A single internal working surface in a periodic jet

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ABSTRACT

We present the numerical simulation of a single internal working surface which forms along the body of a supersonic jet with periodic variations in the magnitude of its velocity. The structure, the evolution and the observational properties of this flow are analysed. We make a qualitative comparison with previous analytic predictions, simulations of the complete jet and observations of Herbig-Haro jets. The apparent lag of the knots of emission with respect to the jet flow (which has been observed in some jets) can be explained by this model.

Key words: methods: numerical – ISM: jets and outflows.

1 INTRODUCTION

In recent years it has become evident that the supersonic flows of atomic gas from young stellar objects are not steady, but rather show bursts in the magnitude of the velocity and changes in the ejection direction. In particular, there is evidence that some strings of Herbig–Haro objects (or stellar jets) have suffered repeated bursts. Jets such as HH34 (Heathcote & Reipurth 1992) and HH111 (Reipurth, Raga & Heathcote 1992) show several aligned, bow-shaped knots which have been interpreted as resulting from repeated increases in the magnitude of the velocity of ejection (Raga et al. 1990). The knots of emission that make up these jets have also been found to have larger proper motions (Eisloffel & Mundt 1992; Heathcote & Reipurth 1992; Eisloffel & Mundt 1994).

Such jets have been modelled as flows with periodic, supersonic variations in the magnitude of the ejection velocity. Raga et al. (1990) developed a model of such a flow in one dimension, and found that a pair of shocks appears in the jet for every pulse of the velocity: one shock which accelerates the slower gas ahead and another which decelerates the faster gas behind. They named these structures internal working surfaces. Kofman & Raga (1992) and Raga & Kofman (1992) further expanded this model.

Some of the recent numerical simulations relevant to this model are Falle & Raga (1993) in which a symmetric pair of shocks is simulated in steady state. Stone & Norman (1993) present 2D and 3D simulations of the complete jet, while in Hartigan & Raymond (1993), a 1D simulation which includes detailed atomic processes is presented. Falle & Raga (1995) carried out simulations of a single internal working surface for different cooling distances using an adaptive grid code. In Biro & Raga (1994) (hereafter Paper I), the complete flow of a jet with a periodic variation in the magnitude of the velocity is simulated in 2D cylindrical symmetry. The structure, time-evolution, emission maps, profiles and emission as a function of distance from the source are presented. Although the simulation in Paper I reproduces the properties of flows such as HH111 and HH34 in general, greater resolution is necessary in order to make more precise comparisons between models and observations.

In this paper we present a simulation of a single internal working surface (IWS) at higher spatial and temporal resolution (as prompted by the results of Paper I). The assumption that the variation in the ejection velocity is periodic (and thus that the separation between successive IWS is regular) is made. This simulation is of one knot in a string of regularly spaced knots. The morphological, kinematic and energetic properties for a single IWS are computed for this model and compared with existing data.

2 THE NUMERICAL METHOD

In order to zoom-in on a single IWS, we assume that, at large enough distances from both the source and the head, the spatial structure of the jet is periodic. In other words, we assume that there is a string of regularly spaced knots, and that two consecutive knots will be at very similar stages of their evolution. If this is the case, then the simulation of a single IWS with periodic boundary conditions in the up and downwind directions is equivalent to that of a string of knots and permits a greater spatial and temporal resolution. The assumption of periodicity appears to be reasonable as, for example, the knots along the body of the jet associated with HH34 are quite evenly spaced (e.g. Eisloffel & Mundt 1992). However, the distance between two successive knots is
important because the closer they are, the more they will interact.

The gas is assumed to be compressible, inviscid, non-adiabatic and with cosmic abundances. We also assume that the flow has cylindrical symmetry and can be described by two-dimensional axisymmetric equations and that the medium surrounding the knot is initially uniform (with constant \( \rho \) and \( P \)).

The scheme used for the numerical integration of the equations that describe this flow is an implementation of the flux-vector-splitting approximate Riemann solver of van Leer (1982). A description of the code and its non-equilibrium cooling function can be found in Biro, Raga & Canto (1995).

2.1 Initial and boundary conditions

The two most important requirements for this case are the periodicity of the flow pattern and the convenience of keeping the knot in a fixed position (to reduce the numerical diffusion associated with rapidly varying shocks). Both these conditions can be met with the right initial and boundary conditions.

Fig. 1 shows the computational grid that was used for this simulation. The parameters chosen are the following: \( T_i = T_e = 10^4 \) K (initial jet and environment temperature), \( P_i = 10^{-10} \) dyn cm\(^{-2} \) (pressure of jet gas), \( n_i = 72.5 \) cm\(^{-3} \) (jet particle density), \( n_i/n_e = 50 \) (jet-to-environment density ratio), \( v_i = 100 \) km s\(^{-1} \) (jet velocity), \( r_j = 2.5 \times 10^{15} \) cm (jet radius), \( v = 50 \) km s\(^{-1} \) (half-amplitude of velocity variation). In general these parameters are the same as those used for the simulation of the complete jet in Paper I. The fact that, in this case, the resolution is higher permits the use of a greater particle density and a smaller jet radius (closer to the values deduced from observations).

We use a grid of cylindrical coordinates with \( n_x \times n_y = 300 \times 300 \) points. This resolution can be handled by a medium-sized workstation. The actual physical size of the domain is of \( x_{\text{max}} = r_{\text{max}} = 2 \times 10^{16} \) cm.

![Figure 1. Initial and boundary conditions for the simulation of a single IWS. The bottom 20 rows of the grid are initialized with the jet variables. A sinusoidal velocity profile is given to the jet gas. The remainder of the domain is initialized with the environment variables. The top and bottom walls have a reflection condition. The left and right walls have a periodic condition.](https://example.com/figure1)

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In this simulation, a velocity distribution is initially set up in space, which results in an IWS like that which appears with a variation in time of the ejection velocity. The jet velocity is given a sinusoidal profile centred in the grid, so that two flows of opposite directions meet in the centre of the domain. In this way, the knot that is formed remains approximately fixed in the centre of the grid, which is equivalent to carrying out the simulation in a frame of reference moving with the IWS. The initial profile of the velocity is given by

\[
V(x) = \Delta V \cos \left(2\pi \frac{x}{x_{\text{max}}} - \frac{\pi}{2}\right).
\]

The velocity of an IWS in a jet with pulses in the velocity is found by Kofman & Raga (1992) to be the average jet velocity. In order to simulate the motion of this IWS relative to the environment, the gas in the environment is given a value equal to the negative of the average jet velocity \( v_e = -v_i = -100 \) km s\(^{-1} \).

A single set of parameters has been used here. In the full jet simulations of Stone & Norman (1993), a variety of amplitudes and frequencies of perturbations are tried, with the general result that greater amplitudes result in a faster evolution (and thus decay) of the IWS. The simulations of a single IWS of Falle & Raga (1995) are quite similar to the one presented here (with greater resolution and no assumption of periodicity), and present several cases with different cooling distances.

The boundary conditions are the reflection on the bottom (to achieve symmetry) and the top wall (which is never reached by the flow). The side walls have a periodic boundary condition. This means that: whatever flows out from the left side flows in from the right, and vice versa. This condition simulates the periodicity of the successive IWS of the flow. Allowing the backflow from the knot to be also the inflow from ahead is equivalent to saying that there is an identical knot ahead and behind the one that is being computed, at a distance equal to the length of the domain \( (x_{\text{max}}) \).

Fig. 2 shows the structure of the flow that results from the initial conditions described above. Superposed on the pressure structure are arrows which indicate the direction of the flow. Material from the jet flows into the IWS through a pair of shocks and then flows out sideways. This gas is swept backwards because of its motion relative to the environment.

3 RESULTS

3.1 Structure and evolution of the IWS

Fig. 3 shows the time-evolution of the pressure structure of this IWS. Contours of equal pressure together with greyscale have been plotted starting at a time \( t \approx 60 \) yr and then at intervals of \( \approx 30 \) yr. The image has been reflected along the symmetry axis. The two-shock structure forms almost immediately. The locus of these shocks is the place where there is a large pressure gradient (many pressure contours together). As material enters the IWS from the jet through both shock surfaces, the knot grows (the separation between the two shocks becomes greater).

The shocked gas trapped between the two shocks has a high pressure and can only escape in the radial direction. The motion of the IWS relative to the environment causes...
this lateral flow to be swept backward. The interaction of these two flows results in the formation of a bow-shaped shock, which initially grows in the radial direction and later on becomes flattened and begins to drift back with respect to the IWS. The reason for this is that the mass inflow into the IWS (and thus out of the sides) decreases with time, so the ram pressure of the external gas is soon greater than the gas pressure of the jet gas being ejected sideways. Since in this case the head of the jet is very far away, its cocoon does not shield the IWS from interacting with the quiescent environment.

Fig. 4 shows the structure of (a) pressure, (b) density, (c) temperature and (d) ionization fraction at a time $t = 120$ yr (which is equivalent to a distance travelled away from the source of $x = 3.4 \times 10^{16}$ cm). As in Fig. 3, the images are reflected along the symmetry axis. In the density stratification, it can be seen that just behind the shocks the gas is not strongly compressed, and only further from them (and toward the centre of the knot) does it condense. The temperature shows exactly the opposite behaviour, there is very hot gas right behind the shocks and then cooler (denser) gas in the middle. This behaviour is typical of shocks with strong post-shock radiative cooling. It should be noted that the highest pressures, temperatures and degree of ionization by this time are not in the knot, but in the bow-shock wings. The importance of this will become evident when maps of emission are presented.

We also measure the growth of the knot as a function of distance. The growth exhibits a linear behaviour with respect to distance

$$\frac{\Delta x}{r_j} = L_N x,$$

where $L_N = 4.01 \times 10^{-17}$ cm$^{-1}$. The growth of the knot as a function of distance is ten times greater than in the simulation of the whole jet described in Paper I. The reason for this appears to be that as the bow-shock drifts back along the top of the IWS, the environment does not permit the lateral flow to continue and thus forces the knot to grow in the axial direction.

### 3.2 Emission of the IWS

The emission in the Hα and [S II]6717 + 31 lines has been calculated from the numerical simulation. From the maps of density, temperature and ionization fraction of hydrogen, the emission coefficient of each of these lines can be calculated. The Hα emission is calculated by considering the contributions arising from the recombination cascade and to the collisional $n = 1 \rightarrow 3$ excitation. In the case of sulphur, the particle density of S II is calculated assuming coronal ionization equilibrium between S II, S III and S IV. The red [S II] lines are obtained from a five-level atom, low-density-regime calculation.

Fig. 5 shows maps of intensity in the Hα line at three different times (a) $t = 75$, (b) $t = 120$, (c) $t = 165$ yr. These maps are calculated using the Hα emission coefficient and by integrating the 3D emission structure through lines of sight perpendicular to the flow direction. Fig. 5(a) shows a bow-shaped emission structure in the centre of the region, in Figs 5(b) and (C), this pattern is seen to drift backwards. This reflects the drifting backwards of the bow-shock that was observed in Fig. 3, and shows that, at this point in the evolution of the IWS, most of the emission comes from the bow-shock and not from the two on-axis shocks. As the bow continues to drift back along the jet, it eventually meets up with the next knot along the line and splatters on to it, making it brighter again. Thus, although the knot moves at the jet velocity, in this simulation the emission moves more slowly (as it drifts in the direction opposite to the advance of the jet).
Figure 3. Pressure structure (contours plus grey-scale) of an IWS at (a) \( t = 60 \) yr, (b) \( t = 90 \) yr, (c) \( t = 120 \) yr, and (d) \( t = 150 \) yr. The formation of the two shocks is seen, as well as a bow shock resulting from the reflection of the inner surface of shocked jet gas with the ambient gas. The bow shock drifts back with respect to the IWS at later times.
Figure 4. (a) Pressure, (b) density, (c) temperature, and (d) ionization fraction structure of the IWS at a time \( t = 120 \text{ yr} \). Logarithmically spaced factors of 2 contours and grey-scales are both used.
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Figure 5. Maps of intensity in the line Ha at times (a) $t = 75$ yr, (b) $t = 120$ yr, and (c) $t = 165$ yr. The bow-shaped peak of emission is seen to drift back along the jet and eventually runs into the next knot.

Figure 6. Value of the ratio of knot velocity to jet velocity ($\zeta = v_k/v_j$) for several times or distances from the source. The values of the radial and tangential velocities of the flow at the knot are measured and used to calculate this ratio. $\zeta$ is found to have values similar to those extracted from observations.

Given the previous result, and the observations of Eisloffel & Mundt (1992) and Heathcote & Reipurth (1992), which show the velocity of the knots along the HH34 jet to be lower than that of the jet itself, we investigate these velocities in our simulation. We measure the tangential and radial velocities ($v_t$ and $v_r$) of the IWS at several times (or distances from the source) in order to make a comparison between the value of the ratio $\zeta = v_k/v_j$ (with $v_k$ and $v_j$ the knot and jet velocities, respectively) found in HH34 and our theoretical prediction. For the purpose of this calculation, an angle of $\alpha = 30^\circ$ between the jet axis and the plane of the sky is assumed. The tangential velocity of a given knot is obtained directly from the displacement of the peak of emission in a given time interval. The radial velocity in the vicinity of the knot is obtained by deriving a line profile for the whole of the integration grid and taking the peak to be the radial velocity. With these velocities, and following Eisloffel & Mundt (1992), we then compute the ratio $\zeta = v_t/v_r \tan \alpha$.

Fig. 6 shows the value of $\zeta$ at several distances from the source. For this simulation, $\zeta$ takes values between 0.4 and 0.8, similar to those derived from observations. Thus we have shown that the drifting back of the bow shock with
The growth of the IWS (given by the ratio $\Delta x/r_0$) is measured and found to be ten times greater than the value found for a knot in the complete jet simulation described in Paper I. The reason for this difference is the strong interaction of the environment with the knot in the single IWS calculation.

Emission maps of the $\text{H}\alpha$ line show that the peak of emission drifts back with respect to the position of the IWS in the jet. We measure the radial and tangential velocities of the flow at the knot for several times (or distances) and find values of the ratio of knot velocity to jet velocity, $\xi < 1$, similar to those obtained from observations (Eislöffel & Mundt 1992).

We follow the emission in the $\text{H}\alpha$ and $[\text{S}\ II]$ lines as a function of the distance of the knot from the source. Both lines peak very near the source and decay soon after that. In this case the peak in $[\text{S}\ II]$ is stronger than that of $\text{H}\alpha$. At greater distances, the $\text{H}\alpha$ is more than twice as strong as the $[\text{S}\ II]$ emission. Neither the fact that the intensities peak so close to the source, nor the value of the $[\text{S}\ II]/\text{H}\alpha$ line ratio appear to agree with the line intensity distributions observed in objects like the HH34 jet or HH113 (Heathcote & Reipurth 1992, Reipurth et al. 1992). The quick decay of the emission may be due to the large amplitude of the perturbation used to produce the IWS (Stone & Norman 1993). Different values of these parameters may result in emission measurements which compare better with observations. Further modelling is clearly necessary to obtain a complete understanding of these objects.

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REFERENCES