fluid in the line and thereby slow the response. Calculations of the centrifugal accelerations compared to the axial acceleration indicate, however, that the centrifugal accelerations are very small. An inspection of the long line, at the conclusion of the testing, was carried out. This inspection did not reveal any significant blockage of the long tube due to burrs caused by transducer placement. The slope of the tail of the measured response of 0.1 psig runs, again, is in good agreement with the analytical predictions. On several of the curves, immediately after the initial rise, the amplitude grows through a point of very slow growth and then continues at a more rapid rate, although less than the initial rate. This type of behavior could have been due to some impulsive content in the input.

Conclusions

To the best of the author's knowledge, this is the first publication of a combined analytical and experimental study of the transient response of a gas-filled circular line. Of particular importance is that, due to a method of solution used, a time domain solution is obtained that is not of a numerical nature. This is in contrast to the works of Brown [6] and Zielke [7] which require a step-by-step calculation of the responses. When compared in the frequency domain with previously available solutions, the analytical solutions obtained here have a limited frequency range. The difference is due to the manner in which the basic equations are satisfied, as discussed in reference [9]. The calculations have demonstrated the validity of using the approximation of unity Prandtl number for air, which has a Prandtl number of 0.71. This is an important point for it greatly simplified the required calculations. From these basic results, the response to other types of transients can be determined.

An analysis of the response of fluid-filled lines to a step modulation of carried frequency is presented. It is believed that this is the first time such calculations have been carried out for fluid lines, with the effects of viscous losses and heat transfer taken into consideration.

The experimental work has verified the theoretical predictions, although an exact correlation was not achieved. Differences between the analysis and the experimental results are explained by recognizing the limitations of the analysis. An effect uncovered by the experimental work is the nonlinearity of the signals at signal amplitudes, unexpectedly as low as 0.2 psi. This pressure change is only on the order of one percent of the mean pressure. The flow velocity would have a maximum value of about 5 percent of the acoustic wave speed for a 0.2 psi step.

Before developing the model presented in this paper, a currently available mathematical model [4] was investigated [9] and found to be inappropriate for use. This is due to the complexity of the solution form in the Laplace domain, and it appears highly unlikely that an inversion of this solution to the time domain is possible. Approximate time domain solutions have been published previously [4] but they were not considered to be appropriate for use here.

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DISCUSSION

F. T. Brown[2]

The author has introduced expansions of the propagation operator and characteristic impedance which are close to being expansions about zero frequency. Their considerable virtue is rapid convergence, better than the traditional zero-frequency expansions used by the writer as the third part of the solution in [5]. This convergence apparently eliminates the need for the messy second part of the solution in [5], used for intermediate frequency components. It does not eliminate the need for the first part, an infinite-frequency expansion, however; the front ends of the step responses given by the author can be improved by the results of [4] and [5]. It is ironic that truncated front-ends are plotted, when the expansion works best for tail-ends (as pointed out by the author). (The rounding of the front-ends in Figs. 11 and 13, not done elsewhere, is misleading.)

It would be helpful if the author would indicate the number of terms (i.e., the maximum value of n) retained in the computations. Comparison of the truncated series to the more exact result from which it springs, made most easily in the frequency domain, also would be of considerable interest.

The question of when each of the many possible available expansions should be used has not yet been resolved. In certain cases, particularly dealing with FM transmission, expansion of the wave operators about a specific frequency probably would be advisable. Expansions in terms of the characteristics of the whole system including terminations, well typified by the product series of Oldenburger and Goodson[3] and discussed further by Oldenburger,[4] represents a different competing class of approximations. The method of characteristics, typified by [6] and [7], represents a third important competing class of approximations with special relevance to nonlinear wave propagation.

The writer would like to correct a false interpretation of his paper [6] given on the seventh page of the present paper. Increasing the amplitude of a *positive* pressure step indeed should cause a faster response at the downstream section. Most of the author's data is for *negative* steps, however, (indicated by *positive* values of the initial pressure inside the tube), for which the re-

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verse is true. The only qualitatively inconsistent data is for the initial pressure of -0.25 psig in Fig. 13. The writer wonders if this effect might have been caused by an entrance loss. Criticism of well-founded nonlinear theory in view of this data would seem ill-advised.

The criticisms above are minor compared with the basic contributions for which the author is to be congratulated.

A. J. Healey[5]

The author is to be commended for obtaining time domain responses to the infinite line problem and it is gratifying to read about his experimental results. This discussion, however, pertains to the aspect of the paper dealing with frequency modulated transient case.

The form of the output—as a time function—as obtained by the author can be extended to that of the instantaneous frequency (the more desirable modulating signal) for certain cases by reworking equation (47). If only cases where $T_0 > 1$ are considered (long lines of small radius) then

$$y(T, T_0) = \frac{\gamma}{\sigma}\left\{\sin \Omega T \int_{T_0}^{T} g(\tau, T_0) \cos \Omega\tau dt - \cos \Omega T \int_{T_0}^{T} g(\tau T_0) \sin \Omega\tau dt\right\}$$

since $\sum_{n,1}^{\infty} \frac{4}{\alpha_n^2} e^{-B_n T_0}$ is small.

Now y is basically sinusoidal and may be expressed as

$$y(T, T_0) = \frac{\gamma}{\sigma} A(T, T_0) \sin (\Omega T - \phi(T, T_0))$$

where

$$A(T, T_0) \sin \phi(T, T_0) = I_2$$

$$A(T, T_0) \cos \phi(T, T_0) = I_1$$

and

$$I_2 = \int_{T_0}^{T} g(\tau, T_0) \sin \Omega\tau d\tau$$

$$I_1 = \int_{T_0}^{T} g(\tau, T_0) \cos \Omega\tau d\tau$$

Thus

$$A(T, T_0) = \sqrt{I_1^2 + I_2^2} \quad \text{and} \quad \tan \phi = \frac{I_2}{I_1}$$

The instantaneous frequency Ω_i is given now by,

$$\Omega_i = \frac{d}{dt} (\Omega T - \phi(T, T_0))$$

$$= \Omega - \frac{d\phi}{dt} (T, T_0)$$

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Differentiating $\tan \phi$ to obtain instantaneous frequency deviations,

$$\frac{d\phi}{dt} (T, T_0) = \frac{(I_1 I_2' - I_2 I_1')\left(1 + \left(\frac{I_2}{I_1}\right)^2\right)}{I_1^2} \qquad (50)$$

where $'$ denotes differentiation with time.

Equation (50) yields the expression for instantaneous frequency deviations. It is controlled by the function $g(T, T_0)$ and as T tends to infinity, $\phi'(T, T_0)$ tends to zero so that the final steady frequency is indeed Ω. For small values of phase shift ϕ,

$$\frac{d\phi}{dt} (T, T_0) \simeq \frac{g(T, T_0) \sin \Omega T}{\int_{T_0}^{T} g(T, T_0) \cos \Omega T}$$

The F.M. transient response time is thus governed strongly by $g(T, T_0)$ which was shown in Fig. 8 of the paper but not from the point of view of instantaneous frequency.

It should be noticed here that the relation between $g(T, T_0)$ and $\sin \Omega T$ is significant and will determine whether the frequency deviation oscillates according to $\sin \Omega T$ or predominantly decays according to $g(T, T_0)$.

Author's Closure

The author is appreciative of the time and effort spent by the discussers in reviewing this paper. With regard to Dr. Brown's discussion several points are taken up here.

1 The truncation of the response plots on Figs. 1, 2, 3, and 4 is not due to an inherent mathematical difficulty, but rather to the increasing calculation time, as the convergence slows with an increase in the ratio of T/T_0.

2 The rounding of the front ends of predicted responses in Figs. 11 and 13 is due to the input transient, which is measured and used to predict the downstream response.

3 Convergence limits on the series summation in equations (31), (32), and (33) were set so that the contribution from the nth term to the summation had to be less than 10^{-6} times the sum. The maximum value of n ranged from 5 to 200, dependent on the values of T and T_0. In reference [9] additional information on this aspect is given.

4 Comparison of the analytical results and those of previous investigators is covered in reference [9], but is briefly stated here. When compared in the frequency domain it is seen that the results given here are in agreement with those of previous investigators [2], [3], and [4] until the nondimensionalized frequency, $\Omega > \alpha_n^2$.

5 With regard to the effect of the step size on the experimentally measured responses, it is believed that the -0.25 psig initial pressure data is as valid as the rest of the data. Since the input pressure transient was measured inside the tube, it is doubtful that an entrance effect played any part in this data.

It is gratifying to the author to see Dr. Healey extending the use of this information.