The orbital evolution of P/Machholz 2 and its debris

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ABSTRACT
We investigate the orbital evolution of the observed components of fragmented comet P/1994 P1 (Machholz 2), and of plausible debris orbits. The cometary orbit is currently quite stable because of its proximity to the 9:4 jovian mean motion resonance and its avoidance of Jupiter. Departures from this stable orbit could occur due to close approaches to one of the terrestrial planets, in particular the Earth. The observed influence of non-gravitational forces upon the brightest component precludes any definitive knowledge of when the comet fragmented, but the evidence suggests that this was within the last two decades, and possibly very soon before its discovery. The most recent intersections between the cometary and terrestrial orbits occurred in two epochs in the 18th and 19th centuries, and we calculate theoretical radiants for the meteor showers which might have been observed then, although for various reasons such observations are unlikely even if P/Machholz 2 had an associated stream at that stage. The next epochs in which such intersections occur are over a millennium away, although some meteoroids ejected with high speeds could meet the Earth within a few centuries. No meteor storms are to be expected. The comet is found to be in an orbit which brings it close to being an octuple-crosser of the Earth, such that differential orbital evolution of the meteoroids could result in eight distinct showers.


1 INTRODUCTION
Intense meteor showers, or meteor storms, are the result of the Earth passing through the densest part of a meteoroid stream released by a comet (see Kresák 1993 for a general discussion). When a comet on an Earth-crossing orbit splits apart, it is to be expected that the many meteoroids released so as to follow broadly the same orbit as that of the comet will produce a meteor storm if their orbital evolution brings their node to a heliocentric distance of \( \sim 1 \) au before they have had time to disperse widely under the effects of differential planetary perturbations, radiative effects, and similar agencies. The last time that an Earth-crossing, short-period comet was observed to fragment in this way was the first half of the 19th century, when P/Biela (now designated 3D/Biela under the new rules of nomenclature; Marsden & Williams 1995) broke into several distinct fragments, eventually producing the prominent Andromedid meteor storms of 1872 and 1885, and a distinct shower in 1892 (see Sekanina 1982, Babadzhanov et al. 1991 and Kresák 1993 for details and pertinent references).

The discovery in 1994 of P/Machholz 2 immediately raised the possibility that this object might spawn meteor showers on the Earth, since almost all other Earth-crossing, short-period comets produce such phenomena. Shortly after discovery it was realized that this comet has a number of co-moving fragments, and therefore it has recently (within years or decades) split, raising the question of whether meteor storms might be anticipated. Any concentration of meteoroids that is recently released (and so still in a reasonably coherent and concentrated grouping), and on an Earth-crossing orbit, may produce meteor storms, which could be frequent if the parent body is on a short-period orbit and thus spending much time in the inner Solar system. The question is whether the Earth is due to pass...
through that concentration of meteoroids in the foreseeable future. Answering that question is the main aim of the present paper.

For reference purposes, it is useful to set down the places where the announcements of the comet’s discovery and nature were made. Its discovery on 1994 August 13 by D. E. Machholz was announced on IAU Circ. 6053, and it was given the preliminary designation 19940; under the new nomenclature rules it is known as P/1994 P1. Subsequent astrometric observations showed it to have a short orbital period (IAU Circ. 6059), soon determined as about 5.2 yr. A nearby, co-moving cometary body, clearly a fragment of the primary object, was discovered on 1994 August 28 (IAU Circ. 6066), and by September 4, third, fourth and fifth components had also been found (IAU Circ 6070, 6071). These components are labelled, in order of discovery, A, D, C, B and E; A appears to be furthest ahead in the orbit, followed by B, C, D and E. The original component is now designated P/1994 P1-A, the other fragments being denoted similarly. Those five seem to fall into two main groups, B being close to A, C and E near D; A and D were the brightest fragments. In 1994 October observations suggested that fragment D was itself made up of at least two distinct components, and the coma produced by D was starting to obscure fragment C, a trail of material also extending beyond fragment E (IAU Circ. 6090). It is clear that the individually identified components are just the brighter fragments among the wider spread of debris produced by the split comet.

By 1994 December, P/Machholz 2 had become substantially fainter, due in part to the decrease in volatile loss as the heliocentric distance increased, and since then it has been beyond the grasp of small telescopes. We obtained images of fragment A at the end of 1995 March with the 3.9-m Anglo-Australian Telescope (AAT), but failed to identify fragment D in the appropriate field although it would easily have been detectable if obeying the conventional formula for cometary magnitudes. Appreciable brightness variations due to outbursts or splittings are not surprising, and our non-detection of fragment D with the AAT is evidence for such processes occurring. For example, fragment D could be intrinsically faint, undergoing substantial cometary activity making it bright in 1994, or perhaps it further disintegrated by 1995 March rendering it unobservable. Our astrometric observations of P/1994 P1-A (Minor Planet Circ. 25097) are used later in the discussion of the cometary orbit.

The splitting of a comet is not an unusual event; 21 cases were known at the time of Sekanina’s (1982) review. More recently, Chen & Jewitt (1994) have analysed images of comets, searching for secondary objects within the coma of the primary components, and confirmed that cometary splitting is a common phenomenon. The frequent occurrence of splitting implies that this is an important factor to be considered in discussions of both the physical nature of comets, and the observed distribution of cometary orbits (e.g. Pittich & Rickman 1994). In view of this, it is to be expected that a major proportion of the interplanetary complex of small bodies is actually made up of the disintegration products of larger cometary objects, a view supported by the observation and interpretation by Rabinowitz (1993) of an extremely substantial population of near-Earth 10–50 m bodies, and the nature of the extraterrestrial influx to Earth has to be understood in this context.

Sekanina (1982) discussed several mechanisms for splitting, including break-up due to the tidal force of the Sun or Jupiter. The best-known case in recent times has been D/1993 F2 (Shoemaker–Levy 9), which collided with Jupiter in 1994 July, the splitting in that case being due to the tidal stress as the comet passed perijove in its jovian orbit. A near passage by Jupiter was also responsible for the splitting of 16P/Brooks 2 in 1889, that comet having been on a heliocentric orbit. Some different mechanism seems to be required for 3D/Biela and the many other comets that have split whilst far from any massive body, precluding any tidal stresses. Fragmentations that are spontaneous (i.e., whose cause is not observed) could be due to rotational instabilities or internal pressure build-ups, and perhaps initiated by meteoroid impacts; the latter was identified as a possibility for the case of 3D/Biela (Babadzhanov et al. 1991). Even since the discovery of P/Machholz 2, there has been another example of a comet splitting far from any planet: 51P/Harrington (IAU Circ. 6089). This comet has q > 1 au, and so is not an Earth-crosser. Another example of an Earth-crossing split comet is that which generated the Kreutz group of Sun­grazers (Marsden 1989). These comets, however, are of long period, and they do not have a node near 1 au. No associated meteors have been identified. As the Kreutz group comets have very small perihelion distances (q < 0.01 au), the splitting in that case could be due to thermal stresses, or tidal disruption within the Sun’s Roche limit. Returning to P/Machholz 2, it is pertinent to ask whether it has undergone a recent near-passage by a planet, thus suggesting that a tidal interaction was responsible for its splitting (as with 16P/Brooks 2), and we discuss this in Section 2.

In this paper we also investigate, using numerical integrations of hypothetical objects with orbits similar to P/Machholz 2, the general dynamical evolution of this parent object (Section 3) and daughter particles assumed to have been released on to free orbits as a result of its recent splitting. Our main interest is an investigation of whether meteoroids so released might be expected to collide with the Earth in the near future, thus producing meteor storms (Section 4). The interval of interest in this respect is only the next few centuries, but we continue our forward integrations over two millennia so as to explore the general pattern of evolution. It is also feasible that P/Machholz 2 has spawned meteoroids in the past, perhaps producing meteor events that might be recorded in historical chronicles; in view of that, we also perform a backwards integration over the past millennium. We have given some limited results from this investigation elsewhere (Asher & Steel 1996).

2 THE OBSERVED FRAGMENTS OF P/MACHHOLZ 2

In this section we consider two facets of the very recent orbital evolution of P/Machholz 2 by addressing two questions.

(i) Are the orbits of any two components known accurately enough so that we can calculate when they separated from each other?

(ii) Are any of the orbits known with sufficient precision
that recent close approaches to any of the planets can be determined with confidence?

Question (i) is of importance in trying to develop an understanding of how cometary fragments move apart after splitting, whilst question (ii) is pertinent in that if there was an extremely close approach to any planet in the recent past, within the Roche limit, that would be suggestive of the recent close approaches to any of the planets can be determined with confidence.

2.1 When did the main splitting occur?

Only fragments A, B and D have been sufficiently well observed to allow individual orbits to be determined (Minor Planet Circ. 24216, orbits by Marsden). The observational arcs of C and E were too short for adequate orbit determinations, although elements could be estimated, for example, by taking the elements of D and adjusting the perihelion passage time \( T \) (recall that components C, D and E form one distinct group); however, such a step would not render orbits for C and E which would be useful in the present context. Using our AAT astrometric measurements, B. G. Marsden (personal communication) and S. Nakano (personal communication to Marsden) were able to calculate new orbits for 1994 P1-A. For each orbit, Table 1 gives the dates of the ends of the arc from which the orbit was determined, the semimajor axis \( a \), eccentricity \( e \), inclination \( i \), longitude of ascending node \( \Omega \), argument of perihelion \( \omega \), and mean anomaly \( M \) at epoch. The orbital period is \( P \approx 5.2 \) yr, and the perihelion distance \( q \approx 0.75 \) au.

The 2–3 month arcs available at the time (1994 November) of MPC 24216 could be fitted by gravitational orbits to within the expected astrometric uncertainties for the three fragments in Table 1. However, the AAT measurements of fragment A were inconsistent with the earlier gravitational orbit by at least 2 arcsec. While such errors sometimes occur in comet and asteroid astrometry, our judgement based on observational experience – in particular, the experience of R. H. McNaught – is that in this case the positions were almost certainly of subarcsecond accuracy. We note in particular that positions consistent with each other were obtained on successive nights. These observations are therefore clear evidence for the action of non-gravitational forces although, with a gap in observations of nearly 4 months, non-gravitational parameters cannot be uniquely determined. In Table 1, the orbit due to Nakano includes a realistic possibility for these non-gravitational parameters; the second orbit due to Marsden considers the post-perihelion arc only, on the presumption that the brightening at discovery would have been caused by break-up before then (and thus the post-perihelion arc is maybe less unrealistically described by a gravitational orbit); and the first orbit from Marsden is a best-fitting purely gravitational orbit to the full observed arc, retaining the 2-arcsec residuals for the March AAT data.

Tracing fragments of a single comet back to see when the orbits meet at a common point in space is notoriously difficult (Sekanina 1982). However, a simple consideration of the rate of change of \( M \), for a given difference in \( a \) between two fragments of a split comet, may sometimes reveal when the pieces were coincident. We will now try such an analysis and show that inconsistent results are derived. We find from a formal error analysis along the lines described by Muinonen & Bowell (1993) that the likely errors in \( a \), based on the arcs until 941109 and assuming purely gravitational orbits, are of order 0.0001 au. Thus the difference \( [a(A) - a(D)] \) between the semimajor axes of fragments A and D, for the orbits from MPC 24216 (see Table 1), is \(-0.001\) 05 au, and is uncertain by about \( \pm 0.0002 \) au. Similarly, \([M(A) - M(D)] = 0.074 \pm 0.001\). Performing calculations involving velocity separations in random directions at various points in the orbit, it is straightforward to show that the separation of A and D is very likely to have occurred at a heliocentric distance \( r < 3 \) au, else differences induced in other elements would have been substantially larger than shown in Table 1, for the observed \([a(A) - a(D)]\). On the other hand, we find that the instantaneous difference in \( M \) produced impulsively is at most \( 0.02 \) if one assumes a fragmentation event at \( r < 3 \) au. As the observed value of \([a(A) - a(D)]\) implies a spreading in \( M \) at a rate of \( 0.19 \) per revolution, this suggests a fragmentation at a time \(~(0.074/0.19)\approx 0.4\ of a revolution prior to the 1994 perihelion passage, and this means at \( r > 3 \) au. Thus there is an inconsistency, and this cannot be accommodated by differential gravitational perturbations over the past orbit.

The explanation for the apparent inconsistency may be deduced from the other orbits in Table 1. The various orbits for 1994 P1-A show that \( a \) (either the osculating value, or the average value that would have led to the separation in \( M \)) is not known as accurately as described above (error analysis of observations assuming a purely gravitational


<table>
<thead>
<tr>
<th>Fragment (Source)</th>
<th>Arc</th>
<th>( a ) (AU)</th>
<th>( e )</th>
<th>( i ) (°)</th>
<th>( \Omega ) (°)</th>
<th>( \omega ) (°)</th>
<th>( M ) (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A (MPC 24216)</td>
<td>940815-941109</td>
<td>3.0119436</td>
<td>0.7501454</td>
<td>12.78662</td>
<td>246.17716</td>
<td>149.26231</td>
<td>357.39719</td>
</tr>
<tr>
<td>B (MPC 24216)</td>
<td>940830-941109</td>
<td>3.0115250</td>
<td>0.7501079</td>
<td>12.78609</td>
<td>246.17341</td>
<td>149.25897</td>
<td>357.38488</td>
</tr>
<tr>
<td>D (MPC 24216)</td>
<td>940830-941109</td>
<td>3.0129903</td>
<td>0.7502288</td>
<td>12.78849</td>
<td>246.18297</td>
<td>149.25125</td>
<td>357.32321</td>
</tr>
<tr>
<td>A (Marsden)</td>
<td>940817-950330</td>
<td>3.0131876</td>
<td>0.7502496</td>
<td>12.78720</td>
<td>246.18154</td>
<td>149.25784</td>
<td>357.39914</td>
</tr>
<tr>
<td>A (Marsden)</td>
<td>940923-950330</td>
<td>3.0141041</td>
<td>0.7503237</td>
<td>12.78729</td>
<td>246.18298</td>
<td>149.25996</td>
<td>357.40012</td>
</tr>
<tr>
<td>A (Nakano)</td>
<td>940815-950330</td>
<td>3.0129586</td>
<td>0.7502309</td>
<td>12.78889</td>
<td>246.18017</td>
<td>149.25999</td>
<td>357.39901</td>
</tr>
</tbody>
</table>
orbit), owing to the existence of non-gravitational forces. It is very difficult to estimate the non-gravitational effect during the observed arc, let alone over the longer, unobserved time-span during which the separation increased. Indeed, the separation of cometary splitting fragments may be better considered in the context of differential radial accelerations rather than, as assumed in the last paragraph, velocities induced impulsively. Sekanina (1977) showed that the sky separations of components of several split comets were well modelled by considering just a separation time and a parameter to quantify the differential radial acceleration resulting from the cometary jet forces.

On the above basis it is therefore possible to make only a broad statement that the comet seems to have split at some point during the past few decades. We do have the following observational constraints (Marsden, personal communication): (a) the comet would have been easily observable three apparitions ago if it had been as bright as in the discovery apparition, so that is unlikely to have been when splitting occurred (i.e., the splitting occurred within the last ∼ 16 yr); (b) it would have been undetectable at the 1989 and 1984 returns, being very near the Sun in the sky; and (c) there was probably an outburst shortly before discovery, otherwise the comet would likely have been discovered earlier, in 1994 June or July – this favours a very recent splitting, especially in view of the decrease in brightness of fragment D by 1995 March. We are unable to deduce exactly when the splitting occurred, but the evidence indicates that this has been at some time within the past decade or so (with a brightening soon after the middle of 1994 due perhaps to some subsidiary fragmentation event), and possibly within the month prior to discovery.

2.2 Has the comet had any recent close planetary approaches?

The next question to be addressed is whether P/Machholz 2 might have had a close approach to any of the planets within the past few centuries (allowing some leeway beyond the ‘past decade or so’ arrived at above). Here we have numerically integrated forward over 100 yr and back over 400 yr the system of the Sun, the eight main planets and each of the orbits in Table 1. Non-gravitational forces were neglected. The integrator (Dormand & Prince 1978) uses a variable step size, which for the most part was about 1–2 d. No approach within 0.05 au of any planet was identified, although we found that exact predictions are impossible even quite near the present. For example, 1994 P1-A will come reasonably close to the Earth in A.D. 2036, but the miss distance derived varies between 0.12 and 0.16 au, depending on which orbit from Table 1 is used; this compares with a closest approach of ∼ 0.32 au during the discovery apparition. A closest approach to within possibly 0.11 au of Venus in 2089 was identified. Regarding possible meteor storms on the Earth, a matter which is discussed in more detail in Section 4, these are not possible for the next few centuries, as even the ellipses themselves (i.e., neglecting the position of each object in its orbit) of the Earth and the comet do not reach a configuration of near-intersection in this time-frame. In such epochs as the comet does have a node near one of the planetary orbits, very close approaches to the Earth or Mars are conceivable, but there is no specific evidence from our integrations for such near misses.

Approaches to Jupiter need not be as close as to other planets to have a comparable effect, since its large mass results in its Roche radius being substantial. There are, however, no passages with 0.8 au of Jupiter predicted by our integrations, in the time-frames considered. The two ellipses (comet and Jupiter) approach somewhat closer than this, but a stable mean motion resonance (Section 3) results in the comet avoiding the planet itself.

Our integrations, therefore, show that there is no evidence for tidal disruption of P/Machholz 2 within the past few centuries because of a transit through any planet’s Roche limit, excluding this as a probable cause of break-up. On the other hand, the lack of any close approaches to the planets within the next few centuries means that we expect the orbital evolution of the large observed fragments to be well behaved.

3 GENERAL DYNAMICAL BEHAVIOUR OF THE COMET

As in the computations described in Section 2, the Sun and eight major planets were included in the integrations described here, which involve the main fragment (P/1994 P1-A), that fragment being taken to be diagnostic of the general dynamical behaviour of the comet. We note, however, that the orbital uncertainty (Table 1) means that the choice of precise starting elements is to some extent arbitrary. The same consideration, and the fact that (a) one cannot know the exact circumstances of past and future planetary encounters, and (b) one cannot know the precise past and future perturbations due to non-gravitational forces, mean that there is no point in integrating the planetary orbits as well as that of the comet. Instead, we used precalculated planetary elements, updated every 500 yr, from Quinn, Tremaine & Duncan (1991). The effect of this is a saving in computational time of about an order of magnitude. The orbit we used, with no consideration of non-gravitational effects in the integration, is the fourth set of elements in Table 1 (Marsden’s orbit for 141 astrometric positions, 940817–950330).

We show in Fig. 1 the main features of the orbital evolution. One cycle of ω, due to jovian secular perturbations and lasting about 3 kyr, is covered, from 1 kyr in the past to 2 kyr in the future. Variations in e (equivalently q) and i are closely correlated with those in ω. The notable sharp changes occurring a few times a century correspond to times when Jupiter has been near (∼ 1 au away) during an aphelion passage by the comet, since its aphelion (Q = 5.27 au) is very near the jovian orbit; with the current values of the perihelion longitudes of Jupiter and the comet, their orbits do not actually cross.

These sudden changes in a are shown at higher resolution in Fig. 2. These regular variations have a period of around 46–48 yr (i.e., four times Jupiter’s orbital period), and appear to be between extremal values near 3.00 and 3.08 au. We note that mean motion commensurabilities with Jupiter exist at a = 2.957 au (7:3), a = 2.997 au (16:7), a = 3.029 au (9:4), a = 3.054 au (20:9), a = 3.075 au (11:5) and a = 3.106 au (13:6). Here the 9:4 resonance clearly dominates: at one point, the comet is at aphelion and Jupiter is slightly ahead of it, so that the planet accelerates it into an orbit of larger...
Figure 1. Orbital evolution of the primary fragment of P/Machholz 2 backwards through 1 kyr and forwards through 2 kyr. Start elements as for Marsden's purely gravitational solution for observations of P/1994 P1-A from 940817 to 950330, as in Table 1. The orbital elements plotted are the semimajor axis ($a$, au), perihelion distance ($q$, au), eccentricity ($e$), inclination to the ecliptic ($i$, deg), longitude of the ascending node ($\Omega$, deg), and the argument of perihelion ($\omega$, deg).

Figure 2. The plot for $a$ from Fig. 1 over the period from the present to 500 yr in the future, allowing the details of the variation in $a$ to be seen. The effects of Jupiter during single aphelion passages, twice per nine orbits of the comet, are clear (see text). The dashed line corresponds to the 9:4 jovian mean motion resonance near $a = 3.029$ au.

Figure 3. The critical argument $\sigma$ referred to the 9:4 jovian mean motion resonance for five test particles with initial values of $a$ spaced by 0, ± 0.002 and ± 0.004 au from the value used in constructing Fig. 1. All eight major planets were involved in the integration.

Another way to study the stability of the resonant effects is to plot the critical argument $\sigma$, which is defined as

$$\sigma = (p + q)\lambda - p\lambda - q\omega$$

for the $(p + q):p$ resonance, where $p$ and $q$ are integers, $\omega$ is the comet's longitude of perihelion ($\omega = \Omega + \omega$), $\lambda$ is its mean longitude ($\lambda = \sigma + M$), and $\lambda$ is the mean longitude of Jupiter. In Fig. 3 the results are shown for five test particles (initially spaced by 0, ± 0.002 and ± 0.004 au in $a$ about the initial value used in deriving Fig. 1, with $q$ fixed); $\sigma$ librates when a particle is resonant. The action of Jupiter is seen to keep each particle resonant, maintaining the $a$-variations in phase over a substantial timespan, except for one particle for which a random energy perturbation as a result of a close approach to the Earth soon after AD 3100 moved it out of the resonance. The effects of the terrestrial planets can be divined from Fig. 4, which corresponds to Fig. 3 except that only the four largest planets were used in the integrations, the terrestrial planets being excluded. The results display less instability; that is, it is close approaches to the terrestrial planets that cause the comet's orbit to become chaotic, at least in the present few millennia whilst the stability of the 9:4 resonance is precluding any approach to Jupiter.
Figure 4. As Fig. 3, except that the terrestrial planets were excluded from the integration. There is less instability in these plots, showing that it is the close approaches to the terrestrial planets that causes any irregularities/chaos in the cometary motion, at least for this idealized particle (non-gravitational forces assumed zero).

4 METEOR STORMS

While the resonant effects are notable, especially in helping to maintain stability for some millennia when Jupiter-approaching orbits might otherwise be substantially perturbed, our first concern was to investigate the effects of precession over time-scales of centuries, in particular to see when meteor production is possible. This orbital behaviour can usually be understood by considering variations in the argument of perihelion $\omega$. For fixed $a$ and $e$, there are four values of $\omega$ permitting Earth-intersection, given to a good approximation – that is, assuming a circular orbit for the Earth, of radius 1 au – by

$$\pm \cos \omega = [a(1-e^2) - 1]/e.$$  

The four values correspond to the northern and southern branches of meteor showers before and after perihelion passage. In Fig. 5, two of these four intersections occur in quick succession, in the 18th and 19th centuries, with the other two being similarly arranged, but in the 32nd and 33rd centuries. The 18th century intersection is pre-perihelion (nighttime shower) at the ascending node, whilst the 19th century intersection is post-perihelion (daytime shower) at the descending node; the 32nd and 33rd century intersections are the complementary pair: respectively, the pre-perihelion descending node and post-perihelion ascending node intersections.

As seen earlier, in Fig. 1, there are variations in eccentricity $e$ (also in inclination $i$) which are closely correlated with those in $\omega$. For some streams, these $e$-variations are large enough that Earth-intersection occurs at a second value of $|\cos \omega|$, so that during a single cycle of $\omega$, eight shower branches can be produced (Babadzhanov & Obrubov 1987, 1992): four branches have one set of $e$ and $i$ values, and four another set. Any object/stream displaying such dynamical behaviour is termed an **octuple crosser** of the Earth. In the present case, the cycle time of $\omega$ is around 3 kyr. In about AD 2500, we find that $e$ will attain values of 0.70 or less, and as a result Earth-intersection will almost – but not quite – occur for the test object integrated here (Fig. 5). It may be that meteoroids on slightly different orbits from the parent comet could be octuple crossers. Interestingly, the first periodic comet discovered by Donald Machholz, 96P/Machholz 1, is an octuple crosser, associated with eight meteor showers including the Quadrantids and $\delta$ Aquarids (Zausaev & Pushkarev 1989; McIntosh 1990; Babadzhanov & Obrubov 1992; Jones & Jones 1992; Wu & Williams 1992; Williams & Wu 1993). We give an extensive list of references here, because the production of eight meteor showers by one comet during a single precession cycle in $\omega$ – as opposed to multiple sets of four showers produced in different $\omega$-cycles with precession in $m$ and hence a shift in $\Omega$ occurring in between (Babadzhanov & Obrubov 1992) – is an important phenomenon to be noted when investigating links between short-period comets and observed meteor showers, and also when considering the possible showers that might result from P/Machholz 2. For another example of an octuple Earth-crossing meteoroid stream, see Obrubov (1995).

A feature of computational modelling of the evolution of meteoroid streams is that there can be a large number of free parameters (e.g., distributions of times and velocities of meteoroid ejection). The present work is motivated by the fact that P/Machholz 2 was observed to contain several macroscopic components and an apparent debris trail. As we have shown, the timing of the splitting event(s) or the cometary activity that has given rise to this material is not tightly constrained by observations. Here it is assumed that separation happened at perihelion, one revolution ago, and a dynamical study is made of the consequences.

Marsden’s orbit for 1994 P1-A based on the whole arc (Table 1) was taken as the reference orbit. This model is fairly representative of scenarios that could have produced...
the observed material. We had performed this study prior to obtaining the AAT astrometry of P/1994 P1-A in 1995 March, using the MPC orbit published in 1994 November, and we drew identical conclusions to those arrived at here; indeed, this was also the case when we used the orbit from the 1994 October MPCs, in this last case letting particles be ejected three revolutions ago instead of one.

Ejection velocities were used varying only in $a$, keeping $q$ and the angular elements fixed (cf. Steel, Asher & Clube 1991). Experience in integrations shows that this is likely to produce the most noticeable variations in orbital evolution. Fig. 6(a) shows results from integrations where the initial values of $\Delta a$ ranged from $-0.0005$ to $+0.0005$ au, spaced in jumps of $0.0001$ au. For this orbit, it turns out that the ejection speed in km s$^{-1}$ is roughly the same as $\Delta a$ in au; thus in Fig. 6(a) the maximum ejection speed is $\sim 0.5$ m s$^{-1}$.

In Fig. 6(b) the values are 10 times greater (speeds up to $5$ m s$^{-1}$), and 10 times greater still in Fig. 6(c) (speeds up to $50$ m s$^{-1}$). Effective separation speeds for splitting comets are generally believed to be up to a few m s$^{-1}$ (Sekanina 1982). We noted earlier in Section 2 that the difference $\Delta a$ between components A and D is likely of order 0.001 au. Based on standard ideas of ejection from comets, with meteoroids swept out by the expanding gas cloud, smaller meteoroids tend to be ejected at higher speeds. Meteoroids that are large enough to produce meteors easily visible by the naked eye are typically $\sim 1$ cm in size. All the ejection speeds chosen here seem physically reasonable in the context of considering a potential meteor storm, being in accord with Jones's (1995) development of the Whipple (1951) theory. Jones also points out that effective speeds could be higher if meteoroids are released with a significant non-gravitational acceleration, as described by Steel (1994).

The integrations were carried out as described in Section 3 (i.e., planetary orbital elements updated every 500 yr). In Fig. 6 we plot for each particle the distance $D$ between components A and D is likely of order 0.001 au. Based on standard ideas of ejection from comets, with meteoroids swept out by the expanding gas cloud, smaller meteoroids tend to be ejected at higher speeds. Meteoroids that are large enough to produce meteors easily visible by the naked eye are typically $\sim 1$ cm in size. All the ejection speeds chosen here seem physically reasonable in the context of considering a potential meteor storm, being in accord with Jones's (1995) development of the Whipple (1951) theory. Jones also points out that effective speeds could be higher if meteoroids are released with a significant non-gravitational acceleration, as described by Steel (1994).

The integrations were carried out as described in Section 3 (i.e., planetary orbital elements updated every 500 yr). In Fig. 6 we plot for each particle the distance $D$ between its elliptical orbit and that of the Earth, as a function of time. Meteoric phenomena (i.e., detectable meteors in the Earth's atmosphere) can only be produced when $D=0$, and even then most particles will not collide with the Earth. Fig. 6, however, does indicate the epochs in which meteor showers/storms may occur. There are four such epochs in each $\omega$-cycle: in the 18th, 19th, 32nd and 33rd centuries, as seen above in Fig. 5, and confirmed here. The dispersal among the meteoroid orbits is greater (i) at times far from the present (in these backward and forward integrations); and (ii) for the larger ejection speeds (i.e., Fig. 6(b) is more dispersed than Fig. 6(a), and Fig. 6(c) substantially more still). Although in Fig. 6(b) the shower epochs in the 18th and 19th centuries, being close to the origin of the integrations (the present), are distinct events, the 32nd and 33rd century intersections, being more than a millennium into the future, are not distinct. This would mean that there would be both nighttime and daytime showers occurring in the same epoch, one at the ascending node and the other at the descending node. This is a characteristic of streams that are old enough so as to have become well dispersed, not only along the orbit (giving an annual shower rather than a periodic storm), but also in the argument of perihelion $\omega$ through differential perturbations upon meteoroids with differing orbital sizes and eccentricities (Williams 1990; Babadzhanov & Obrubov 1992; Steel 1994). The meteoroids with large ejection speeds (Fig. 6(c)) are so swiftly dispersed by differential perturbations that they begin to intersect the terrestrial orbit as soon as 300–800 yr into the future. This does not correspond to one of the four clear-cut shower branches, but results from the near octuple-crossing nature of P/Machholz 2, as discussed above. There are two shower branches possibly formed around this time (pre- and post-perihelion, both corresponding to the descending node; Fig. 5), but given the dispersion in orbits in Fig. 6(c), and the proximity of $q$ to 1 au, the two branches are not very distinct. Since these intersections occur only for meteoroids ejected at high speeds from the comet and hence widely dispersed, a meteor storm would not be expected.

Another point to note from Figs 6(b) and (c) is the fact that, whilst a few particles deviate from the norm (such as the one in Fig. 6(b) attaining large values of $D$) due to close approaches to the terrestrial planets, the majority remain in broadly similar orbits, with a proclivity to remain in one of the jovian mean motion resonances between $a=2.95$ and $3.11$ au. This argues for the longer-term persistence of the orbit of P/Machholz 2, unless some agency such as strong, non-gravitational forces drives it away from this island of stability.

These results demonstrate that the next few centuries are not an epoch in which to expect detectable meteor activity produced by P/Machholz 2. Looking backwards in time, this comet could have spawned meteoroids producing observable showers within the past few centuries, if the comet has been on the same basic orbit as at present for such a time and our integrations suggest that it has been, owing to the stability of the 9:4 resonance, although we cannot be sure that past activity might not have resulted in a substantial non-gravitational force acting for some time — and it was actively releasing meteoroids over such a period, or at least
within the past millennium or so. That second condition is more dubious, given that the comet has only recently been discovered, but nevertheless it is worthwhile to calculate the radiants from which meteors might have appeared to emanate in the 18th and 19th centuries. The approximate radiants are given in Table 2.

The daytime shower in the last century is unlikely to have been observed because the radiant is so close to midday: a shower with a radiant on the dayside of the Earth, but with transit closer to dawn or dusk, might have been observed, especially since the activity date is near mid-winter in the northern hemisphere (where most potential observers were located). The nighttime shower in the 18th century has a radiant rather deep in the southern sky which, again combined with the date, makes observation from the northern hemisphere unlikely. On the other hand, written records of observations from such a shower might exist from southern hemisphere sources, South Africa being a possibility. Of course, there may not even have been a meteoroid stream derived from P/Machholz 2 in existence at that stage, so the question is moot.

5 CONCLUSIONS

The action of non-gravitational forces on P/Machholz 2 precludes a unique determination of when its fragmentation into at least five macroscopic components plus numerous smaller debris particles occurred. A date within the last ~ 16 yr seems likely, and the separation may have been very recent indeed. Both the discrepancy in the motion of P/1994 P1-A relative to a gravitational orbit, and the disappearance or fading of P/1994 P1-D, are evidence of substantial cometary activity, outbursts or break-up. P/1994 P1 might well have been a quite small comet, but it is an important reminder of the phenomenon of fragmenting comets.

Objects on Jupiter-approaching orbits are unpredictable on fairly short time-scales (Carusi et al. 1985b; Nakamura & Yoshikawa 1991; Pittich & Rickman 1994; Tancredi 1995). However, our integrations identify the 9:4 jovian resonance as currently exerting a remarkable stabilizing influence on P/Machholz 2. If the comet does not disintegrate completely, random gravitational impulses through close approaches to the inner planets (most likely the Earth), or the action of non-gravitational forces, will eventually displace it from that resonance, although possibly merely into another one nearby: the 7:3, 9:4, 11:5, 13:6, 16:7 and 20:9 resonances are all within 0.1 au of the semimajor axis of P/Machholz 2.

We have also calculated when meteor showers derived from P/Machholz 2 may occur. It is possible that meteor activity which could be associated with this comet has been observed in the past few centuries, which would give an important indicator regarding the comet’s history of activity, but we consider it unlikely. Looking into the future, the stream of debris released by the comet is not expected to intersect the Earth for at least several centuries, and the dispersal of meteoroids in space implies that no meteor storms similar to the Andromedids, produced through the break-up of 3D/Biela, are to be expected from P/Machholz 2.

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