option, in which the NLP solver restarts several times from different initial points, and also the option of invoking the global solver were utilized. The global solver employs the methods of range bounding (e.g. interval analysis and convex analysis) and range reduction techniques (e.g. linear programming and constraint propagation) within a branch-and-bound framework to find global solutions to non-convex models. It runs, however, considerably slower than the default nonlinear solver. As another check, the model B2’s solution obtained by the PSO was introduced to the LINGO’s solver as the initial point. It was found that the LINGO solver improved the objective value by less than 1%, showing the local search ability of the gradient-based algorithm used in the LINGO.

For the special case of TarREL = 1 and assuming that the optimal reservoir capacity is greater than zero, one can drop all the binary variables from the model formulation and convert it to an NLP. After several attempts, LINGO was able to converge to a feasible solution but reporting that it may not be a local optimum one. The solution was as follows: the objective value $= -23,963$, $P_{cap} = 982$ MW and normal water level $= 830$ m above sea level (masl). The same solution was obtained when solving this special case with multistart and global options of the solver. This shows the possibility of the solution to be the global optimum solution. More evidence supporting this possibility is that the special case problem was also solved by the PSO algorithm and the best objective value achieved was equal to $-23,790.4$, which is 99.3% of the best objective value of the LINGO.

In order to show the complexity of the search space of the problems, the variation of the objective function of model A2 against the variation of the model’s decision variables is shown in Figure 4. This figure has been obtained by fine discretization of the variables of production capacity and minimum reservoir operating level at the optimum normal water level $= 830$ masl. Then the objective function

![Figure 4](https://iwaponline.com/jh/article-pdf/12/3/275/386456/275.pdf)
was evaluated for any discrete point of the meshed space using the simulation model. As seen in Figure 4, the objective function is a multimodal function with many local optima, making it a kind of challenging optimization problem. Note that models A1 or A2 are the simplest models among the ones developed and their fewer number of decision variables has made it possible to perform such objective function evaluations. However, this would be enough to show how hard it could be to solve model B2 with a 495-dimensional search space.

**SUMMARY AND CONCLUSIONS**

This paper presents an application of the PSO algorithm to design and operation optimization of the Bakhtyari Dam hydropower system in Iran. Different formulations of optimization models were tested. Model B2, with the largest degree of freedom to search for the best solution in which both design and operational variables are optimized, was found to be slightly better than models with only design decision variables. However, it was realized that the significance of using such a more complicated model is marginal. In other words, models A1 and A2, which optimize only the design variables, perform as well as model B2, optimizing design and operational variables of parametric release rules. The reason behind such a result was explored and it was shown that model A1’s operating policy may be a near-optimal policy for model B2, as long as the energy deficits are linearly penalized. To verify this, the policy may be a near-optimal policy for model B2, as long as severe energy deficits were penalized, nonlinearly. It was found that the importance of optimizing the system operation becomes highlighted and model B2 performs better than models A1 or A2 in this case. It was also verified that a combination of the branch-and-bound algorithm and gradient-based optimization techniques employed in the LINGO solver fails to solve the studied problems.

**REFERENCES**


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