

plot of  $EI$  versus  $V^2I/E$  gave  $EI$  values several orders of magnitude lower. Nondimensional plots made of the work now being concluded indicate approximate agreement with the  $BI/V$  versus  $V^2I/E$  curve, but the  $EI$  versus  $V^2I/E$  data are similarly several orders of magnitude lower. It should be pointed out that the data for the plots referred to by Mr. Lord were obtained for arcs in atmospheric pressure air. In addition, it is important that the potential gradient used should be that of the arc column itself and not include the voltage drops in the electrode sheath regions. Therefore the technique used to experimentally determine the potential gradient should be specified in order to compare related data.

I hope that these comments by the discussers and this closure will serve to stimulate additional interest in the magnetically balanced electric arc in a transverse gas flow. To my knowledge, the problem of the internal behavior of such a balanced arc has not been experimentally attempted, although several theoretical models have been proposed. Only preliminary measurements have been made of the properties in the region adjacent to such a balanced arc. More work is needed to give a greater degree of fundamental understanding to the present empirical procedure necessary in the design of devices where an electric arc interacts with crossed magnetic and convective fields.

## The Radial-Equilibrium Equation of Turbomachinery<sup>1</sup>

Joseph Davids<sup>2</sup>

I would like to congratulate the author on a very excellent and interesting paper.

One of the most interesting parts of this paper is the physical interpretation given for the various terms of the radial equilibrium equation, which enables you to determine the magnitude and influence of the various terms involved. It appears that the solution must be iterative and primarily based upon the initial assumed meridional streamline curvature and flow distribution. This presents the questions: How does the author determine the final shape of the streamline curvature and flow distribution? Also, has there been any experimental verification on compressors or turbines of the radial pressure distribution obtained from this method?

Max J. Schilhansl<sup>3</sup>

L. H. Smith assumed a linear relationship between the blade surface velocities and the average velocities and described the averaging procedure in Appendix 1. It seems, however, that it is necessary to use a quadratic approximation, for instance,

$$w_u = (w_u)_p + \left( \frac{\partial w_u}{\partial \theta} \right)_p (\theta - \theta_p) + \left[ \frac{(w_u)_p - (w_u)_s}{(\theta_s - \theta_p)^2} - \frac{\left( \frac{\partial w_u}{\partial \theta} \right)_p}{\theta_s - \theta_p} \right] (\theta - \theta_p)^2$$

for the following reason.

Let us consider an element as of an infinitely thin blade of a

<sup>1</sup> By L. H. Smith, Jr., published in the JOURNAL OF ENGINEERING FOR POWER, TRANS. ASME, Series A, vol. 88, 1966, pp. 1-12.

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straight cascade making an angle  $\beta$  with a perpendicular to the cascade axis. The surface velocities are  $w_p$  and  $w_s$  at pressure and suction side and have the components  $w_p \sin \beta$  and  $w_s \sin \beta$ , respectively, parallel to the cascade axis.

If the angle  $\beta$  is zero, as it is in the last 10 percent of the blade length (see Fig. 10), then the average component  $\bar{w} = 0$ , while a quadratic approximation supplies

$$(\bar{w}_u)_{\beta=0} = \frac{1}{\theta_s - \theta_p} \int_{\theta_s}^{\theta_p} \left( \frac{\partial w_u}{\partial \theta} \right)_p (\theta - \theta_p) \times \left[ 1 - \frac{\theta - \theta_p}{\theta_s - \theta_p} \right] d\theta \neq 0$$

The method of Smith implies that the flow is "impact free" along the entire loading edge. Such a flow is among the possible flow patterns, but it has not yet been proven that it is the actual one.

## Author's Closure

The author would like to thank Mr. Davids and Dr. Schilhansl for their comments.

Mr. Davids has inquired about how the initial assumption of meridional streamline shape affects the final calculated flow distributions. The answer is that the initial streamline assumption does not affect the final flow when the iterative calculation procedure is carried to convergence. Convergence here means that each streamline (line connecting points having the same streamfunction at different axial stations) has at each axial station the curvature and slope that were employed when the radial-equilibrium equation was used to find the streamfunction distribution at that station. It is implied here that the numerical iteration procedure *does* converge. It is the author's experience that it does converge if (a) a solution exists (a solution does not exist, for example, if a specified flow area is smaller than that needed to choke the flow) and (b) adequate damping (limitation on change from one iteration to the next) is provided. When the station spacing is small compared with the annulus height, large damping is necessary in order to prevent divergence of the iteration procedure.

The author cannot cite specific comparisons of this theory with experiment, nor does he wish to imply that exact comparisons exist but cannot be disclosed. However, on several occasions, measured hub and casing pressures have been compared with design values with satisfactory agreement, although this is not always the case. Of course, frictional effects are not completely accounted for, nor are leakage flows and regions of flow separation, so some deviations are to be expected.

Dr. Schilhansl is correct in pointing out that flow properties do not vary linearly in the  $\theta$ -direction, and his example demonstrates this. In the main body of the paper, leading up to equation (50), linearity was not assumed. Later, in order to evaluate the  $G$ -functions, it was necessary to assume some form of variation with  $\theta$ , and the linear variation was chosen for simplicity. Since the  $G$ -functions evaluated by this means turned out to be small it is hoped that this assumption will not lead to serious errors, although there is no proof of this without further investigations.

It is not clear to the author why the theory should be limited to so-called "impact-free" operation. Cascade tests have shown that airfoils can operate over a moderate range of incidence angles with good performance, presumably without significant boundary layer separation. Velocity peaks of moderate magnitude do occur, but the present analysis has at least the potential for including their effects. It would be worthwhile to evaluate the  $G$ -functions for such a case using actual flow distributions to see if they then become significant, but the author has not carried out such an analysis.