RESEARCH ARTICLE | MAY 01 1990

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Comput. Phys. 4, 280–284 (1990)
https://doi.org/10.1063/1.168367

Analysis of high-perveance electron guns

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(Received 5 July 1989; accepted 8 January 1990)

High-power microwave tubes such as traveling wave tubes and klystrons still make use of electron guns as the main source of current. The design of these guns can become complicated, and the use of computer-aided design (CAD) techniques will ease and enhance their design. A CAD system based on a supermicrocomputer with an integrated array processor was used to analyze the electric fields in the electron gun of a traveling wave tube. The analysis of the fields was motivated primarily to assess the sensitivity of the space-charge and current distributions to the focus and anode electrode structures and the applied electric field. The space charge has a great influence on the formation of the electron beam and as such on overall device performance. An iterative procedure based on the boundary-element method (BEM) was used to determine the space-charge distributions and the electrostatic fields. The method and its underlying assumptions will be discussed in detail. Electron trajectories are determined by solving the equation of motion using Hamming's predictor–corrector method, allowing for the effects of space charge. Due to the computational demands of these methods, the inclusion of an array processor in the workstation was essential to accelerate the calculations. The algorithms had to be vectorized to make optimal use of the array processor. The analysis of the electron gun is mainly concerned with shaping the focus and anode electrodes to optimize the electron-beam current entering the interaction area of the tube where amplification takes place. Focusing of the electron beam in the interaction area is taken over by a parallel magnetic field. The optimization of the beam in the electron gun includes finding the correct perveance of the beam, minimizing the interception current to the anode, and finding the correct current distribution of the electron beam where it enters the interaction area. The above CAD analysis techniques allow an accurate design of these parameters, and modifications are easily made.

INTRODUCTION

Traveling wave tubes require well-collimated electron beams for their operation. Electron guns are normally used as the current source which provides a suitably shaped beam of high current to the interaction area of the tube. Figure 1 shows a schematic of the essential parts of a traveling wave tube.

The design of high-perveance electron guns used in traveling wave tubes has been based on the concentric sphere model with the outer sphere as cathode.[1] For the initial design of guns, this model has proven to be very successful. Testing and refining the design can be done by numerical, analogical, or empirical methods.[2]

Most of the existing code developed for ray tracing in electron guns is based on mesh techniques such as finite-difference and finite-element methods.[3–6] The codes using these methods have been developed to be highly efficient and are used routinely in gun design and analysis.

A large amount of progress and ongoing research in the computation of electromagnetic fields and particle trajectories is being made.[7] With the advent of fast microcomputers with array processors, the boundary-element method of field computation has become a practical means for simulating electromagnetic devices with a high degree of accuracy using relatively simple, although computationally demanding, algorithms. In particular, it will be shown how the boundary-element method can be used in the analysis and design of high-perveance electron guns.

The boundary-element method has the advantage that the initialization procedures for the determination of the elements is considerably simpler than those required for finite-element or finite-difference methods. The specification of the nodal points in mesh techniques is usually rather complex, especially if high accuracy is required during ray tracing, where different mesh sizes may be required for different regions. In addition, the memory storage required is also considerably less than in the case of mesh techniques as far fewer boundary elements are required for a given accuracy than finite-element or finite-difference nodes. Various interpolation methods are used to determine electric fields in mesh techniques. With the boundary-element method, the electric field can be calculated directly at any point in the gun.

A computer-aided design (CAD) system initially developed for cylindrically symmetric electric devices[8] has been upgraded to include the effects of space charge. The system runs on a Masscomp MCS-510E eight-slot workstation with the AP-501 integrated AP board. Calculations within the array processor can run at a maximum computational speed of 10 Mflops. As desk top computers become faster, the boundary-element method will become more attractive due to the greatly simplified problem initialization and because the accuracy in ray tracing will be a function of the differential equation solver used and not the mesh size discretization. In effect, for greater accuracy reinitialization of the mesh will be required in mesh techniques, whereas only an improvement in the relative error control of the differential equation solver is required in the boundary-element method.

I. METHOD OF SOLVING POISSON'S EQUATION

Assuming that there are no time-dependent fluctuations in the electron beam of an electron gun, the electrostatic field in the electron gun can be found by solving Poisson's equation:

$$\nabla^2\phi = -\rho/\epsilon_0,$$

where ϕ is the electrostatic potential; ρ is the space-charge density; and ϵ_0 is the free-space permittivity constant. As the system is cylindrically symmetric, the Laplacian is in cylindrical coordinates with no angular dependence.

In order to apply the boundary-element method, the electrode boundary is divided into small, discrete ring elements, and the contained space where the beam is formed is divided into m space annuli.[9] An iterative procedure is employed to determine the correct space-charge distribution for a given geometry and applied potentials.[2] Initially, Laplace's equation is solved for the electrostatic potential assuming no space charge. A charge distribution assumed to be concentrated at the center of each electrode ring element is found by solving the following set of linear equations:

$$GQ = V,$$

where V is a vector with n components defining the potentials on each element. Q is the n component charge vector that has to be determined. G is a $n\times n$ matrix determined by the geometry of the system and with elements

$$G_{ij} = \frac{1}{4\pi\epsilon_0}\int \frac{d\bar{r}_j}{|\bar{r}_i - \bar{r}_j|}.$$

G is a full, symmetric, positive definite matrix, and the Cholesky method is used to solve this system of equations. Using the determined charge distribution, the electrostatic potential and fields can be calculated at any point in the enclosed space. Trajectories can be calculated by solving the equation of motion.

Assuming that the current density is uniform around the trajectory, each trajectory is assumed to represent an amount of current emerging from an annular area on the cathode. The current density is calculated near the cathode surface based on a least-squares method[10] where the potential near the cathode surface has the following form:

$$\phi = As_i^{4/3}(1 + 8s_i/15r_c),$$

where s_i is the distance from the cathode surface and r_c is the radius of curvature of the cathode.

By choosing a number of points near the cathode, a residual term is set up as follows:

$$R = \sum_i \left[As_i^{4/3}\left(1 + \frac{8s_i}{15r_c}\right) - \phi_i\right]^2.$$

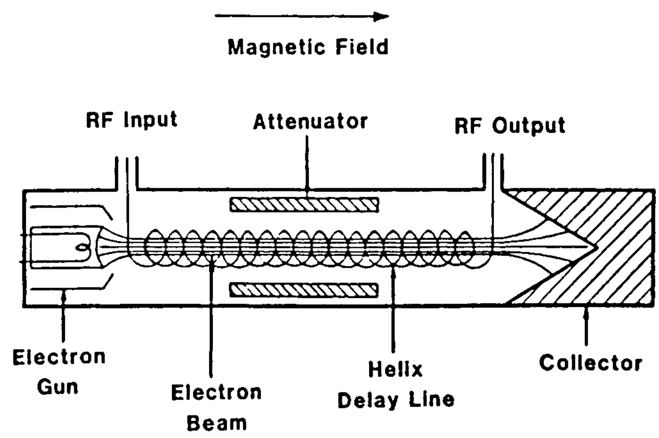

FIG. 1. Basic structure of a traveling wave tube.

Minimizing with respect to A results in A being equal to

$$A = \frac{\Sigma_i\, s_i^{4/3}(1 + 8s_i/15r_c)\phi_i}{\Sigma_i\, s_i^{8/3}(1 + 8s_i/15r_c)^2}.$$

Using this value of A, the current density is then determined as

$$J = \tfrac{4}{9}\epsilon_0\sqrt{2\eta}A^{3/2},$$

where η is the charge-mass ratio of an electron. Magnetic fields can be incorporated into this method.[10]

The trajectories are traced through the space-charge elements, and each space-charge element has a total charge determined according to the current associated with the trajectory and the time that the trajectory spends in the element.

Having determined the space-charge distribution, the boundary-element charge distribution has to be redetermined by solving the following set of linear equations:

$$\sum_j G_{ij}q_j = V_i - \sum_k q_k D_{ki},$$

where i and j refer to the boundary elements and k to the space-charge elements. The terms D are similar to G.

With the new boundary-charge distribution and space-charge distribution, the trajectories are recalculated. The procedure is repeated until some convergence criterion is satisfied. We have used a chosen percentage change in the current density as our stopping condition.

II. COMPUTER IMPLEMENTATION OF METHOD

The main advantage of using the boundary-element method on the Masscomp supermicrocomputer with an integrated array processor is the fact that the calculation of the electric field components can be done entirely in the array processor, accelerating the calculation of trajectories by a factor of 100. The calculation of the electric field is easily vectorized as the same formula is used for each element. Each element's contribution to the field is done in parallel and finally summed to obtain the total field.

The programs are written in FORTRAN 77 under a UNIX operating system which makes file interfacing

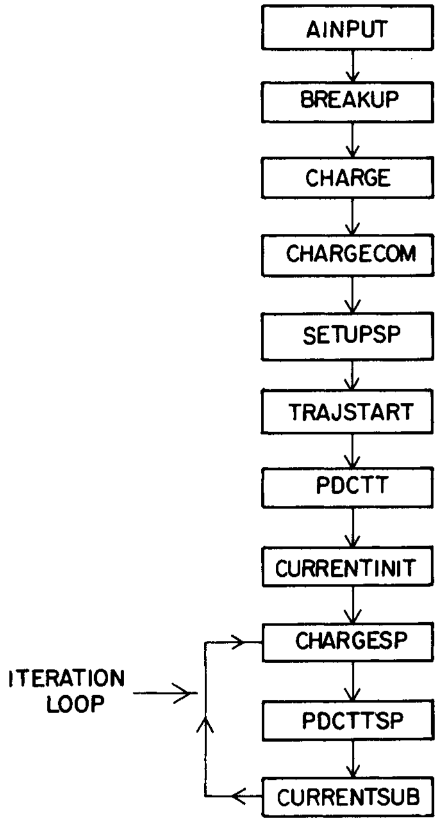

FIG. 2. Components of the CAD system.

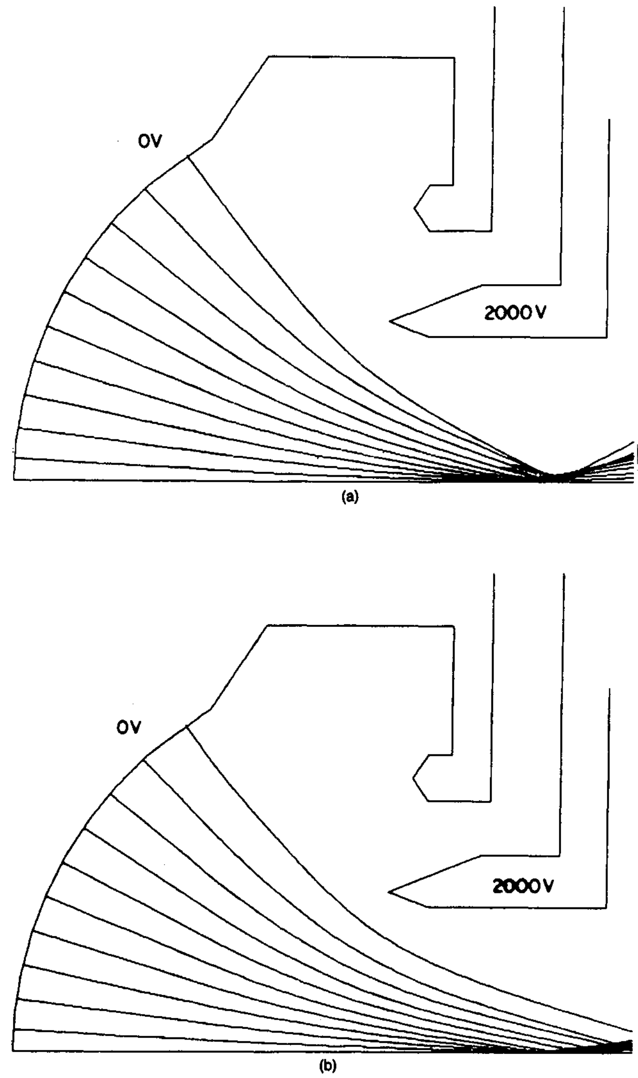

FIG. 3. Trajectory plot of 2.2-perv electron gun: (a) ignoring space charge and (b) including space charge.

between the programs particularly suitable. The array processor is accessed by directly callable FORTRAN subroutines. The array processor has a maximum 16 384 word memory, some of which is required as temporary storage for some array-processor functions. This places a limit on the maximum number of elements one can use effectively in the system. The basic structure of the system used for the electron gun simulation is shown in Fig. 2.

AINPUT allows one to enter the electrode structure in the form of lines and arcs. BREAKUP splits the electrode boundary into ring elements, determining their central coordinates, width, and angle to the z axis. CHARGE determines the charge distributions on the electrodes, with unit potential placed successively on each electrode and zero placed on the remaining electrodes. For 200 boundary elements, CHARGE takes 2 min to solve for the charge distribution. CHARGECOM combines these distributions according to the actual potentials placed on the electrodes. SETUPSP determines the central coordinates, volumes, and areas of the space-charge annuli. TRAJSTART determines the starting conditions of the trajectories. PDCTT calculates the trajectories using a modified Hammings predictor–corrector method with a Runge–Kutta starter.[11] This is a highly accurate integrator of the equation of motion developed for electron optical imaging applications. A less accurate integrator can be used in gun applications which will accelerate the calculations considerably. CURRENTINIT determines the current density associated with each trajectory and determines the space charge in the space annuli by tracing the trajectory through the space-charge annuli. CHARGESP calculates the new boundary-charge distribution, allowing for the effects of the space charge. PDCTTSP recalculates the trajectories, allowing for the presence of space charge. CURRENTSUB is similar to CURRENTINIT, but checks for convergence of the iterative procedure.

III. APPLICATION OF METHOD TO SEVERAL GUNS

In order to test the program, the two guns with experimentally determined perveances analyzed by Radley *et al.*[10] were used as test cases. Figure 3 shows the gun with an experimentally determined perveance of 2.2 μpervs. Figure 4 shows a gun with a measured perveance of 1.9 μpervs. Our calculations agree in both cases. Figures 3(a) and 4(a) show the trajectories, ignoring the effects of space charge, whereas Figs. 3(b) and 4(b) include space charge.

The calculated values are dependent on the distance from the cathode at which current densities are calculated. At positions very close to the cathode, errors in the calculation of the potential can become large, and boundary-ele-

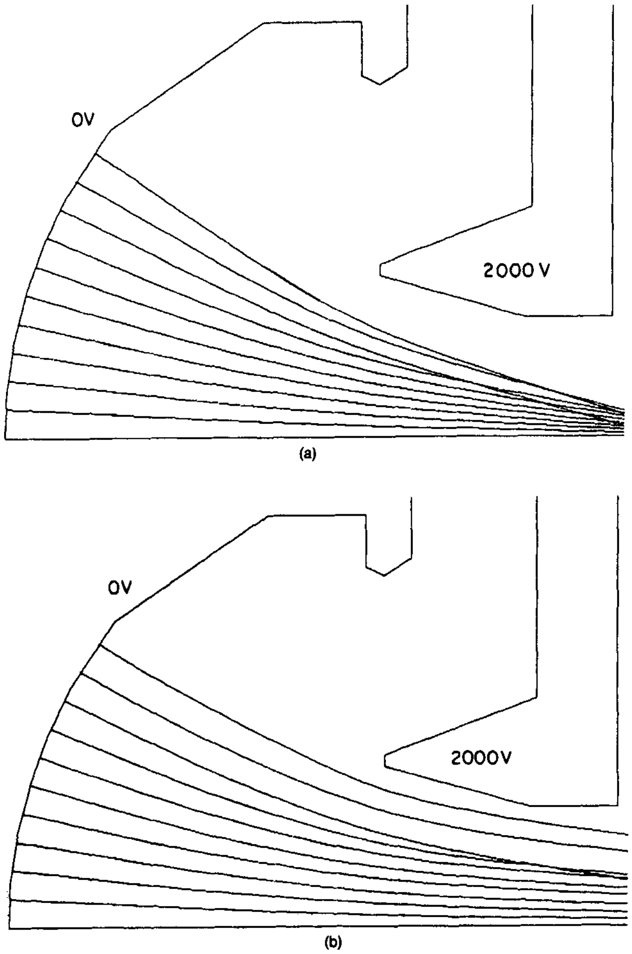

FIG. 4. Trajectory plot of 1.9-perv electron gun: (a) ignoring space charge and (b) including space charge.

ment refinement is required. At positions far from the cathode surface, the least-squares method can start to break down, although by a judicious choice of the intervals and the inherent smoothing effect of this method, reasonably accurate results are obtainable as far as 50% of the radius of curvature from the cathode. Any position between these extremes can be used with an error of less than 5%.

Determining the optimal region for current-density calculations has to be considered. This region will be a function of the geometry of the gun and the number of boundary and space-charge elements. The lower bound to this region is determined by the size of the boundary elements on the cathode. When the distance is comparable to the size of the elements, the potential calculation will be inaccurate. The lower bound should be about four times the length of an element. As mentioned above, an upper bound of about 50% of the radius of curvature can be used. Most accurate results seem to be obtained at a midinterval distance of 20% of the radius of curvature from the cathode.

The iteration procedure converges fairly smoothly as shown in Fig. 5. Convergence is ensured by choosing sufficient boundary and space-charge elements, preferably in excess of *100*. Insufficient elements will lead to large errors in potential and field values, so that divergence can result, and in the case of convergence, inaccurate results will be obtained. Figure 5 shows that about ten iterations are required to obtain a stable value of the current.

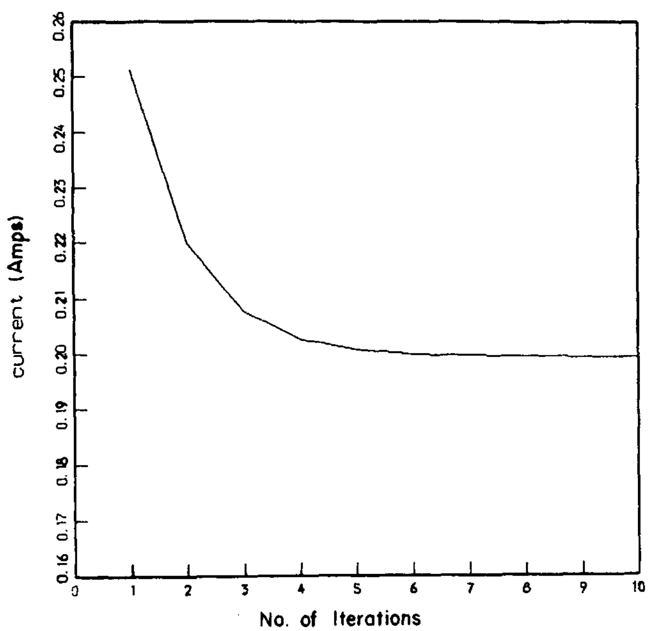

FIG. 5. Calculated current versus number of iterations for gun in Fig. 3.

Using 200 boundary elements and 100 space elements each iteration takes 40 min, giving a total computation time of 7 h. About 6.5 h of this time is required for the calculation of the trajectories, as a rather stringent error control was used. If the error control in the integrator is relaxed, the overall time is reduced to to 2 h. Any further relaxation

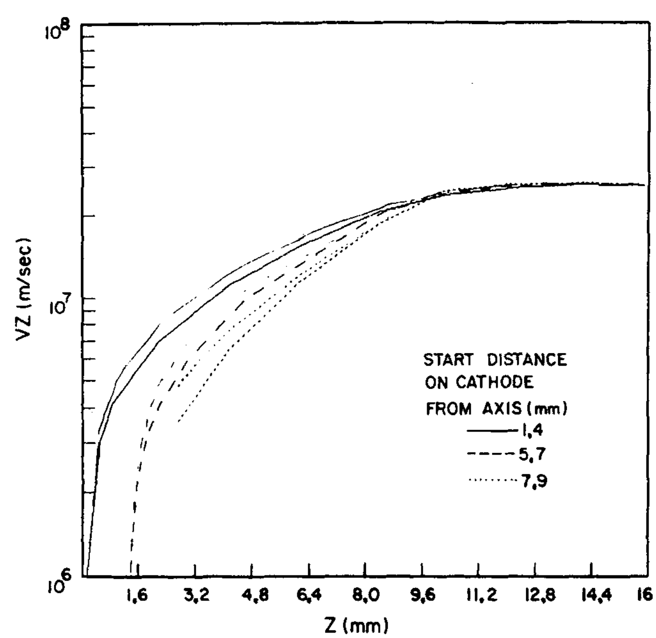

FIG. 6. Velocity in axial direction versus position along axis for different starting positions (upper graph for each position ignores space charge).

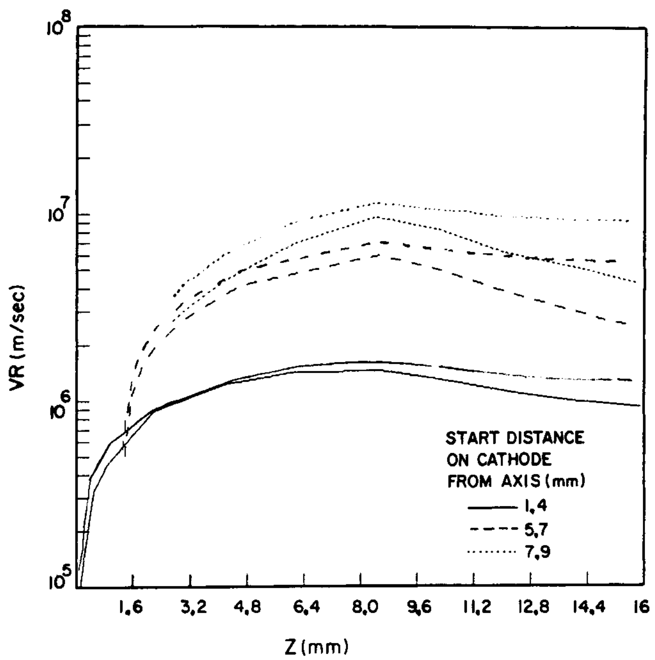

FIG. 7. Velocity in negative radial direction versus position along axis for different starting positions (upper graph for each position ignores space charge).

in the error control leads to erroneous results. Most of the time is consumed in calculating the trajectories. Ten trajectories are calculated at each iteration.

Figure 6 shows a plot of the velocity of the electrons in the z direction against their z coordinate for the gun in Fig. 4. Three points of origin on the cathode were chosen, and the plot shows the velocity variation with and without space charge. The axial velocity is reduced due to the effects of the space charge initially, but reaches the same end speed due to the accelerating voltage. Figure 7 shows a similar plot for the negative radial component of the velocity. The absolute values of these velocities are also reduced due to space charge. The reduction is stronger for electrons originating further from the axis.

IV. CONCLUSION

The inclusion of the array processor has made the boundary-element method a viable technique for analyzing high-perveance electron guns on workstations. We have successfully applied the above system to determining construction tolerances in electron guns. The addition of more postprocessing routines will enable a more detailed analysis of the electron beam, providing cross-sectional current-density profiles at any point along the beam or the analysis of the interception currents with the body of the tube. Although the system as described is useful, it is rather time consuming. Ideally, a computer with a speed of the order of 1 Gflop would bring the total calculation time to within a minute.

REFERENCES

1. J. R. Pierce; *Theory and Design of Electron Beams* (Van Nostrand, New York, 1954).
2. P. T. Kirstein, G. S. Kino, and W. E. Waters; *Space-Charge Flow* (McGraw-Hill, New York, 1967).
3. W. B. Herrmannsfeldt, "Electron trajectory program," SLAC-226 (1979).
4. C. B. Thorington, IEEE Trans. Electron Devices **ED-33**, 1883 (1986).
5. R. True, IEEE Trans. Nucl. Sci. **NS-32**, 2611 (1985).
6. A. C. Paul, IEEE Trans. Nucl. Sci. **NS-26**, 4242 (1979).
7. C. W. Trowbridge, IEEE Trans. Magn. **MAG-24**, 13 (1988).
8. M. F. Calitz and A. G. du Toit, IEEE Trans. Magn. **MAG-24**, 427 (1988).
9. A. Renau, F. H. Read, and J. N. H. Brunt, J Phys. E **15** (1982).
10. D. E. Radley, D. Dirmikis, and A. B. Birtles, Proc. IEE **122**, 62 (1975).
11. E. Kasper, Optik **69**, 117 (1985).