Fundamental nonlinearities of the reactor-settler interaction in the activated sludge process

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ABSTRACT

The activated sludge process can be modelled by ordinary and partial differential equations for the biological reactors and secondary settlers, respectively. Because of the complexity of such a system, simulation models are most often used to investigate them. However, simulation models cannot give general rules on how to control a complex nonlinear process. For a reduced-order model with only two components, soluble substrate and particulate biomass, general results on steady-state solutions have recently been obtained, such as existence, uniqueness and stability of solutions. The aim of the present paper is to utilize those results to formulate some implications of practical importance. In particular, strategies are described for the manual control of the effluent substrate concentration subject to the constraint that the settler is maintained in normal operation (with a sludge blanket in the thickening zone) in steady state. Such strategies contain how the two control parameters, the recycle and waste volumetric flow ratios, should be chosen for any (steady-state) values of the input variables.

Key words | clarifier, continuous sedimentation, modelling, partial differential equation, thickener

INTRODUCTION

Mathematical models have been used frequently for the study of each of the different sub processes within the activated sludge process (ASP) in biological wastewater treatment (WWT). The biological reactors have been modelled in detail by, if considered completely mixed, a system of ordinary differential equations (ODEs) (Henze et al. 2000; Gujer 2008). The secondary settling tank (SST), where the separation of water from sludge occurs, can be modelled with partial differential equations (PDEs) (Chancelier et al. 1994; Diehl 1996; Bürger et al. 2005; De Clercq et al. 2008; Bürger et al. 2011). Many simulation models have been presented also for the entire ASP, see references in Diehl (2012). However, regardless of the successfulness of a simulation model, it cannot give general rules on how to control the SST and hence neither the ASP. Strategies for control therefore need analysis of the mathematical model, i.e. the coupled ODE-PDE system.

Attir & Denn (1978) presented an interesting simulation model of the ASP, consisting of two ODEs for the biological reactor, modelling one substrate and one biomass component, and a numerical algorithm for the SST. With the same two material components, Sheintuch (1987, 1993) performed a limited steady-state analysis. Since then, in publications with the aim to analyse (not by simulations) and control the reactor-settler interaction, the SST has either been modelled in an excessively simple way (e.g. with a single ODE) or it has been assumed to always behave in a certain way, for example, it never becomes overloaded and there always exists a sludge blanket in the thickening zone independently of the operating conditions (Charef et al. 2000; Georgieva & Ilchmann 2001; Ito 2004; Kumar et al. 2006, 2009; Liu et al. 2006; Zhao et al. 2006; Tzoneva 2007; Koumboulis et al. 2008; Boulkroune et al. 2009; Francisco et al. 2011; Serhani et al. 2011). Of course, a variation of the input variables or a change in the control parameters will influence the SST, which may become over- or underloaded. An analysis has to include this complication automatically without making any a priori assumption on the behaviour of the SST. Such results have been presented for a stand-alone SST (Diehl 2008). These results have been utilized in the underlying analysis of the simple ASP model in the present paper. The model consists of two ODEs for the biological reactor and two PDEs for the SST, where the concentrations of one
particulate and one soluble component are the unknowns. The mathematical analysis of such a system of ODEs and PDEs is remarkably complex and comprehensive; see Diehl & Farås (2012a, b, submitted). In those mathematical papers, steady-state solutions of the ASP are found and conditions for their uniqueness given, they are classified by means of the input variables, their stability is investigated, and their dependence on the volumetric flow control parameters (the recycle and waste ratios) is given.

The aim of the present paper is to present some implications of more practical importance regarding the fundamental nonlinearities of the reactor-settler interaction. A natural control objective is to keep the effluent concentration of substrate below a reference value subject to keeping the SST in normal, or optimal, operation with a sludge blanket level within the thickening zone. We focus on steady-state situations and will provide a tool for making strategies on how to adjust the control parameters as the influent volumetric flow or concentrations are increased or decreased.

A REDUCED-ORDER ASP MODEL

An idealized ASP is shown in Figure 1. The soluble substrate is represented by the $S$-variables in the figure and the particulate component represented by the $X$-variables. Input variables are $S_{in}$, $X_{in}$ and $Q$, and the control parameters are the recycle ratio $r$ and the waste ratio $w$. All variables depend on time. Within the SST, the concentrations depend also on the depth $z$: $S = S(z, t)$ and $X = X(z, t)$. We make two constitutive assumptions. The dissolved oxygen concentration in the reactor is assumed to be sufficiently high so that the growth rate of biomass depends only on the available substrate in the reactor; $\mu = \mu(S_f)$. This function is normally given by a Monod relation and we choose the batch-settling flux function for the hindered settling as the following:

$$\mu(S) = \frac{\mu_{max}}{K + S_f} , \quad f_b(X) = v_0(e^{-nX} - e^{-nX_{max}})X.$$

Note that the analysis is not restricted to these specific choices. For numerical calculations we let $\mu_{max} = 0.2$ h$^{-1}$, $K = 0.05$ kg/m$^3$, $v_0 = 10$ m/h, $n = 0.35$ m$^3$/kg and $X_{max} = 15$ kg/m$^3$. The height, depth and cross-sectional area of the SST are $H = 1$ m, $B = 3$ m and $A = 1,500$ m$^2$, respectively. The volume of the reactor is $V = 3,000$ m$^3$.

The conservation of mass yields the model equations (where $S = S(z, t)$ and $X = X(z, t)$):

$$V \frac{dS_f}{dt} = QS_{in} + rQS_u - (1 + r)QS_f - V \frac{\mu(S_f)}{Y} X_f$$

(substrate in reactor)

(1)

$$V \frac{dX_f}{dt} = QX_{in} + rQX_u - (1 + r)QX_f + V(\mu(S_f) - b)X_f$$

(biomass in reactor)

(2)

Figure 1 | An idealized activated sludge process (ASP) with a completely mixed biological reactor and an SST. The indices stand for $f$ = feed, $e$ = effluent, $u$ = underflow, $r$ = recycle and $w$ = waste.
\[
A \frac{\partial S}{\partial t} + A \frac{\partial}{\partial z} \left[ F^S (S, z, r, w, Q) \right] = (1 + r)QS_0 \delta(z) \\
(\text{substrate in SST})
\]

\[
A \frac{\partial X}{\partial t} + A \frac{\partial}{\partial z} \left[ F^X (X, z, r, w, Q) \right] = (1 + r)QX_0 \delta(z) \\
(\text{biomass in SST})
\]

For numerical calculations and plots, we have in (1) chosen the yield factor \( Y = 0.7 \) and in (2) the death rate \( b = 0.01 \) h\(^{-1}\). In the PDEs (3)–(4), the source terms with the delta function \( \delta(z) \) model the SST feed inlet, and the flux functions are:

\[
F^S (S, z, r, w, Q) = \begin{cases} 
- (1 - w)QS/A & \text{for } z < 0 \\
(1 + w)QS/A & \text{for } z > 0
\end{cases}
\]

\[
F^X (X, z, r, w, Q) = \begin{cases} 
- (1 - w)QX/A & \text{for } z < -H \\
A f_b (X) - (1 - w)QX/A & \text{for } -H < z < 0 \\
f_b (X) + (1 + w)QX/A & \text{for } 0 < z < B \\
(1 + w)QX/A & \text{for } z > B
\end{cases}
\]

In accordance with \( F^X \), the z-axis is divided into four zones; see Figure 1. Equations (1)–(4) need initial data at \( t = 0 \) for their solution and the PDEs (3)–(4) need a so-called entropy condition (Diehl 1996). Note that Equations (3)–(4) are defined for \( - \infty < z < \infty \) and boundary concentrations must not be imposed as such are outputs obtained from the solution; the effluent and underflow concentrations are the values of the solutions \( S \) and \( X \) in \( z < -H \) and \( z > B \), respectively.

**THE STEADY STATES OF THE ASP**

Assume now that the input and control variables \( S_{in}, X_{in}, Q, r \) and \( w \) are constant in time. Equations for stationary solutions are easily obtained for the ODEs (1)–(2); simply set the time derivatives to zero. For the PDEs, however, the situation is far more complicated. Equation (5) yields the natural fact that every stationary solution has a constant substrate concentration in the whole ASP; \( S = S_t = S_u = S_e \). With this fact, the steady-state equations for the ASP model can be written as:

\[
Q(S_{in} - S) - V \frac{\mu (S)}{Y} X_t = 0
\]

\[
QX_{in} + rQX_u - (1 + r)QX_t + V (\mu (S) - b) X_t = 0
\]

\[
(1 + r)QX_t = (1 - w)QX_e + (r + w)QX_u.
\]

In (7), the limiting flux function is given by:

\[
\Phi_{lim}(X_t, (r + w)Q) = \min \left( A f_b (X) + (r + w)QX \right).
\]

For a given feed concentration \( X_t \), the limiting flux \( \Phi_{lim} \) is the largest flux that can be conveyed through the thickening zone (in steady state); see Diehl (2008). If the feed flux \((1 + r)QX_t \) equals the limiting flux, then the SST is said to be critically loaded. The four Equations (5)–(8) have the four unknown variables \( S, X_t, X_e \) and \( X_u \). It turns out that there are two types of stationary solution with respect to the reactor concentrations:

- The normal steady-state solution satisfies \( 0 < S < S_{in}, 0 < X_t \leq X_{max} \).
- The wash-out steady-state solution satisfies \( S = S_{in} \) and \( X_t = X_e = X_u = 0 \), i.e. the biomass has been washed out. This also models the situation in a new plant before biomass has been added.

Whether the normal steady-state solution implies an under-, over- or critically loaded SST depends on whether the feed point \((X_t, (1 + r)QX_t)\) lies below, above or on the graph of the limiting flux function \( \Phi_{lim}((r + w)Q) \); see the operating chart in Diehl (2008, Figure 2).

It is of practical importance to know whether a steady-state solution of a system of equations is stable or not, i.e. what happens if there is a disturbance in the reactor concentration \( S \) or \( X_t \). A comprehensive analysis (Diehl & Farás 2012a) shows that when a normal solution exists, it is always stable. In order to give a condition when a normal solution exists in terms of the input variables \( S_{in}, X_{in} \) and \( Q \), and the control parameters \( r \) and \( w \), we define the following time variables:

\[
\tau_X = \frac{V(r + w)}{Q w(1 + r)} \quad \text{and} \quad \theta_X = \frac{V X_t}{(1 - w)QX_e + wQX_u}.
\]
The variable \( \theta_X \) is the sludge age of the ASP and it depends on some solution variables \( (X_f, X_e, X_u) \), in contrast to \( \tau_X \). These two variables coincide when the SST is not overloaded. This can be seen by setting \( X_e = 0 \) and using (8) to conclude that \( \theta_X = \tau_X \) holds. If \( X_{in} > 0 \), a normal solution always exists and no wash-out solution is possible. In the more practically interesting case \( X_{in} = 0 \), there always exists a wash-out solution; however, a normal solution exists in addition if and only if the input and control variables satisfy the following inequality (see Figure 2):

\[
(\mu(S_{in}) - b)\tau_X > 1 \iff w < \frac{V(\mu(S_{in}) - b)\tau_X}{Q(1 + r) - V(\mu(S_{in}) - b)}. \tag{10}
\]

A real ASP does not work properly if the sludge age is too low. This is in accordance with the present model. Assume that the SST is in a normal steady state and not overloaded. Then \( X_e = 0 \) holds and, as concluded above, \( \theta_X = \tau_X \) holds. From the formula for \( \theta_X \), we get an equality without \( r \): \( \tau_X w Q X_u = V X_f \). Using this to substitute \( X_u \) in Equation (6) one arrives at:

\[
Q X_{in} + V \left( \mu(S) - b - \frac{1}{\tau_X} \right) X_f = 0. \tag{11}
\]

For the most interesting case when \( X_{in} = 0 \) holds, we thus have (compare with (10)):

\[
\mu(S) = b + \frac{1}{\tau_X} \iff S = \mu^{-1} \left( b + \frac{1}{\tau_X} \right). \tag{12}
\]

As \( \mu \) is increasing this means that \( S \) is a decreasing function of \( \tau_X \); see its graph in Figure 3.

A TOOL FOR CONTROL STRATEGIES

We now focus on a normal steady-state solution of Equations (5)–(8). To guarantee that the SST is critically loaded and a sludge blanket in the thickening zone is possible, we add the following equation, which states that the feed flux to the SST equals the limiting flux of the thickening zone:

\[
(1 + r)Q X_f = \Phi_{lim}(X_f, (r + w)Q). \tag{13}
\]

Letting \( r \) and \( w \) be variables, we have in total five equations and six variables: \( S, X_f, X_e, X_u, w \) and \( r \). Hence, there is generally a one-parameter solution. For a normal steady state, an extensive analysis yields that \( r \) can be used as the parameter. Of particular interest is the

Figure 2 | As \( X_{in} = 0 \), a normal solution exists only for \((r, w)\) below the curves, which are obtained for \( Q = 1,000 \) m³/h (left) and \( Q = 600 \) m³/h (right). The ‘sludge age’ is \( \tau_X = 6 \) h along both curves. In real ASPs, the point \((r, w)\) lies mostly below the curves where \( \tau_X > 6 \) h (cf. Figure 3).

Figure 3 | The substrate concentration, which is the same in the entire ASP in steady state, is shown as a function of the sludge age for a critically or underloaded SST; \( X_{in} = 0 \) kg/m³ and \( Q = 1,000 \) m³/h. Note that \( \tau_X > 6 \) corresponds to the lower region of Figure 2 (left) and that the boundary case \( \tau_X = 6 \) h and \( S = S_{in} = 0.4 \) kg/m³ occurs on the curve in Figure 2 (left).
dependence of $S$ on $r$ and $w$; see the control curve in Figure 4. The equations for the calculations are given in the Appendix.

As Figure 4 (left) shows, for fixed inputs $S_{in}$, $X_{in}$ and $Q$, the concentration $S$ is decreasing with $r$ along the curve (the corresponding values of $w$ are given by the right curve). For small values of $r$, $S$ decreases fast as $r$ increases, whereas for large $r$, $S$ is hardly reduced. As large values of $r$ mean a high cost of energy for pumping the suspension within the plant, an optimum value balancing energy consumption and waste discharge can easily be found.

The control curve depends on the input variables $S_{in}$, $X_{in}$ and $Q$; see Figure 5 (for simplicity, we consider here only the case $X_{in} = 0$). The control curve moves monotonically upwards in the $rw$-plane as $S_{in}$ is increased, however, its dependence on $Q$ is slightly more complex. With the aim of such curves manual control strategies can be found. For example, given a steady state with a critically loaded settler; if $S_{in}$ is increased, then the SST will be overloaded in its new steady state unless a control action is made; e.g., for fixed $r$, increase $w$ until $(r, w)$ lies on the new curve. If, in addition, a reference value of $S$ should be maintained, one has to adjust both $r$ and $w$ as is done in the example in the next section.

A DYNAMIC SIMULATION WITH AN EXAMPLE OF A CONTROL ACTION

The numerical scheme by Bürger et al. (2005) has been used for the dynamic simulation of Equations (1)–(4) shown in Figure 6. The simulation starts in steady state with a critically loaded SST and a sludge blanket in the thickening zone. The values of the control parameters and the substrate concentration are such that the point $(r, w, S) = (0.5, 0.0084, 5.02 \text{g/m}^3)$ lies on the control curve in Figure 4 and $X_{in} = 0 \text{kg/m}^3$ through the whole simulation. At $t = 5 \text{h}$, $Q$ is increased from 1,000 to 2,000 m$^3$/h and $S_{in}$ is decreased from 0.4 to 0.2 kg/m$^3$. Thus,
the mass flux into the plant \( QS_{in} = 400 \text{ kg/h} \) is unchanged. The large step increase in \( Q \) implies that the underflow concentration \( X_u \) makes a jump downwards immediately. Because of the reactor-settler interaction, interesting nonlinear dynamic behaviours of the reactor concentrations \( X_f \) and \( S_f \) occur during the first day. The sludge blanket rises initially and then decreases slowly. It would decrease until it reached the bottom unless a control action is performed. The step changes in \( Q \) and \( S_{in} \) at \( t = 5 \text{ h} \) imply that the control curve in Figure 4 changes to the one in Figure 7. At \( t = 5 \text{ days} \), a control action is made in which new values of \( r \) and \( w \) are computed with the initial substrate concentration \( S = 5.02 \text{ g/m}^3 \) as a reference value. This results in the point \((r, w, S) = (0.61, 0.0047, 5.02 \text{ g/m}^3)\) on the new control curve in Figure 7. As is shown by the simulation, the sludge blanket is stabilized and a new steady state is reached with the initial substrate concentration regained.

**CONCLUSIONS**

Previous publications on strategies for control of the ASP have included assumptions that the SST will behave well irrespective of variations in the inputs and control variables. This is unsatisfactory. By utilizing results for nonlinear PDEs, in particular Equations (7) and (13), it is possible to obtain general results for the steady-state solutions of a reduced-order model of an ASP. The comprehensive mathematical analysis and proofs are not presented here, instead we show some practical implications of the more theoretical results. Inequality (10) involves the input variables and the sludge-age variable \( \tau_X \). If (10) is not satisfied (normally for \( \tau_X \) too small), then only the wash-out solution with no biomass is possible. For larger \( \tau_X \), a normal solution, which is stable under disturbances in the reactor, also exists and Figure 3 shows how the steady-state (effluent) substrate concentration depends on \( \tau_X \) in the case of a critically or underloaded SST (in the interesting case \( X_{in} = 0 \)). We have presented how strategies can be formulated for the manual control to obtain a critically loaded SST with a sludge blanket in the thickening zone in steady state. This is done with the aim of the control curve, which is a unique relationship between the three variables \((r, w, S)\); see Figures 4 (left) and 7 (left). Given a reference value on the effluent substrate concentration \( S \), the point \((r, w)\) is uniquely determined. Each set of input variables yields a
new control curve. This can be used to determine how the control parameters \( r \) and \( w \) should be adjusted when a change in an input variable has occurred (rather, some suitable time average for periodically varying inputs with not too large amplitudes). It is a research challenge to obtain similar fundamental relationships for a model including more material components and a more advanced SST model, which takes compression and dispersion into account.

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APPENDIX

Given values of the input variables $S_{in}$, $X_{in}$ and $Q$, the equations for the calculation of a control curve are given here. The equations for obtaining a critically loaded SST in steady state are (5)–(8) and (13). Diehl & Farås (2012b) have proved that $r$ can be used as a parameter so that the other variables can be expressed as a function of $r$. This is particularly convenient, as $r$ happens to be the main control variable. The problem is, however, that the other variables cannot be expressed as explicit functions of $r$, they can only be given implicitly in the following way. For a normal solution, Equation (5) yields a unique correspondence between $X_f$ and $S$:

$$X_f = f_1(S) \text{ where } f_1(S) = \frac{QY(S_{in} - S)}{V \mu(S)}.$$

Define the following function:

$$f_2(S) = \left( \mu(S) \left( 1 + \frac{X_{in}}{Y(S_{in} - S)} \right) \right) \frac{V}{Q}.$$

Then Equations (5)–(7) and (13) can be used to express $w$ as a function of $S$ and $r$ in the following way:

$$w = f_3(S, r) \text{ where } f_3(S, r) = \frac{f_2(S)r}{1 - f_2(S) + r}.$$  \hspace{1cm} (14)

Equation (13) can now be written in terms of $S$ and $r$ only:

$$Qf_1(S) = \Phi_{lim}(f_1(S), (r + f_3(S, r))Q).$$  \hspace{1cm} (15)

For each given number $r \geq 0$, Equation (15) can be solved for a (unique) value $S$ with some root finding algorithm. These values of $S$ and $r$ are plugged into (14) to obtain the corresponding value of $w$. Then the triple $(r, w, S)$ is a point on the control curve in three dimensions. Doing this for a vector of $r$-values, the control curve is obtained as in Figures 4 (left) or 7 (left). The projection of the control curve onto the $rw$-plane is simply obtained by plotting only the pairs $(r, w)$ as in Figure 4 (right), 5 or 7 (right).