Table 3

K AS A FUNCTION OF (a/h)

$$K \doteq \left(2 - \left\{\left[1 + 2\left(\sqrt{\frac{2(1-2v)}{(1-v)^2}} - 1\right)\frac{h}{a} - \left(\frac{2(1-2v)}{(1-v)^2} - \sqrt{\frac{2(1-2v)}{(1-v)^2}} - 1\right)\frac{h^2}{a^2}\right] \mathrm{erf}\sqrt{\frac{a}{2h}} - \sqrt{\frac{2}{\pi}}\frac{(1-a/h)}{\sqrt{a/h}}\exp\left(\frac{-a}{2h}\right)\right\}\right)^{-1/2}$$

	K	
$\frac{a}{h}$	$v = .30$	$v = 0$
1.00	0.789	.982
1.11	0.891	1.021
1.67	1.031	1.141
2.00	1.091	1.201
2.86	1.131	1.150
5.00	1.089	1.141
10.00	1.029	1.049
20.00	1.013	1.020
100.00	1.004	1.002
1000.00	1.000	1.000

NOTE: A similar table in Reference (1) is in error; it ignored the numerical factor 2 preceeding the curly brackets.

$$q(r) = \frac{G(1-v)a^2}{(1-2v)hR}\left\{\frac{r^2}{a^2}\,erf\left[\frac{a}{2h}\left(1-\frac{r^2}{a^2}\right)\right]^{1/2} + \left[\left(\sqrt{\frac{1-2v}{2(1-v)^2}} - 1\right) + \frac{r^2}{a^2} + \left(\frac{1-2v}{2(1-v)^2} - \frac{1}{2}\sqrt{\frac{1-2v}{2(1-v)^2}}\right)\frac{h}{a}\right] \times \frac{1}{\sqrt{\pi}}\left[\frac{a}{2h}\left(1-\frac{r^2}{a^2}\right)\right]^{-1/2}\exp\left[\frac{-a}{2h}\left(1-\frac{r^2}{a^2}\right)\right] - 1\right\} \tag{A7}$$

where $q(r)$ is the contact stress, r is the radial coordinate, G is the shear modulus of the film, v is Poisson's ratio of the film, h is the film thickness, a is the contact radius, and R is the radius of curvature of the sphere (or spherically shaped segment).

Integrating the contact stress over the contact area gives the total normal load, i.e.,

$$P = 2\pi\int_0^a q(r)\,rdr$$
$$= \frac{Ga^4(1-v)}{2R(1-2v)h}\left(2 - \left\{\left[1 + 2\left(\sqrt{\frac{2(1-2v)}{(1-v)^2}} - 1\right)\frac{h}{a} + \left(\frac{2(1-2v)}{(1-v)^2} - \sqrt{\frac{2(1-2v)}{(1-v)^2}} - 1\right)\frac{h^2}{a^2}\right] erf\sqrt{\frac{a}{2h}} - \sqrt{\frac{2}{\pi}}\frac{\left(1-\frac{a}{h}\right)}{\sqrt{(a/h)}}\exp\left(-\frac{a}{2h}\right)\right\}\right) \tag{A8}$$

A similar equation presented in reference (1) is in error; it left out a factor of two from the large parenthesis in the foregoing. The results of reference (1) are still valid, however.

Rearranging equation (A8), the area of contact is given by

$$A = \pi a^2 = K\sqrt{\frac{2\pi P(1-2v)hR}{G(1-v)}}. \tag{A9}$$

As the value of K is never far from unity (see Table 3), one may simplify this expression to

$$A = \sqrt{\frac{2\pi P(1-2v)hR}{G(1-v)}}. \tag{A9'}$$

The average contact stress, σ, is just the load divided by the area of contact, hence

$$\sigma = \frac{P}{A} = \sqrt{\frac{PG(1-v)}{2\pi(1-2v)Rh}} \tag{A10}$$

The equations given by Appendix 1 and Appendix 2 should satisfy a very great need. Their existence should mean the complete abandonment of the Hertz equations in analyzing solid lubricated contact.

DISCUSSION

R. L. Adamczak[2] and B. Rubin[2]

Dr. Finkin is to be commended for his careful attempt to reduce to engineering practice the empirical theories of wear. The bonded dry film lubricants have long been plagued with the uncertainty aspects over the question of probable performance life. A start in this direction must inevitably increase the early design specification and use of this lubrication technique.

The author has suggested a wear equation modeled after the one proposed by Burwell[3,4] and has recommended a limitation to a mild low contact stress adhesive wear situation. Other recent work[5] reports the equation of the type suggested by the author to generally apply over much broader contact stresses, and, in fact, concludes that correlation of various wear situations including the high contact bench wear tests and operational data can be specified through knowledge of calculated wear coefficients.

It is noted that in Table 1 of the paper that the wear coefficients vary inversely with the contact stress in five out of the six examples where both 200 and 10,000 psi are available for the same material and test conditions. What is the author's explanations of this apparent relationship?

Dr. Finkin suggests that for preliminary design estimation that the wear equation has an order of magnitude validity and subsequent application will necessitate testing to determine exact film life. We have examined this assumption against some recently reported life test data on journal bearing test specimens coated with dry film lubricants.[6] The dimensions were such that a maximum volume of 0.008 in^3 bonded lubricant could fill the clearance space. The volume loss by calculation, using the wear coefficients proposed in Table 2, are approximately two orders of magnitude. This would mean that this wear coefficient approach could tend to lead one far astray in choice of bonded lubricants. Since the factor K, represents the tendency of the contacting surfaces to wear by adhesive bonding between the opposing surfaces and includes a great many intrusive and external variables of the bonded film and the film substrate interface, intuitively it would appear that the factor K be subdivided perhaps between two coefficients between film characteristics and interface characteristics K_1 and K_2 as a refinement. Would the author care to comment on the possibilities of improving the correlation between the test data and calculation based on test data?

Wright-Patterson AFB, Ohio.

[3] Burwell, J. T., and Strang, D. J., *Applied Physics*, Vol. 23, 1952, p. 18.

[4] Burwell, J. T., "Survey of Possible Wear Mechanisms," *Wear*, Vol. 1, 1957, p. 119.

[5] Haines, C. E., and Dufur, C. W., "Wear Coefficient Correlation of Test Methods and Design Calculations for Solid Film Lubricants," *ASLE Trans.*, Vol. 11, 1968, p. 261.

[6] Hopkins, V., and Campbell, M., "Friction and Wear—Life of Selected Lubricant Films at −100°F, R. T. and 400°F," ASLE Preprint No. 69AM6C-1 24th ASLE Annual Meeting, May 5–9, 1969.

Table 4 Calculated volumetric loss of bonded film

Film	Film wear coefficient	Calc volume loss
MoS_2—Graphite epoxy	10^{-10}	0.29 in.^{3}
MoS_2—Graphite phenolic	10^{-10}	0.11
MoS_2—Sb_2O_3 polyimide	10^{-11}	0.003
MoS_2—Graphite Na-silicate	10^{-10}	0.13
MoS_2—Graphite mixed oxides	10^{-9}	0.42

Test conditions
Temperature—Air ambient
Load—3000 psi (projected)

Author's Closure

The apparent correlation, observed by the discussers, between contact pressure and wear coefficient, cannot presently be satisfactorily explained. This suggests a topic requiring further investigation and the discussers are invited to participate. The reason for the phenomenon may lie in stress-controlled film structure aspects, including possibly the degree of compaction.

Looking at Figs. 7 and 8 of the discussers' reference [6], one sees a range of mean wear life, for the many solid lubricant combinations tested, of about a factor of 10. This helps to substantiate the author's assertion that one can estimate wear life to within this accuracy. Any disagreement arises from the assumed value of the wear coefficient, K. It must be borne in mind that later quantity is not a universal constant, but rather lumps together all aspects of environment, temperature, substrate materials and their properties, surface pre-treatment, and the like. The more closely the system used to determine K resembles the intended use, in the foregoing characteristics, the better will be the agreement between prediction and reality.