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Modelling of Unsaturated Flow in Heterogeneous Soils

Part II: Stochastic Simulation of Water Flow over a Field

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K. Høgh Jensen and M. B. Butts
Technical University of Denmark, Copenhagen

Modelling of soil moisture conditions in spatially variable fields is treated using stochastic methods. Spatial variability of moisture content in a vegetation covered field is partly caused by field variability in soil physical parameters. In the present study a physically based model is coupled with a statistical description of retention properties and saturated hydraulic conductivity respectively to simulate moisture conditions in heterogeneous soils.

Results are compared with measurements obtained from two 0.5 ha field sites.

Simulations based on the variation in retention properties are shown to account for much of the observed variation in soil moisture conditions with some deficiencies evident close to the soil surface. Variations in saturated hydraulic conductivity alone give an incomplete description of observed variability in soil moisture conditions.

Introduction

The behaviour of flow and transport processes in the unsaturated zone involves a complex interaction of climatic, plant and soil parameters. One of the major complications is the heterogeneity of natural soil formations. Nevertheless, the accurate description of these processes is important in many areas of hydrology and agriculture.

Within a particular soil profile the flow process is governed to a large extent by the soil hydraulic properties. Over a field or catchment, these hydraulic properties exhibit a large degree of spatial variability as a result of soil heterogeneity (Nielsen *et al.* 1973, Byers and Stephens 1983, Russo and Bresler 1981). Concequently, the distribution and movement of soil water will also vary appreciably.

Despite these observations it is usual when modelling flow in the unsaturated zone to assume the distribution of hydraulic parameters is uniform over the area of interest and to treat the soil as a homogeneous medium characterised by "equivalent" hydraulic properties. These "equivalent" properties are usually established by sampling over a few locations and appropriately averaging. Since the flow processes in the unsaturated zone are highly non-linear the notion of an "equivalent porous media" is questionable. Recent studies by Dagan and Bresler (1983) and Mantaglou and Gelhar (1985) conclude that for heterogeneous soils "equivalent" hydraulic properties depend on the soil-moisture conditions and are therefore no longer useful. Smith and Hebbert (1979) demonstrate by Monte Carlo simulation that by introducing the spatial variability of soil hydraulic properties the hydrological response of a catchment is different from the response obtained by simply treating the catchment as a homogeneous unit.

In recognising the heterogeneity of field soils several practical questions arise concerning the movement of soil water:

- how large are spatial variations in soil hydraulic parameters and which are the most important of these parameters?
- what are the consequences of this variability on the soil water flow integrated over a field or catchment?

A possible framework for approaching some of the problems related to field variability is offered by stochastic modelling. In this approach the soil properties and flow variables are interpreted as stochastic variables because the deterministic variation cannot be known in all details and therefore is subject to uncertainty. The variables are consequently defined in terms of their statistical moments and the actual field is interpreted as a realization of the ensemble of fields which all have the same properties at the sampling points as the given field.

Various techniques have been used for the stochastic analysis of the heterogeneity problem ranging from spectral methods (Yeh *et al.* 1985) and numerical methods (Smith and Schwartz 1980) to a more simple statistical averaging procedure (Dagan and Bresler 1983; Bresler and Dagan 1983).

The approach adopted here is a statistical averaging procedure, where the statistical moments of the flow variables are predicted on the basis of the moments of the soil properties for given boundary conditions. The data are obtained from the research field described by Hansen *et al.* (1986).

Statistical Analysis of Retention Properties

The two basic relationships which determine unsaturated flow processes are the retention function and the hydraulic conductivity function.

In the first approach to describe field variability, the effect of soil heterogeneity

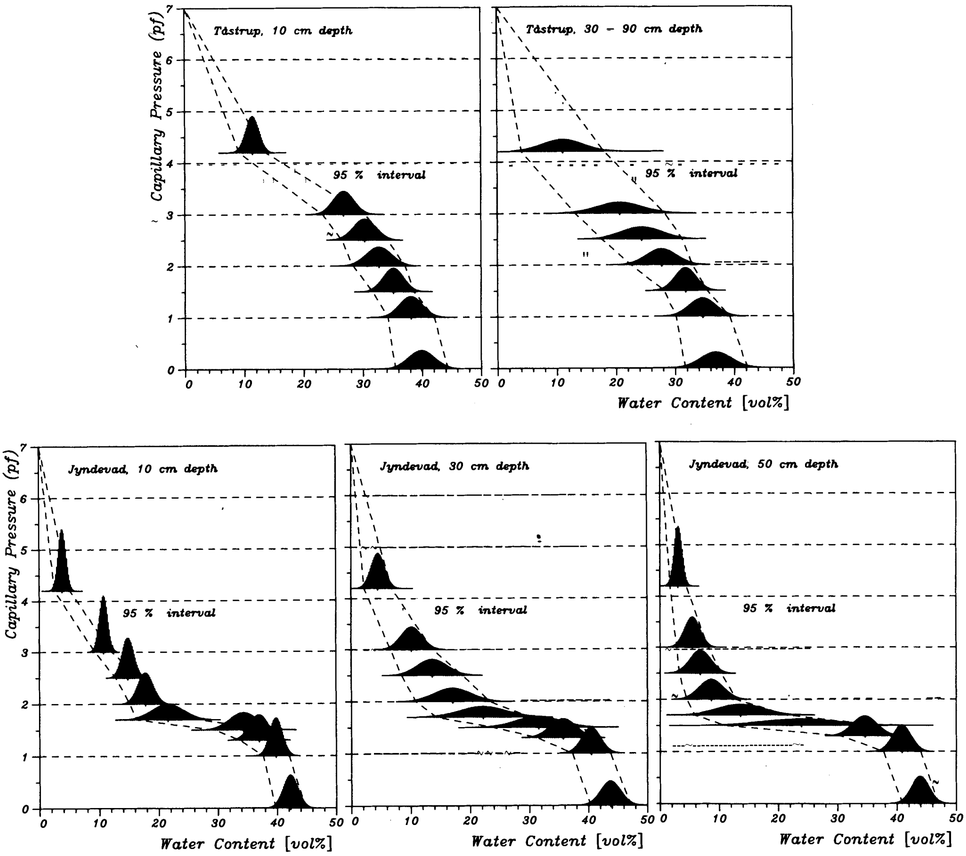


Fig. 1. Spatial variability of retention properties within the field sites.

is evaluated on the basis of the retention function alone, and the hydraulic conductivity properties will partly be derived from the information embodied in this function following the method by Kunze *et al.* (1968).

When applying this method it is recommended to introduce a matching factor in order to improve the accuracy of the predicted values. A comparison analysis between measured and predicted saturation conductivities suggest a matching factor of 0.02 for the Taastrup site and 0.10 for the Jyndevad site. Although, in principle, this method should establish the complete hydraulic conductivity function, practical applications have shown that the method possesses some shortcomings. Instead, the procedure described in Part I (Jensen 1986) is applied by utilizing the concept of field capacity in combination with the predicted saturation conductivities.

The spatial variation of retention properties within the two field sites are shown in Fig. 1. For each of the suctions applied in the laboratory analysis the frequency distributions of the corresponding moisture contents are shown.

For the Taastrup site two horizons of different characteristics have been recognized and analyzed separately. As shown by the figure, the soil properties are more homogeneous in the upper soil horizon than in the lower. It is emphasized that the larger variability is a result of a larger horizontal variability and not caused by the larger depth interval.

For the Jyndevad site soil sampling has been performed only at three depths and the corresponding frequency distributions are shown for all three horizons. Although the research field is situated in an alluvial outwash plain which generally is considered homogeneous in geological terms, a pronounced small-scale variability is observed. It is evident from the figures that the soil properties are more homogeneous in the upper soil horizon than in the lower. Particularly for the deepest horizon a considerable variability is recognized notably for pF-values between 1.5 and 2.0. Since the moisture content at this suction interval represents field capacity a similar large variability in field moisture conditions is expected.

Statistical Analysis of Saturated Hydraulic Conductivity

A more traditional approach is to describe field variability in soil moisture conditions assuming that the variability can be accounted for by variations in the saturated hydraulic conductivity K_s alone. While in general both the retention curve and hydraulic conductivity function will exhibit some spatial variability this assumption recognises that K_s may change by several orders of magnitude within a field whereas other hydraulic parameters vary within narrower limits (Russo and Bresler 1982).

Saturated hydraulic conductivity has been determined on 100 cm^3 undisturbed soil cores (triplicate sampling) at 24 locations at several depths for each of the two field sites.

The statistical distributions of saturated conductivities measured at the two field sites Taastrup and Jyndevad are presented in Fig. 2. Table 1 lists the important statistical parameters. In both cases the saturated conductivity K_s was found to be log-normally distributed over the field, Fig. 2. The soil at Jyndevad is a coarse sand and the distribution of K_s is relatively uniform, whereas the sandy loam at Taastrup has a smaller average conductivity and larger variability.

Experimental values for the saturated conductivity obtained from the two field sites, Taastrup and Jyndevad, range between $1.6 \times 10^{-8} - 6.5 \times 10^{-4}$ (m/s) and $6.3 \times 10^{-7} - 1.4 \times 10^{-3}$ (m/s).

No experimental results are available so far on the hydraulic conductivity for lower moisture contents. To determine hydraulic conductivity over the whole regime a power function will be adopted. Suitable values for the parameters in such a function will be obtained by calibration on the individual profiles, (Part I, Jensen 1986).

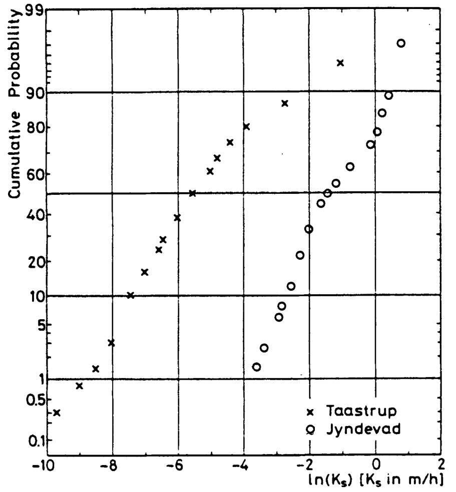


Fig. 2.
Statistical distribution of saturated conductivity K_s for the two experimental sites, Taastrup and Jyndevad.

Table 1 – Statistical parameters of saturated conductivity distribution K_s [m/h].

Site	Mean value of ln K_s μ (ln K_s)	Standard deviation of ln K_s σ^2 (ln K_s)
Taastrup	−5.18	1.99
Jyndevad	−1.23	1.31

Statistical Analysis of Moisture Content

The moisture content has been measured regularly in the field in order to analyze the spatial variability. Figs. 3 and 4 illustrate the horizontal variability with time for various depths. The shaded area between the two lines represents twice the standard deviation. For a normal distribution of the moisture contents these intervals represent 95 % of the probability mass.

The variability pattern is somewhat different between the two sites. For Taastrup (Fig. 3) the horizontal variability is largest in the top layer, somewhat smaller at 30 cm depth after which the range of variation is more or less of the same magnitude. No marked seasonal variation in the variability pattern is observed.

The observations from Jyndevad (Fig. 4), however, show a significant seasonal variation for the upper two layers, where the horizontal variability becomes less during periods with high water stress. Further, as opposed to the Taastrup results, the general range of variation in moisture content close to the soil surface is of similar magnitude to the variation deeper in the profile. At some levels the variation is even larger here.

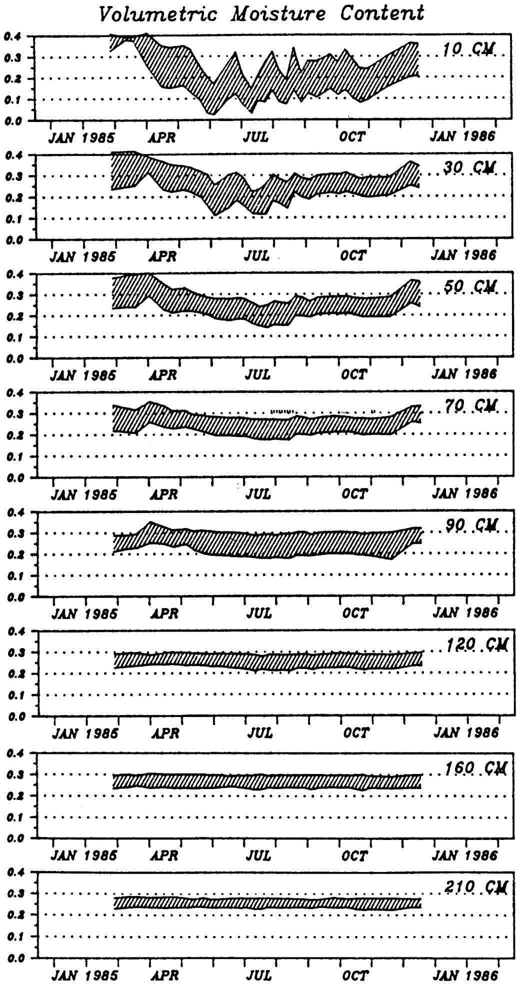


Fig. 3.

Horizontal variability in measured moisture content at Taastrup.

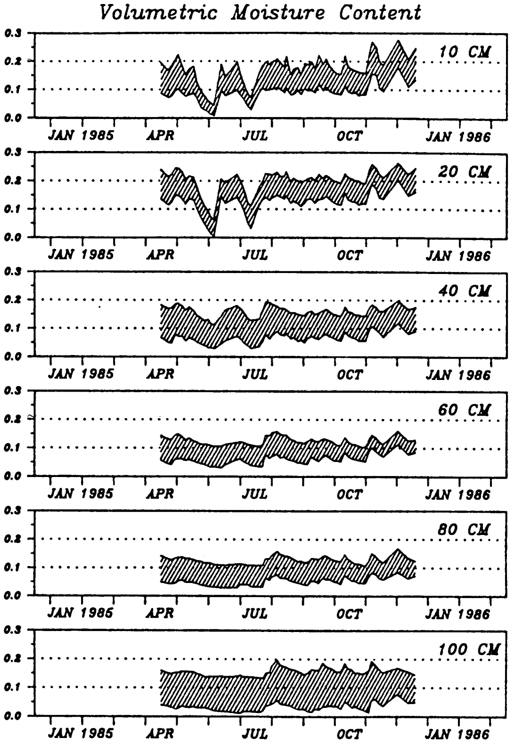


Fig. 4. Horizontal variability in measured moisture content at Jyndevad.

Range of variation shown by twice the standard deviation.

Stochastic Modelling of Field Soils

In practice, measurements of soil hydraulic parameters are limited to a few point values. The distribution of hydraulic properties over a field or catchment is difficult to predict from these values because of the complex heterogeneity of natural soils. A useful approach is to treat soil properties as stochastic variables such that their distribution in space can be conveniently described by a few statistical moments. It is generally assumed that statistical stationarity and the ergodic hyopthesis apply which means that ensemble averages and space averages can be interchanged. Hence, the expectation of a variable represents the average over the field and similarly the standard variation will represent the variability over the field.

Stochastic modelling combines this stochastic description of the distribution of hydraulic properties with the unsaturated flow equation to predict the distribution of soil moisture conditions. In many applications it is not required to give a detailed description of the flow processes within a field. Instead by also treating the flow variables as stochastic the useful information about their distribution is contained in a few statistical moments. Just as the variance of the hydraulic parameters measures the spread of values, the variance of water content gives a measure of variability over the field.

The general treatment of this problem is a complicated topic in stochastic differential equations, so further reduction of the problem is required. Perturbation methods (Andersson and Shapiro, 1983, Yeh *et al.* 1985) and Taylor series expansions (Dettinger and Wilson 1981) assume the variations in hydraulic properties are small. Usually this is not the case and the validity of this assumption has not been satisfactorily tested against field data.

More widely applicable is the Monte Carlo method, which takes sample values of the hydraulic properties and solves the flow equations for each sample. The results are combined to estimate the distribution of the flow variables. However, the flow equations must be solved many times to obtain useful results and this makes substantial demands on computer resources.

The present analysis combines a comprehensive numerical model of unsaturated flow with an averaging procedure to determine the distribution of soil moisture.

The model used to predict unsaturated flow is developed by Jensen (1983) and briefly outlined in Part I (Jensen 1986). The important feature is that the movement of water is described by the Richards equation by assuming essentially vertical flow. Horizontal gradients are caused by soil heterogeneity, but these are assumed to be small in comparison with the vertical gradients.

Since the flow is vertical the field is represented as an ensemble of vertical soil columns, each column representing a possible soil profile. The hydraulic properties are distributed amongst the ensemble according to the distribution found in the field. Any spatial structure in the distribution of soil properties is neglected by assuming the columns are statistically independent.

Based on these assumptions the predictions of soil moisture conditions over the fields are obtained by dividing the cumulative distributions of either retention properties or saturated hydraulic conductivity into a number of classes, each class corresponding to a single profile. For each of these profiles the soil moisture conditions are determined from the unsaturated flow model described previously using the hydraulic properties which represent the indvidual classes. The statistical moments for the moisture content can subsequently be derived from the model predictions.

Modelling Based on Variability in Retention Properties

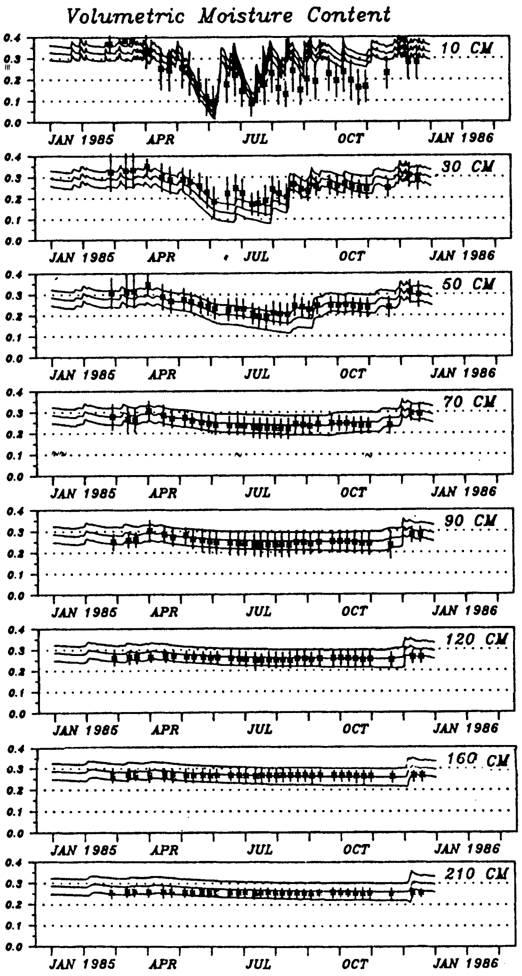
The following analysis represents an attempt to predicting the statistical moments of the soil moisture content on the basis of the moments of the water retention properties. The probability functions of the water content for the individual suctions, Fig. 1, are divided into 10 equal classes. As a first approach it is assumed that full correlation exists between water contents at the different suctions, thus justifying a linkage between similar probability fractiles. This procedure leads to 10 retention curves, each representing a soil class within the field with hydraulic characteristics as embodied in the retention curve. For a given retention curve the hydraulic conductivity function is established from the procedure described earlier.

The statistical analysis of the spatial variability in water retention characteristics has identified two horizons in Taastrup and three in Jyndevad each having different retention properties, Fig. 1. Hence, the procedure described above will be carried out for the individual horizons. In defining 10 different soil columns a retention curve has to be determined for each soil horizon. Preliminary analyses have shown that there is a rather weak correlation betwen soil properties of the individual horizons implying that high retention properties in one horizon may well coexist with low retention properties in another horizon and vice versa. However, as a first approach it is assumed that the soil properties have the same nature throughout the profile.

Having established 10 statistically independent soil profiles each representing possible field conditions, the soil water flow model is used for simulating the flow conditions in each.

The stochastic simulation results are presented in Figs. 5 and 6 and compared with observations. The average moisture content for the individual layers and the range of variation given by twice the standard deviation are used for this comparison.

The simulation of the spatially averaged moisture contents shows some of the same discrepancies close to the soil surface as experienced for the individual sampling profiles. Some of these discrepancies may be explained by an insufficient mod-



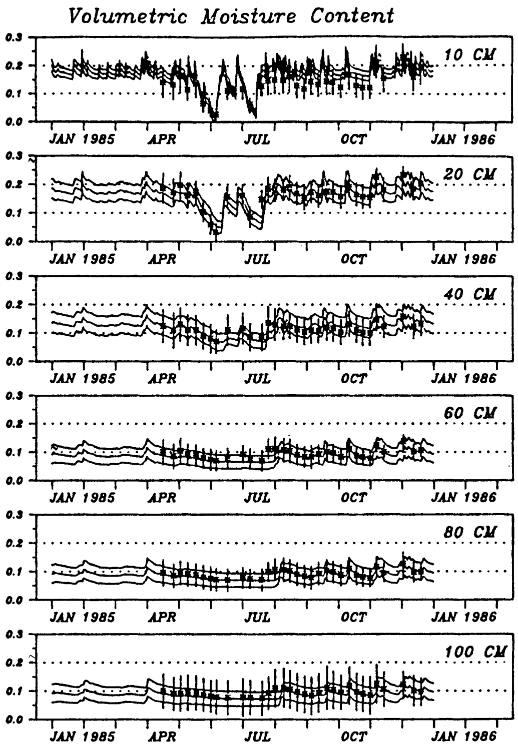


Fig. 5.
Measured and simulated variability in moisture content at Taastrup.

Fig. 6. Measured and simulated variability in moisture content at Jyndevad.

* Measured range of variation.
= Simulated range of variation based on variability in retention properties.

elling of the evapotranspiration processes. However, difficulties in calibrating the neutron probe close to the surface may also contribute. The simulations of the spatially averaged moisture content deeper in the soil are very accurate.

A further analysis of the simulation results also shows that the simulation based on the average retention curve corresponds rather closely with the spatially averaged values. This is surprising because of highly non-linear phenomena inherent in soil water dynamics and in view of the investigations reported by e.g. Bresler and Dagan (1983), who question the validity of the concept of "equivalent soil properties". One reason for this apparent disagreement may be the very different flow conditions considered. Bresler and Dagan (1983) consider short-term, rather abrupt flow events, whereas the present investigation analyses naturally occurring time series of unsaturated flow. Under such conditions the non-linear effects may tend to balance out implying that "equivalent soil properties" may constitute a reasonable concept.

Simulations of the range of variation in moisture content compare in general well with measurements at lower levels, while discrepancies are evident close to the soil surface. Here the predicted range is too small in particular at Taastrup, but also at Jyndevad, except during periods with high water stresses where both the measured and simulated ranges are very small.

The reason for the simulated range generally being too narrow is obviously due to the small variation in retention properties, Fig. 1. Hence, the spatial variation in moisture content cannot alone be explained by the variation in soil physical properties, using the retention curves as the basis. Other factors are apparently affecting the variability in this horizon. This may well be a soil physical property which is not accounted for in details by the retention curve such as the hydraulic conductivity. However, in addition, variations in plant related variables like the density of the ground cover, the green active material, albedo conditions and root growth will certainly also contribute to the observed variations.

In the soil horizons below the upper measurement level nearly all the observed variability in moisture content can be predicted by the variation in retention properties, using the simple stochastic modelling approach adopted in the present study.

Modelling Based on Variability in Saturated Hydraulic Conductivity

The second analysis assumes that the saturated hydraulic conductivity K_s is the only spatially variable soil hydraulic property. Since a power function is adopted to describe the hydraulic conductivity over the whole regime (Eq. (4) in Part I, Jensen 1986) the variability introduced through K_s applies for the conductivity at all water contents.

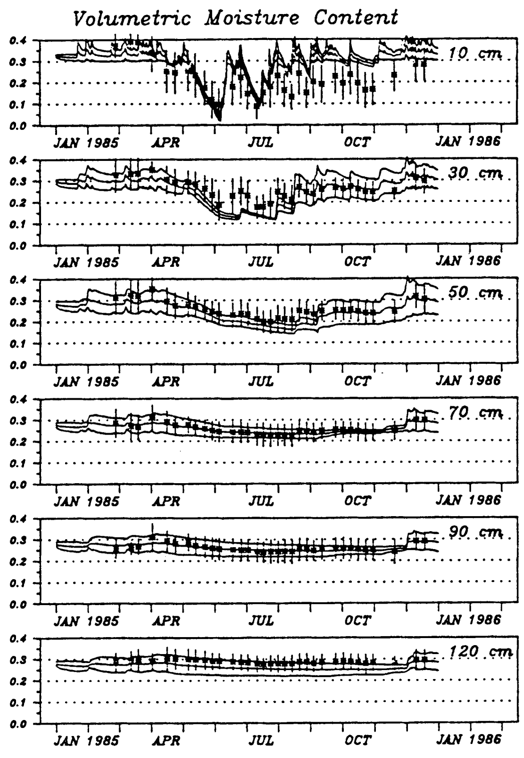


Fig. 7. Measured and simulated variability in moisture content at Taastrup.

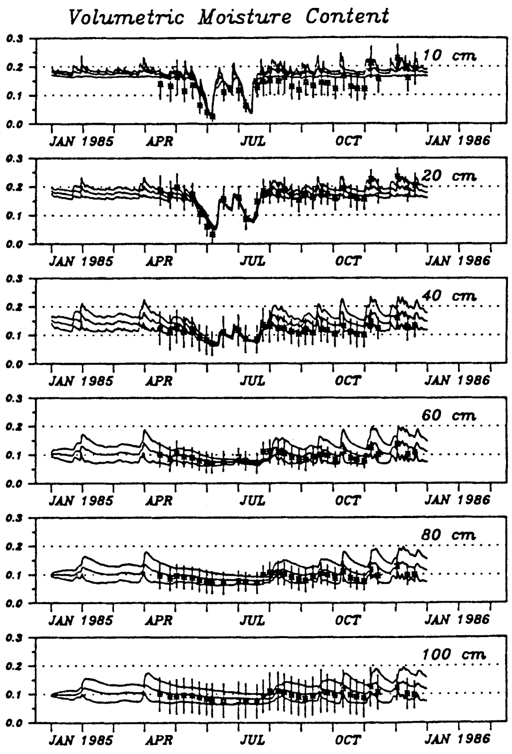


Fig. 8. Measured and simulated variability in moisture content at Jyndevad.

* Measured range of variation.

≡ Simulated range of variation based on variability in hydraulic conductivity.

The cumulative distribution of K_s, Fig. 2, is divided into 10 classes, each class corresponding to a single profile. The numerical model is subsequently applied to each profile assuming that the conductivity function is constant with depth. The mean retention properties for the defined soil horizons are applied to all profiles.

The simulated and measured moisture contents are compared in Figs. 7 and 8. The simulation of the mean values are of similar quality as in the first approach: obvious shortcomings close to the soil surface and accurate predictions at lower depths.

The range of variation in moisture content is predicted less accurately than in the first approach. In general a smaller portion of the observed variations can be explained by the variation in saturated conductivity alone than is explained by variation in retention properties.

This is particularly noticeable in periods when little flow occurs. Changes in the saturated conductivity affect the rate of flow through the soil. This can be readily seen in Fig. 8. The upper line in the simulated range represents low conductivity values, where the water moves at a slower rate through the soil. This leads to higher water contents at each level and larger peaks in response to rainfall events. The lower line represents high saturated conductivities. The variation in water content is much smaller because the water passes rapidly through the soil. The peaks occur earlier for the same reason. Since the conductivity controls the flow rate, variations in the flow predicted on the basis of the conductivity function alone will decrease as the amount of water moving through the soil decreases. Hence, a complete description of soil moisture conditions it seems should also include the variation in retention properties.

Previous analyses by Dagan and Bresler (1983) and Yeh *et al.* (1985) have also examined unsaturated flow in heterogeneous soils on the basis of variability in saturated conductivity alone. However, these studies have been essentially theoretical without direct comparison with field conditions.

Conclusions

The study has shown that soil variability has a large influence on water and flow conditions in the field, and it is consequently of importance to account for this phenomenon in solving hydrological and agricultural problems.

The present study is a first and preliminary approach to the problem of simulating the flow conditions in heterogeneous fields. In one approach an attempt is made to simulate the field variability of moisture content on the basis of the variability embodied in the retention properties. Below 30 cm from the soil surface the mean moisture content and much of the observed variation can be explained by this approach. Close to the surface other factors become important, such as the variability of root extraction and plant cover as well as the natural variation in hydraulic conductivity. Also hysteresis may be more important in this horizon.

The model simulations have indicated that the concept of "equivalent" hydraulic properties may be useful in applications over areas the size of agricultural fields where only the average response is required.

An approach which incorporates only the variability in saturated hydraulic conductivity appears somewhat more incomplete. It has often been assumed that the saturated conductivity is the important parameter governing flow in the soil, and while water is moving the simulations have shown that the parameter has a significant influence on the variability in moisture content. However, when there is little movement of water other factors become more dominant.

The two approaches are to some extent complementary. Where the first one excludes the full range of variation in the hydraulic conductivity properties, the second one excludes the natural variation in retention properties. The goal of future work is therefore to combine these two complementary methods to obtain a more accurate and more physicaly sound description of unsaturated flow in heterogeneous soils.

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Address:
Institute of Hydrodynamics and Hydraulic Engineering,
ISVA,
Technical University of Denmark,
Building 115,
DK-2800 Lyngby, Denmark.