
Square-Root Relations Between Main Saccadic Parameters

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Purpose. To derive and evaluate two equations in which saccade duration and peak velocity are proportional to the square root of saccade amplitude.

Methods. A population of horizontal visually guided saccades in a range of amplitudes from 1.5° to 30° was recorded by means of electro-oculography in eight normal adult subjects. The peak velocity–amplitude data of this population were fitted to four models: inverse linear, exponential, power law, and square root. To approximate the duration–amplitude relation, the square root was tested against the linear and power law models. For each model, the best-fit values of its parameters were estimated by the method of least squares.

Results. When the entire population was used, all tested models displayed comparable goodness of fit, but when different subranges of this population were used, only the square root equations appeared to be robust and acceptably accurate.

Conclusions. In a restricted range of saccade amplitudes from 1.5° to 30°, the square root model has some advantages over the others commonly used: to express peak velocity and duration as functions of amplitude, it requires the estimation of only two parameters, whereas the others require four. Because of its robustness, this model can be used to evaluate populations of saccadic eye movements with different ranges of amplitudes. The two parameters of the model equations allow a simple and clear physical interpretation. *Invest Ophthalmol Vis Sci.* 1996;37:2750–2758.

Saccades have been described as fast movements of the eyes that bring to the foveal region a new part of the visual field.¹ The relationships of saccade amplitude to saccade peak velocity and duration are known as the main sequence for normal saccades or, simply, main sequence. To stress the importance of these relationships in studying saccadic eye movements and their neurophysiological control Bahill, Clark, and Stark² borrowed from the astronomy term main sequence, by which astronomers refer to the fundamentally important relationship between the brightness of a star and its temperature. Significant changes in the saccade main sequence may result from impairment in certain brain structures or to drug effects.^{3,4} Because of the stochastic nature of these relationships, correlation and regression analysis are used widely.

Sometimes in saccadic studies it is necessary to

compare two populations of saccades with nonoverlapping or poorly overlapping amplitude ranges. For instance, Van Gisbergen and coworkers⁵ studied in monkeys a population of macrosaccades (2° to 20°) and microsaccades (0.2° to 0.5°). Another example is the Van Gelder et al⁶ study of catch-up and anticipatory saccades in smooth pursuit tasks. In such cases, it is desirable to be able to describe data accurately with as few parameters as possible and with a good ability to predict the data outside the range used to estimate parameters.

Three types of theoretical curves have been proposed to approximate the peak velocity–amplitude relationship. One is inverse linear,^{7,8} also known as the Michaelis–Menten equation⁹

$$Vp = \frac{Va \cdot A}{A + Ao}$$

where Vp is peak velocity, Va is an asymptotic maximum of peak velocity (the saturation value), A is saccade amplitude, and Ao is the half-maximum amplitude, i.e., the amplitude value at which 50% of the peak velocity is reached.

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Another function often used to fit the data is of the exponential type,^{10,11}

$$Vp = Va \cdot \left[1 - \exp\left(-\frac{A}{Ao}\right) \right]$$

where *Va* again denotes the saturation value, and *Ao* represents the amplitude for which peak velocity reaches 63% of its saturation value.

The third suggested function is a power law

$$Vp = V_1 \cdot A^X$$

where *V₁* denotes the velocity of a saccade with one degree of amplitude, and the exponent *X* is on the order of 0.38 to 0.6.^{12,13}

For the duration–amplitude relationship, the linear function

$$D = D_0 + d \cdot A;$$

and the power law

$$D = D_1 \cdot A^p$$

have been suggested.⁸

THE SQUARE-ROOT MODEL

In this article, we show that for horizontal saccadic eye movements in a range of amplitudes from 1.5° to 30°, it is possible to approximate both saccade duration and peak velocity in a form

$$F = F_1 \cdot \sqrt{A}$$

where *F* is either duration or peak velocity, and *F₁* is the corresponding parameter value for the amplitude of 1°. The quality of this approximation is examined against the commonly used models mentioned above.

Interdependence of Main Saccadic Parameters

From the literature, it is known^{14,15} that a strong linear relationship ($r \geq 0.98$ ^{16,17}) exists between mean velocity *Vav* and peak velocity *Vp* of saccadic eye movements. Mathematically, this may be expressed in a form

$$\frac{Vav}{Vp} = K \approx Const. \tag{1}$$

In a wide range of saccadic amplitudes (from 5° to 60°), *K* takes values in the range 0.52 to 0.72.¹⁵ Taking into account that by definition

$$Vav = \frac{A}{T}, \tag{2}$$

where *T* is saccadic duration, it is possible to write equation 1 as

$$\frac{A}{T \cdot Vp} = K \approx Const. \tag{3}$$

which reflects the dependence among the three main observed parameters of saccadic eye movements. We will use equation 3 as a base to derive the other relationships of interest. For instance, Yarbus¹³ suggests the power law for the duration–amplitude relation:

$$T = 0.021 \cdot A^{0.4} \tag{4}$$

and

$$V = \frac{\pi A}{2T} \text{Sin} \frac{\pi}{T} t \tag{5}$$

for the angular velocity profile. From equations 4 and 5, we derive

$$Vp = 75 \cdot A^{0.6} \tag{6}$$

Alternatively, substitution of equation 4 into equation 3 for *T*, and letting *K* take its middle-range value, *K* = 0.64, gives the same result. It should be noticed that in deriving equation 6 from Yarbus, we used the shape of the velocity profile, whereas the relationships of equation 3 are free of this assumption.

It has been shown¹⁵ that the substitution of a linear function for *T* into equation 3 leads to the inverse linear dependence of peak velocity on amplitude. Thus, a linear duration–amplitude function yields the inverse linear dependence for peak velocity–amplitude, whereas a power function for the duration–amplitude relation yields a power function for peak velocity–amplitude. For example, $T = T_1 \cdot A^X$ for the duration–amplitude relation leads to $Vp = V_1 \cdot A^{1-X}$ for the peak velocity–amplitude relation. Yarbus¹³ reported *X* = 0.4.

Another example of the usage of equation 3 comes from the fact that for saccades of small amplitude, their peak velocity is proportional to their amplitude.^{1,15} One of the suggested approximations¹⁵ is $Vp = 60 \cdot A$. The substitution of this into equation 3 and, as before, letting *K* take its middle-range value, *K* = 0.64, leads to *T* = 0.026 second, which is in good agreement with data reported elsewhere.¹⁵ It follows now that by the relationships between main sequence parameters (equation 3), all saccades may be categorized into three different ranges of saccade ampli-

tudes: the small-amplitude range, in which duration remains fairly constant and increase in the amplitude is caused by an increase in the peak velocity; the mid-amplitude range, in which the increase in the amplitude is caused by an increase in saccade duration and peak velocity; and the large-amplitude range, in which peak velocity saturates—that is, an increase in the amplitude is primarily caused by duration.*

The mid-amplitude range, in which peak velocity and duration increase with amplitude, has been considered to be from 2°–3° to 30°–36°.10,18 The vast majority of naturally occurring saccadic eye movements belongs to this range of amplitudes, and they are made without head movement.1,13,19 Saccades in this range display certain specific properties:

1. Peak velocity does not reach its saturation value, which begins with amplitudes of 35° to 40°8,11 or approximately 30° in monkeys.19
2. The velocity profile may be considered as unimodal, i.e., velocity reaches its peak value at one point in the course of time.8,13 Here we deal exclusively with horizontal, visually guided saccades with amplitudes from 1.5° to 30°, which we assume to be in the mid-amplitude range.

Basic Equations

To approximate the peak velocity–amplitude relation, the power function was fitted to the population of 786 horizontal visually guided saccades in a mid-amplitude range, described below. The exponent of the power function was strikingly close to 0.5 ($X = 0.4954$, $SE = 0.0014$, $R^2 > 0.92$, $P < 0.001$). We use this empirical fact to formulate the first model equation:

$$Vp = V_1 \cdot \sqrt{A} \quad (7)$$

Substitution of equation 7 into equation 3 yields an equation for the duration–amplitude relation

$$T = T_1 \cdot \sqrt{A} \quad (8)$$

where

$$T_1 = (V_1 \cdot K)^{-1}$$

* Carpenter¹ provides the following vivid analogy: “For saccades less than some 5° in amplitude, the pulse is very short and the response is dominated by the mechanical properties of the eye, resulting in the more nearly constant duration associated with small saccades; under these circumstances, the peak velocity varies in proportion to the amplitude. The situation is rather like that of a man falling through the air: if his drop is a long one, the duration of his fall will be in proportion to its height, since for most of the way he will be falling at his terminal velocity. In shorter falls, acceleration will dominate his performance, and his peak velocity will depend on the distance he falls.”

The two model parameters are values of peak velocity and duration for a saccade of 1° amplitude. Note also that by logarithmic transformation, both equations can be reduced to a linear regression form, $y = a + 0.5 \cdot x$ (e.g., $y = \log T$, $x = \log A$).

From equations 7 and 8, it follows that a strong linear relationship must exist between peak velocity and duration in this range of the amplitudes:

$$\frac{Vp}{T} = \frac{V_1}{T_1} = y \approx Const \quad (9)$$

Similarly, it can be shown that from equations 7 and 8, equation 3 also follows.

This provides an alternative way to derive the square root equations. If we accept equations 3 and 9 as empirical facts, equation 9 may be rewritten in a form $Vp = \gamma \cdot T$. By substituting this into equation 3, the duration–amplitude relation takes the form of equation 8. The substitution of equation 8 into equation 3 leads to the peak velocity–amplitude relation in the form of equation 7.

Thus, we have shown that for the existence of the square root model, it is necessary and sufficient that equations 3 and 9 be held simultaneously. Both equations 3 and 9 may be examined directly from the data, and we are going to use them to justify the square root model.

MATERIALS AND METHODS

Subjects

Eight normal subjects were run as part of a larger oculomotor study.⁶ Their ages ranged from 22 to 34 years (mean, 26.1 years). None reported oculomotor, neurological, or psychiatric conditions likely to influence eye movement. All reported normal or corrected visual acuity at the normal reading distances required of their occupations (e.g., research staff of other laboratories). Procedures followed the tenets of the Declaration of Helsinki; informed consent was obtained before their participation and was approved by the New York University Medical Center Institutional Board of Research Associates.

Recording Procedure and Experimental Setup

Eye movements were recorded using electro-oculography—a horizontal channel obtained with silver–silver chloride electrodes at the outer canthus of each eye (summing the movement of the two eyes). Head restraint was achieved with chin and forehead rests and lateral head stops. The differentially amplified signals were digitized in real time at 250 Hz and were observed on a separate monitor.

The subject monitor was 30° across. The target

was an uppercase X, 0.34° wide by 0.6° high. Each trial began with a 3-point calibration sequence—a fixation target on for 1.5 seconds each at center, right, and left edges of the screen. The trial concluded with another calibration sequence in reverse order. Each subject was given four trials, of 20 seconds each, of 1° to 17° horizontal target jumps in pseudorandom directions, with intervening pauses of 0.8 second each. The trials were given in two pairs, interspersed with trials of smooth pursuit tasks of the same duration and with the same standard calibration procedure. Instructions were simply to “follow the moving X.”

Measurement and Analysis of Saccadic Parameters

Saccades were detected automatically by computer algorithm and were edited interactively to remove artifacts and to delete all saccades other than those responding to the target jumps. The result was a population of 786 horizontal visually guided saccades in a range of amplitudes from 1.5° to 30°. The two large calibration saccades per trial provided the 25° to 30° saccades. These were augmented by calibration saccades from the pursuit tasks. Rightward and leftward saccades were combined for the current analysis. Gain linearity across the full amplitude range was confirmed by waveform plots, where no systematic deviations were seen between target and point of gaze during saccade-free portions of the waveform.

The peak velocity–amplitude data of this population were fitted to four models: inverse linear, exponential, power law, and square root. To approximate the duration–amplitude relationship, we tested the square root against the linear and power law models. For each model, the best-fit values of its parameters were estimated by the method of least squares. The error term ERR was computed as a ratio of the sum of squared deviations of the data points from a given curve to the total sum of squared data points and was expressed in percent:

$$ERR = \frac{\sum_1^N [F(A_k) - Y_k]^2}{\sum_1^N Y_k^2} 100 \tag{10}$$

where $F(A)$ is a model equation, A_k is the amplitude value at a point k , and Y_k denotes the measured peak velocity or duration for that point.

RESULTS

We have shown that equations 3 and 9 are necessary and sufficient for the existence of the square root model. Equations 3 and 9 allow direct empirical testing. Because equation 1 is equivalent to equation 3, we examined equations 1 and 9 on the entire popula-

tion of saccades in the range of amplitudes from 1.5° to 30°. The regression function of mean velocity on peak velocity was $V_{av} = 0.60 \cdot V_p$, with standard error 0.002 and coefficient of determination $R^2 > 0.98$. The regression function of peak velocity on duration was $V_p = 5.42 \cdot T$, with standard error less than 0.05 and $R^2 > 0.94$. In addition, from equation 9 it follows that the ratio of V_p to T must be of the order of the ratio of V_1 to T_1 from equations 7 and 8. Calculated from the same population, the parameters (peak velocity and duration, respectively, for a saccade of 1° amplitude) were $V_1 = 97.58$ and $T_1 = 17.36$, and the ratio was 5.62, which is close enough to 5.42. Figure 1 illustrates the results.

Next, we used the entire population to estimate the parameters of all models. We refer to these estimates as the population parameters. Results are presented in Tables 1 and 2 (Pop Val) and are illustrated in Figures 2A and 2B. It can be seen that all curves may be used to approximate both velocity–amplitude and duration–amplitude scatterplots. The coefficient of determination R^2 was greater than 0.98 for all peak velocity–amplitude curves and greater than 0.97 for all duration–amplitude curves.

Then we tested the dependence of each model on the place and length of the amplitude range used to calculate the model parameters, i.e., how close the parameters would be to their population values, how stable they were, and how well they would predict data outside the range from which the parameters were estimated. To answer these questions, 14 subranges were drawn from the entire population. Each subrange was used to calculate parameters of a given curve, and the error term was calculated from the entire population. Results are summarized in Tables 1 and 2. Both tables have the same structure. The first five rows from the top show the effects of progressively deleted large amplitudes when the left edge was fixed at 1.5°, the next four rows (6 to 9) show the effects of progressively deleted small amplitudes when the right edge was fixed at 30°, and the next five rows (10 to 14) show samples of 5° ranges shifted from one to another sequentially by 2°. The bottom row shows the standard deviations of the parameters computed from these 14 ranges.

Several things are seen in the tables. First, both show that the square root parameters have little variability compared to all others. For instance, from Table 1, the maximum deviation of the square root parameter (range, e.g., 9 to 14) from its population value does not exceed 5% (rounded to the nearest integer). At the same time, the parameters of the power law (range, 13 to 18), exponential, and inverse linear models (range, 25 to 30) exceed 80%, 140%, and 200%, respectively. From Table 2, the maximum deviation of the square root parameter (range, 1.5 to 7) does not exceed 12%,

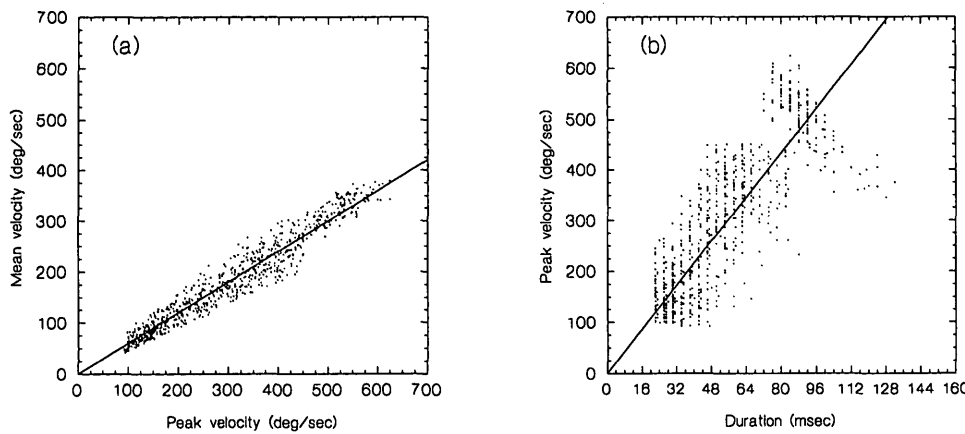


FIGURE 1. Scatterplot and regression function of saccade (a) mean velocity on peak velocity and (b) peak velocity on duration. The entire population of saccades in the amplitude range from 1.5° to 30° was used to estimate parameters.

but the maximum deviation of the parameters of the power law and linear models (range, 25 to 30) do exceed 110% and 146%, respectively.

Second, it can be seen from Table 1 that when the left edge of the ranges was fixed and the right edge progressively increased, both the inverse linear and exponential models provided unacceptably low values of the parameters and then monotonically approached the population values with the increase of the right bound-

ary. On the other hand, if the right edge of the ranges was fixed and the left edge progressively decreased, these models provided unacceptably high parameter values that approached their population values with the decrease of the left edge amplitudes. The same conclusion is valid for the error term. The power law model does not display such a property, and the square root model remains fairly stable. Two limiting cases are illustrated in Figure 2: the very left range of amplitudes, 1.5° to

TABLE 1. Peak Velocity–Amplitude Relations

Amplitude	Models										
	Inverse Linear			Exponent			Power Law			Square Root	
	V	Ao	Err	V	Ao	Err	V	X	Err	V	Err
1.5–20	624	9.0	1.4	442	7.1	2.3	87	0.56	2.0	100	1.4
1.5–17	622	9.0	1.5	438	7.0	2.3	86	0.57	2.1	100	1.4
1.5–15	620	8.9	1.5	430	6.8	2.5	84	0.59	2.6	100	1.4
1.5–12	572	7.8	1.9	395	5.9	3.8	83	0.59	2.7	98	1.4
1.5–7	426	4.9	5.9	292	3.7	11.6	85	0.57	2.0	94	1.5
10–30	710	11.8	1.3	544	11.2	1.6	136	0.39	2.1	98	1.4
15–30	760	14.5	1.6	572	13.5	2.1	130	0.40	1.9	96	1.4
20–30	1136	35.5	4.6	759	25.6	5.2	75	0.57	2.0	95	1.4
25–30	2154	92.1	8.1	1264	55.6	8.3	39	0.76	5.8	95	1.4
5–10	644	10.2	1.5	429	7.4	2.8	84	0.58	2.4	99	1.4
7–12	800	14.6	1.5	522	10.2	1.5	80	0.61	3.2	102	1.6
9–14	834	15.8	1.6	550	11.4	1.6	84	0.58	2.6	103	1.7
11–16	675	10.5	1.3	492	9.1	1.6	121	0.44	1.8	103	1.7
13–18	558	6.2	2.0	451	7.1	2.1	180	0.29	4.4	102	1.5
Pop Val	690	10.8	1.3	524	9.7	1.5	99	0.50	1.4	98	1.4
Standard deviation	438	23.8	2.5	234	13.8	3.6	34	0.12	1.7	3.2	0.1

$$\text{Inverse: } Vp = \frac{V \cdot A}{A + Ao}$$

$$\text{Exponent: } Vp = V \cdot \left[1 - \exp\left(\frac{-A}{Ao}\right) \right]$$

$$\text{Power: } Vp = V \cdot A^X$$

$$\text{Square-root: } Vp = V \cdot \sqrt{A}$$

$$\text{Err} = \frac{\sum_1^N [F(A_k) - Y_k]^2}{\sum_1^N Y_k^2} 100.$$

TABLE 2. Duration–Amplitude Relations

Amplitude	Models							
	Linear			Power Law			Square Root	
	<i>T</i> ₀	<i>T</i> ₁	Err	<i>T</i> ₁	<i>X</i>	Err	<i>T</i> ₁	Err
1.5–20	24	2.9	3.4	20	0.43	2.4	17	2.3
1.5–17	24	2.9	3.4	20	0.43	2.4	17	2.3
1.5–15	24	2.9	3.4	21	0.41	2.7	17	2.3
1.5–12	23	3.1	4.4	21	0.40	2.8	18	2.3
1.5–7	21	3.8	10.4	22	0.39	3.2	19	2.8
10–30	34	2.0	3.0	16	0.52	2.3	17	2.3
15–30	43	1.7	5.4	20	0.46	2.3	17	2.3
20–30	59	1.3	12.4	26	0.38	9.2	17	2.3
25–30	69	0.8	24.2	40	0.24	9.2	17	2.3
5–10	23	3.3	5.6	17	0.51	2.4	17	2.3
7–12	30	2.4	2.6	20	0.42	2.6	17	2.3
9–14	32	2.2	2.7	20	0.44	2.4	17	2.3
11–16	28	2.0	2.8	15	0.56	2.6	17	2.3
13–18	24	2.8	3.1	12	0.64	3.5	17	2.3
Pop Val	28	2.3	2.4	19	0.46	2.2	17	2.3
Standard deviation	15.4	0.8	7.2	6.7	0.1	2.8	0.6	0.1

Linear: $T = T_0 + T_1 \cdot A$

Power-law: $T = T_1 \cdot A^X$

Square-root: $T = T_1 \cdot \sqrt{A}$

$$Err = \frac{\sum_1^N [F(A_k) - Y_k]^2}{\sum_1^N Y_k^2} 100$$

7°, in Figures 2C and 2D, and the very right range of amplitudes, 25° to 30°, in Figures 2E and 2F. It is seen from these cases that inverse linear and exponential models, despite their fit within the range used for parameter estimation, are of no use outside the range of estimation. Their parameters do not allow any reasonable interpretation, and the error terms are unacceptably large. The estimates of the power law model based on the range 1.5° to 7° predict fairly well the values outside the range of estimate up to 20°, in accord with Yarbus.¹³ The estimates of the power law model based on the range of amplitudes 25° to 30° behave as poorly as the inverse linear and exponential models. The square root parameters remain almost the same. Notice also how close the square root curves are to the population means (open circles), which were computed independently for each integer amplitude.

Third, from rows 10 to 14 of Table 1, it follows that the estimates of the parameters of all models except the square root are unstable with respect to a small shift of the amplitude ranges from which they were drawn. For instance, parameters of the inverse linear model drawn from the interval 11° to 16° are less than 4% away from their population values, but a shift to the left of just 2° (range, 9 to 14) gives an overestimate of more than 20% for the *V*-parameter, and more than 46% for the *A*₀-parameter. The shift

of 2° to the right (range, 13 to 18) leads to the underestimate of more than 19% in *V*-parameter and more than 42% in *A*₀. The same order of behavior comes from comparison of rows 11 versus 10 for the exponential model and rows 13 versus 14 for the power law. The square root remains fairly stable. Similarly, even more instability is seen in Table 2 if one compares, for instance, rows 12, 13, and 14.

In both tables, we see that the standard deviations for the square root parameters are less than 4% of their population values, but for all others, the standard deviations are greater than 30% of the population values of the model parameters.

Next, we examined all the models with respect to individual subject data. For three of eight subjects, square root showed the best fit; for three other subjects, one or more of the other models was better; and for two subjects, the square root was as good as the best-fit model. Figure 3 illustrates the results for the subject where the square root model shows the largest error compared with the best-fit model. For the duration–amplitude relation, the best was linear regression, where $R^2 = 0.976$, whereas $R^2 = 0.973$ for square root. For peak velocity–amplitude, the best was inverse linear, where $R^2 = 0.998$, whereas $R^2 = 0.984$ for square root. It can be seen from Figure 3 that the behavior of the models fitted to this subject remains

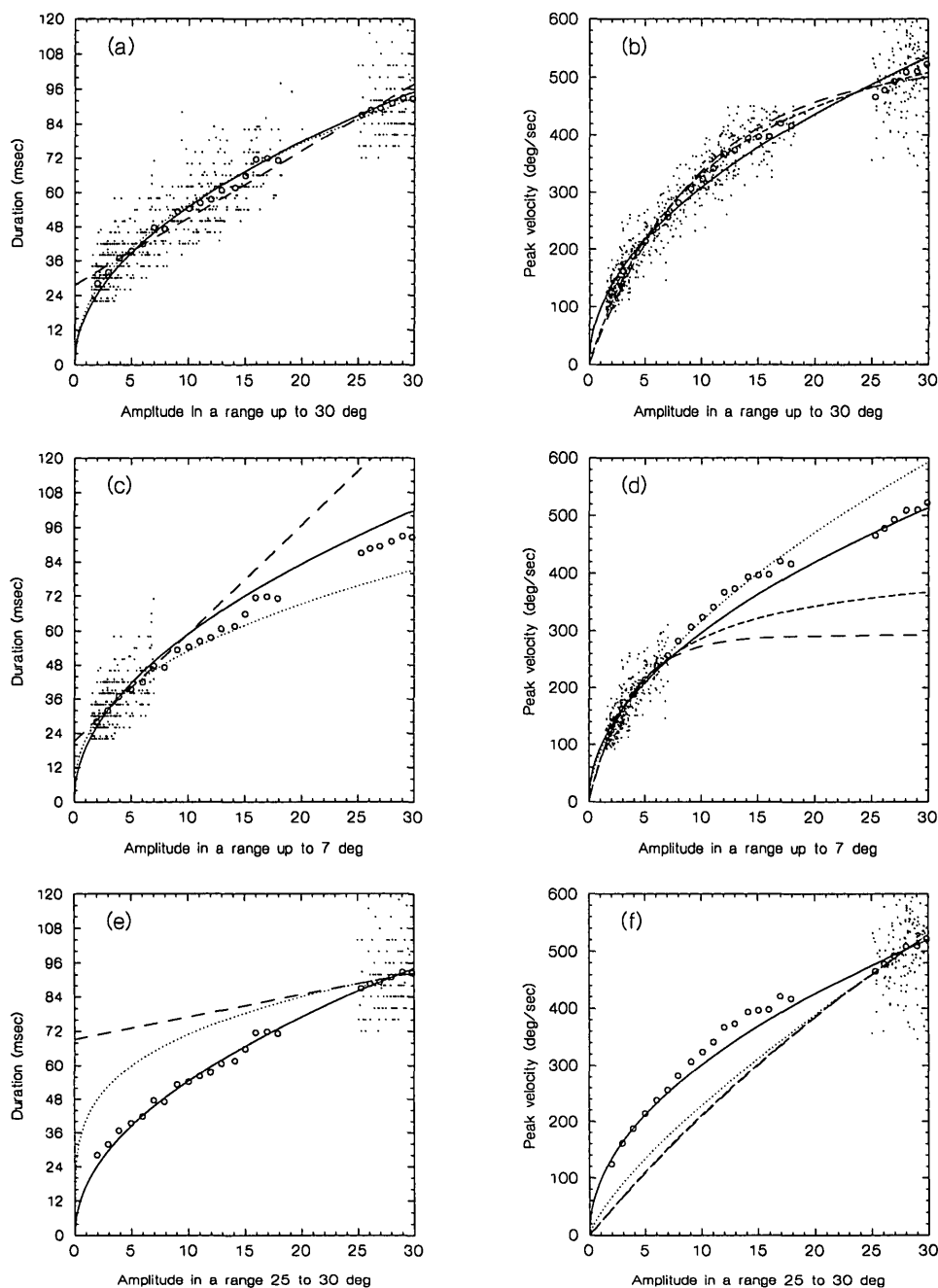


FIGURE 2. Examples of different models used to approximate dependence between saccadic parameters, calculating the parameter values using data in the entire range of amplitudes up to 30° (a,b), only up to 7° (c,d), and amplitudes between 25° and 30° (e,f). A linear function (*large dashed line*) is shown for the duration–amplitude relations (a,c,e); the exponential (*large dashed line*) and inverse linear (*short dashed line*) models are shown for the peak velocity–amplitude relationships (b,d,f); and the square root (*solid line*) and power law (*dotted line*) models are shown for both relationships. Shown are the data used for the models' parameter estimations (*small filled circles*) and mean values of the corresponding saccadic parameter estimated for 1° amplitude ranges centered on integer values (*large open circles*).

the same as when fitted to the data combined over subjects, i.e., of the entire population. No subjects showed peak velocity saturation within the population range. Such saturation would have improved the relative fits of the inverse linear and exponential models.

Finally, it could be argued that the square root model is only appropriate for a population of saccades whose amplitudes span the entire mid-amplitude range and that one of the other models might be better for the smaller and more typical eye move-

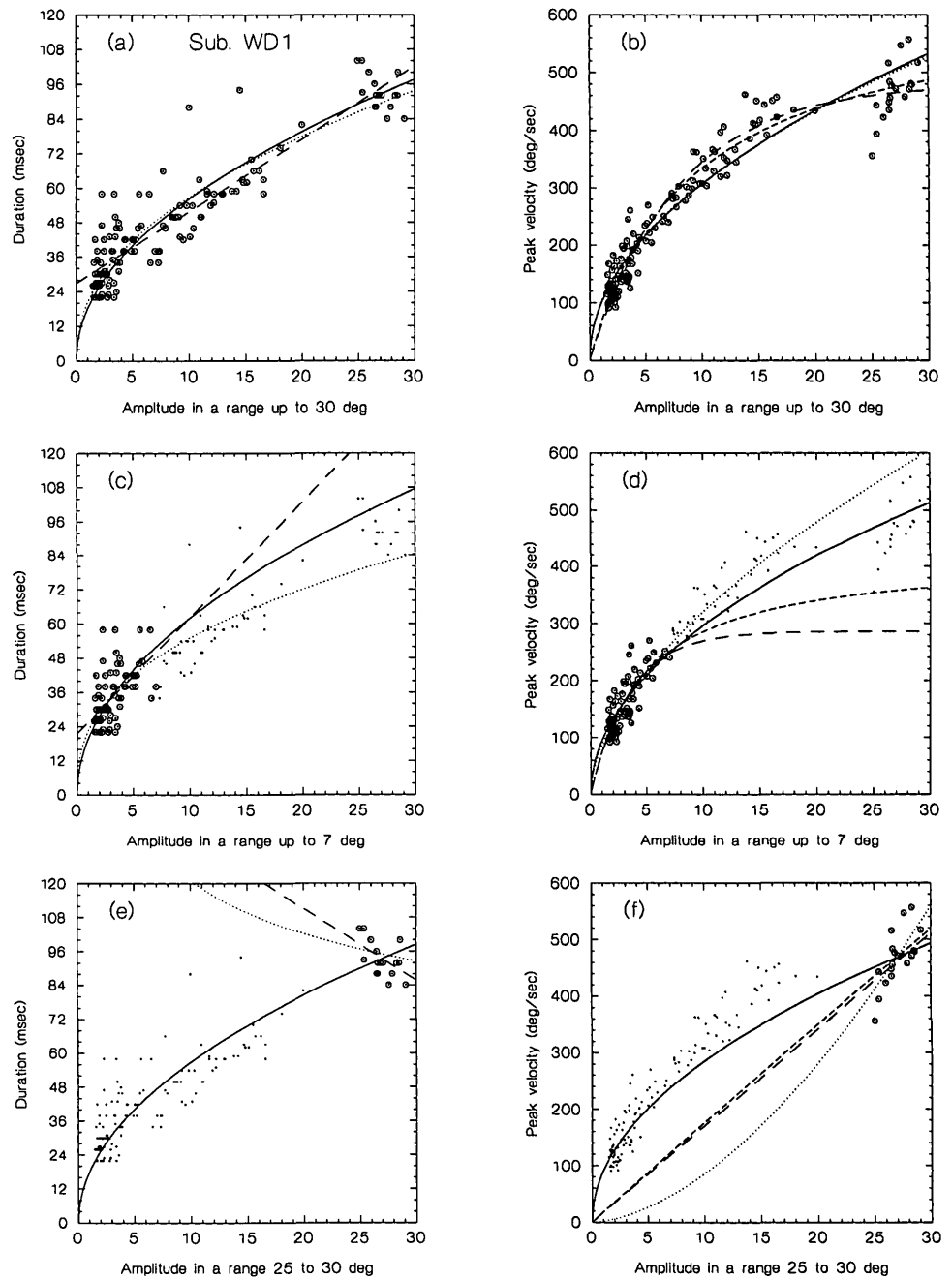


FIGURE 3. Examples of different models used to approximate dependence between saccadic parameters for data obtained from subject WD1. Amplitude ranges and functions estimated are the same as those for Figure 2. Shown are the data used for the parameter estimations (*large dotted circles*) and the rest of the data recorded from the subject (*small filled circles*).

ments. Indeed, this is true. For instance, in a range of amplitudes up to 15°, we found that the power law serves better than the square root model with respect to the goodness of fit and to stability of the model parameters estimated from the different subranges. Figures 2C and 2D also show the power law's excellent predictions of amplitudes up to 15° from the 7° subrange. Inverse linear and exponential models remain unacceptably unstable.

DISCUSSION

Comparison of Data With Prior Research

To approximate the duration–amplitude relationship, Yarbus¹³ used the empirically found formula T

$= 0.021 \cdot A^{0.4}$, and he examined this formula in a range of saccadic amplitudes up to 20° using a small mirror fixed to the eye.¹³ Our estimate, calculated from the same range of amplitudes and obtained by the electro-oculography method, gives $T = 0.020 \cdot A^{0.43}$. We derive from Yarbus for peak velocity–amplitude $Vp = 75 \cdot A^{0.6}$, and our estimate from the range of amplitudes up to 20° (Table 1) is $Vp = 87 \cdot A^{0.56}$.

Collewijn et al,¹¹ using a search coil recording technique, reported for the peak velocity–amplitude relationship $Vp = 520 \cdot [1 - \exp(-A/11.2)]$, with amplitudes ranging to 80°. Our result, in a range up to 30°, is $Vp = 524 \cdot [1 - \exp(-A/9.7)]$ For duration–amplitude and amplitudes to 50°, they found $T = 23$

+ $2.7 \cdot A$, and our data give $T = 28 + 2.3 \cdot A$. When the peak velocity–amplitude parameters were calculated from the range of amplitudes up to 20° , our saturation value was underestimated as $Vp = 442 \cdot [1 - \exp(-A/7.1)]$, which is close to the estimate obtained by Collewijn et al¹¹ of $Vp = 450 \cdot [1 - \exp(-A/7.9)]$ in a range up to 30° .

Finally, in a recent study of horizontal saccades performed with the infrared limbus-tracking technique,²⁰ the main sequence in the reference adults was in agreement with most previous reports. In a range of amplitudes up to 30° , they found $Vp = 488 \cdot [1 - \exp(-A/9.17)]$ for peak velocity–amplitude and $T = 28.7 + 2.59 \cdot A$ for duration–amplitude.

It follows that our data are in a good agreement with representative parameter values of others obtained by different recording techniques.

CONCLUSIONS

Five conclusions can be reached from this study. First, the square root model seems to be as accurate as other models in characterizing saccadic data over the entire range of amplitudes up to 30° . This is seen in Figures 2A and 2B and in error terms for population values in Tables 1 and 2. Second, only the square root model appeared robust; it remained fairly constant over the subsets of amplitude ranges. This is seen in Tables 1 and 2 and is illustrated in Figures 2C to 2F. Third, the model requires the estimate only of two parameters, whereas all others require four. These parameters allow simple and clear interpretation. Fourth, by the use of a logarithmic transformation, the model equations can be written in a simple linear regression form. Fifth, the model can be used only in a range of saccade amplitudes in which the peak velocity saturation value has not been reached. The vast majority of naturally occurring saccades belongs to this range of amplitudes.

Key Words

amplitude, duration, peak velocity, saccadic eye movements, velocity

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