Cosmic ray acceleration in pulsar-driven supernova remnants: the effect of scattering

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ABSTRACT

In an earlier paper, we showed that cosmic rays (CR) might be accelerated to very high energy (> 10^{15} eV) as they drift through the ordered electric and magnetic fields expected in a supernova remnant driven by a rotating pulsar. We ignored scattering by magnetic fluctuations. Here we show that strong scattering inhibits the acceleration process, but that weak scattering increases the maximum CR energy.

Key words: acceleration of particles – scattering – shock waves – pulsars: general – cosmic rays – supernova remnants.

1 INTRODUCTION

A good case can be made that cosmic rays (CR) with energies up to 10^{15} eV are diffusively accelerated at shock fronts surrounding supernova remnants (SNRs) (Axford, Leer & Skadron 1977; Krymsky 1977; Bell 1978; Blandford & Ostriker 1978; Drury 1983), but that CR cannot be accelerated to energies above this because the ratio of the diffusion length to the radius of the SNR or the acceleration time to the SNR lifetime is too large (Lagage & Cesarsky 1983). Nevertheless, the shape of the CR spectrum suggests a Galactic origin at energies above 10^{15} eV (Wdowczyk & Wolfendale 1989).

It was suggested by Bell (1992, hereafter Paper I) that CR might be accelerated beyond 10^{15} eV in SNRs driven by pulsars. On general grounds (Hillas 1984) this is feasible, because the magnetic fields and characteristic velocities are both higher than in other SNRs. The large magnetic field originates at the surface of the neutron stars, and the pulsar wind is driven to relativistic velocities by rapid rotation of the star. The reader is referred to Paper I for a detailed description of our model. A related model has been proposed by Berezhko (1991, 1994).

In Paper I we took the Crab nebula as an example. We proposed that CR might be accelerated to energies approaching 10^{15} eV at the outer shock where the SNR encounters the interstellar medium (ISM), and then in a second stage, which was the subject of Paper I, accelerated to higher energies within the nebula where the initially relativistic stellar wind slows to match the expansion velocity of the outer edge of the SNR. The stellar wind and the undisturbed ISM are separated by a shell of swept-up interstellar plasma. A CR accelerated by the first stage at the outer shock is available for second stage acceleration if it diffuses through the shell into the stellar wind. Once inside the wind, it can drift around in the intranebular electric and magnetic fields and gain energy, as described in Paper I. Within the wind the radial flow of thermal plasma produces a \(-v \times B\) electric field, which results in a potential difference between the pole (defined by the rotation axis of the neutron star) and the equator of the nebula. CR gain energy from this electric field. We calculate the energy gain by tracing CR trajectories through the nebula. It was assumed in Paper I that the magnetic field follows the pattern outlined in Rees & Gunn (1974) and Coroniti (1990), and that the wind meets an inner termination shock at around one-tenth of the radius of the SNR. At the inner shock, the wind is slowed to one-third of the speed of light, and then decelerates more gradually beyond that (velocity \(\propto\) radius^{-2}) to match the expansion velocity of the outer edge of the nebula. The magnetic field is azimuthal about the pole, with an amplitude varying as \(\sin^2 \theta \cos \theta\), where \(\theta\) is the angular distance from the pole. The electric field at any point is perpendicular to both the magnetic field and the radius, as shown in Fig. 1(a). The spatial dependence of the field assumed both here and in Paper I is

\[
B = \begin{cases} 
(B_0/3)(R/R_n)^{-1}(R_{\text{shock}}/R_n)^2 \sin^2 \theta \cos \theta, & 0 < R < R_{\text{shock}} \\
B_0(R/R_n) \sin^2 \theta \cos \theta, & R_{\text{shock}} < R < R_{\text{sh}, n} 
\end{cases}
\]

\[
E = (cB_0/3)(R/R_n)^{-1}(R_{\text{shock}}/R_n)^2 \sin^2 \theta \cos \theta, & 0 < R < R_{\text{sh}, n}
\]

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inner shock
rotation axis
outer edge of nebula

Figure 1. (a) Schematic representation of the structure of the nebula and the magnetic and electric fields; (b) contour plot (contour interval $2 \times 10^{-5}$ G) of the magnitude of the azimuthal magnetic field in the region of the dotted box (dimension $1.5 R_{\text{shock}}$) in (a).

where $R$ is the distance from the neutron star, $R_{\text{shock}}$ is the radius of the inner (termination) shock, and $R_n$ is the radius of the nebula. Fig. 1(b) plots the magnitude of the magnetic field in the region of the inner shock. Suitable parameters for the Crab nebula are $R_n = 5 \times 10^{16}$ m and $R_{\text{shock}} = R_n/10 = 5 \times 10^{15}$ m. Our choice of $B_0 = 2.6 \times 10^{-3}$ G gives a maximum magnetic field of $10^{-3}$ G at the edge of the nebula, and a maximum magnetic field of $10^{-4}$ G at the inner shock, which accords with observation (Swinbank & Pooley 1979). Depending upon the sign of $B_0$, a proton gains energy as it progresses either from the pole to the equator or from the equator to the pole. We choose to examine the case in which a proton gains energy by moving from the pole to the equator. The energy gain is

$$\Delta E = e \int E \cdot d\vec{B} = 4 \times 10^{35} \text{ eV},$$

although a realistic value of $\Delta E$ might be somewhat higher (Paper I).

The magnetic field gradients around the pole cause a VB drift, which carries the CR deep into the nebula. If the CR penetrate far enough into the nebula, they pass upstream of the inner termination shock of the stellar wind before recrossing the shock and being carried back to the outer edge of the nebula with the thermal plasma by the $E \times B/\beta^2$ drift. Some sample trajectories are shown in Fig. 2, which is reproduced from Paper I. The CR trajectories are tracked in three dimensions but projected on to the $(R, \theta)$ plane in Fig. 2 (and similarly in Fig. 4), where $R$, $\theta$, and $\phi$ are the usual spherical coordinates, and the magnetic field is in the $\phi$ direction. The symmetry of the fields allows us to place reflecting boundaries for the CR trajectories at $\theta = 0$ and $\pi/2$.

In Paper I, we found an energy gain which comes in two parts. First, the single encounter with the inner termination shock increases the CR energy in a manner which is analogous to shock drift acceleration (e.g. Jones & Ellison 1991). Secondly, the CR pitch angle is such after recrossing the shock that the CR gain energy adiabatically through magnetic field compression as the shocked stellar wind slows to match the expansion speed at the edge of the nebula. Both these processes can be considered in terms of the CR gaining energy from the electric field. CR which do not penetrate to the inner shock still gain energy from the second process of magnetic field compression.

Drift acceleration at the inner shock is enhanced by the existence of the magnetic null around the pole. CR passing through the inner termination shock near the null can penetrate a large distance upstream before the relatively weak polar magnetic field deflects them and they are swept back.
through the shock. In passing a long way upstream they have more time to drift in the plane of the shock and thereby gain more energy. This process has been analysed in more detail by Lucek & Bell (1994).

In the concluding section of Paper I, it was remarked that the calculations assumed a magnetic field which was perfectly free from small-scale fluctuations which might scatter the CR. Magnetic field is rarely perfectly uniform, and the consequent scattering, notably in pitch angle, might strongly affect the acceleration process. The purpose of this present paper is to include scattering in the model, and to determine whether it enhances or hinders acceleration.

2 THE EFFECTS OF SCATTERING

It is immediately clear that scattering will hinder the adiabatic acceleration as the CR pass out through the nebula in the compressing field. The density of the background plasma remains approximately constant during the outward pass beyond the inner shock. It is compressed in the radial direction as the plasma slows to meet the outer edge of the nebula, but it expands in the other directions perpendicular to the radius because of the increase in the surface area $4\pi R^2$ of each expanding shell. CR with motions predominantly in the $\phi$ direction parallel to the azimuthal magnetic field experience an expansion and are adiabatically decelerated. CR with motions predominantly in the $(R, \theta)$ plane perpendicular to the magnetic field experience a compression and are adiabatically accelerated. When averaged over an isotropic CR distribution, the two effects cancel out and there is no net energy gain or loss. In Paper I, the CR passed into the nebula around the pole and conservation of CR angular momentum about the pole meant that their azimuthal velocity $v_{\phi}$ was small during the outward pass. Consequently, their motion lay predominantly in the $(R, \theta)$ plane, and their energy increased instead of decreasing or staying constant. A small amount of CR scattering by field fluctuations would be sufficient to isotropize the distribution and remove any net energy gain. The required amount of angular scattering is so small that it must be virtually certain that this part of the acceleration process is ineffective.

If scattering has a strong effect on the outward pass of the CR, it may also have a strong effect on the inward pass. The outward $E \times B/B^2$ drift tends to carry the CR at the velocity of the background plasma. The CR make headway against the wind through the VB drift which carries the CR inwards. Immediately outside the shock the wind velocity is $c/3$. The VB drift has a similar magnitude in the opposite direction and allows a net inward drift if the azimuthal velocity $v_{\phi}$ of the CR is small, i.e., CR which have velocities nearly perpendicular to the magnetic field can drift inward to the inner shock. However, CR are also subject to an outward curvature drift because of their azimuthal motion along the curved field lines. If a CR velocity is not predominantly perpendicular to the magnetic field, the combination of the outward curvature and $E \times B/B^2$ drifts wins over the inward VB drift, and the CR cannot penetrate the inner shock. The overall consequence is that for a CR to penetrate the inner shock its pitch angle must be such that its azimuthal velocity is small throughout the time that the VB drift is carrying it against the strong plasma wind immediately outside the inner shock. In the absence of scattering, the pitch angle can remain at the required value, but scattering will cause the pitch angle to vary, making it likely that at some stage during the process the outward drifts will dominate and the CR will not reach the inner shock. The effectiveness of the acceleration will be reduced if the scattering is sufficiently strong. Weak scattering may have little effect.

The previous two paragraphs have outlined ways in which scattering reduces acceleration. However, there is a process by which scattering can be expected to increase acceleration. The main acceleration to high energy takes place during the interaction with the inner shock. As can be seen in trajectory D in Fig. 2, the CR moves in the $\theta$ direction while passing upstream of the shock, and consequently gains energy from the electric field ($E = -v \times B$). If the CR undergoes scattering whilst upstream it can recross the shock at a larger value of $\theta$ and hence gain more energy. Furthermore, scattering may allow the CR to pass upstream of the shock more than once, which will further increase the energy. As the scattering increases, the acceleration process begins to take on the character of diffusive shock acceleration, and large energy gains at the shock become possible.

Hence there are at least two identifiable processes by which scattering inhibits acceleration, and one identifiable process by which scattering enhances acceleration. We investigate the competition between these processes by calculating CR trajectories through the nebula with scattering. The trajectories are integrated numerically for a random sample of particles using the numerical method described in Paper I.

3 THE SCATTERING MODEL

We assume that CR are scattered by small fluctuations in the magnetic field. We assume that in the frame moving with the background plasma there is no associated electric field; the scattering is elastic in the wind frame. The CR trajectory is Lorentz-transformed into this frame for application of the scattering algorithm. Upwind of the inner shock we assume a wind Lorentz factor of 10. We expect the magnitude of the fluctuating field $\delta B$ to be smaller than, or at most comparable with, that of the local underlying field $B$. Consequently, the fluctuating field cannot scatter the CR through a large angle during one Larmor gyration. We therefore describe the scattering process in terms of a series of small-angle scatters. At each time-step during the trajectory integration, we call a subroutine which scatters the CR velocity through a small random angle. We assume that the direction of scatter is uncorrelated between successive time-steps. The magnitude of the angular scatter $\psi$ is chosen such that

$$\langle \psi^2 \rangle = \frac{\omega_k}{\Delta t}$$

(3)

where $\Delta t$ is the time-step, $h$ is a fixed constant, and $\omega_k$ is the CR gyrofrequency. This makes the scattering frequency proportional to the gyrofrequency, which seems reasonable since the fluctuating magnetic field can be expected to be large where the background field is large, and small where the background field is small. $h$ can be identified as the Hall parameter $\omega_k \tau$, where $\tau$ is a scattering time.
In practice, the scattering may be considerably more complicated, since the magnetic fluctuations are probably not spatially uncorrelated. The opposite assumption of occasioned large-angle scattering on a time-scale less than a gyration time is unlikely, since this would require a highly localized fluctuating magnetic field with a magnitude exceeding that of the background field. Nevertheless, Zachary and coworkers (Max et al. 1986; Zachary & Cohen 1986; Zachary et al. 1986) have shown that scattering by magnetic fluctuations exhibits some aspects of large-angle scattering when viewed on a time-scale larger than one gyropериод. We adopt the small-angle scattering model, because it is the simplest physical model which seems realistic.

We investigate the effect of scattering by choosing a value of $h = \omega_0 \tau$ and then integrating the trajectories of many particles and compiling statistics. The scattering is monitored to make sure that the cumulative effect gives the required $\omega_0 \tau$, and that the correct conservation relations are obeyed for elastic scattering in the wind frame.

The Bohm diffusion constant (Chen 1984) is known to apply very approximately to the spatial diffusion of a thermal plasma across magnetic field in a variety of early laboratory experiments. Consequently, we choose this as our reference point for the strength of scattering. The Bohm constant is

$$D_B = \frac{kT}{10eB} \frac{r_x^2 \omega_x}{24}, \quad (4)$$

where $T$ is the plasma temperature, and $r_x$ is the gyroradius of a particle with a velocity $v = \sqrt{3kT/2m}$. This gives the same spatial diffusion across the magnetic field as equation (3) if the Hall parameter $h = \omega_0 \tau$ is equal to 12. Hence we choose $\omega_0 \tau = 12$ as our reference scattering strength, corresponding notionally to Bohm diffusion. Of course, the Bohm diffusion coefficient is only a rough guide to the expected scattering. Moreover, it refers to a non-relativistic Maxwellian distribution rather than the relativistic non-thermal CR distribution.

4 INJECTION AT 4 TIMES THE SHOCK RADIUS

In Paper I, we used the drift approximation to calculate the outward CR trajectory when it had passed well beyond the inner shock. In that part of the trajectory the gyroradius is much smaller than the nebular radius, and it is sufficient to calculate the drift motion of the guiding centre; there is no need to calculate the detailed gyrations of the particle. We cannot do this easily when there is scattering, because the motion is more complicated. Hence, in this paper, we calculate the details of the trajectory and do not make the drift approximation. However, it would be computationally expensive to follow the detailed CR trajectory through the outer part of the nebula, and we therefore calculate the motion only at radii less than 4 times the radius $R_{\text{shock}}$ of the inner shock. In Paper I, we injected CR at 10 times the inner shock radius, since this is thought to be the ratio of the nebular radius to the shock radius (Rees & Gunn 1974; Kennel & Coroniti 1984a, b). In this paper, we inject them at $4R_{\text{shock}}$ and follow their trajectories until they reach that radius again. This is equivalent to assuming that the nebula radius is $4R_{\text{shock}}$. Unless the scattering is extremely weak, there would be little energy gain beyond $4R_{\text{shock}}$. It is also within this inner region that the CR experience most difficulty in entering the nebula, since the outward flow velocity of the background plasma is greatest there. Hence an examination of trajectories within this region should give us the information to assess the effect of scattering on acceleration.

We also reduce the computational cost by terminating the calculation of the outward part of a CR trajectory if the CR is well outside the inner shock and its gyroradius is very much smaller than the distance from the inner shock. The energies of these low-energy CR would not change significantly beyond the point of termination.

The CR were injected uniformly (with area weighting) and randomly around the rotation axis in the angular range $0 < \theta < 0.1 \, \text{rad.}$ The $VB$ drift at $\theta > 0.1 \, \text{rad}$ is not sufficient to carry the CR far into the nebula, and they are ejected with little energy gain. The CR are initially placed slightly within the outer radius at $0.98 \times 4R_{\text{shock}}$ so that they do not immediately exit the nebula within the first gyro-orbit. The pitch angle and gyrophase are chosen randomly from an isotropic distribution. The initial CR energy is 0.01 times the electric potential difference $\Delta E$ between the pole and the equator, i.e., its energy would increase by a factor of 100 if it passed from the pole to the equator and there were no scattering. In Paper I, the injection was additionally weighted by the inward drift velocity to represent the expected inward flux, but we do not include the velocity weighting here, since the inward flux is determined by diffusion as well as systematic drifts. Following injection, the CR are followed through the nebula until their radius exceeds $4R_{\text{shock}}$ (or until the trajectory is terminated as described above), at which point their energy is recorded and a spectrum compiled.

Fig. 3 presents histograms of the CR spectrum for various $\omega_0 \tau$. The number of CR sampled is $10^6$ in the case of no scattering ($\omega_0 \tau = \infty$), and $4 \times 10^8$ in the other cases. The bins are logarithmically spaced, and the number of CR in each bin is scaled by being multiplied by the square of the energy. This means that if the CR differential spectrum were $f(E) \propto E^{-3}$, the histogram would be flat. The histograms show that there is very little acceleration when $\omega_0 \tau = 12$ or 3.80. The isolated peak at $E/E_0 \approx 10$ for $\omega_0 \tau = 3.80$ corresponds to a single CR calculated to gain this energy. For these small values of $\omega_0 \tau$ the scattering is strong and the CR distribution is isotropized in a few gyrations about the magnetic field. Even near the pole, the $E \times B/B^2$ and curvature drifts dominate the inward $VB$ drift, and all except a very few CR are ejected from the nebula without penetrating to the inner shock. Consequently, the acceleration process is inefficient. However, when the scattering is weaker, $\omega_0 \tau = 37.95$, acceleration is much more efficient. In fact, it is more efficient than in the total absence of scattering, $\omega_0 \tau = \infty$. When $\omega_0 \tau = 37.95$, the scattering is not sufficient to stop substantial numbers of CR penetrating the inner shock by the $VB$ drift, and those that do penetrate the inner shock can have their energy gain increased by scattering, as discussed in Section 2. A few CR even have energy gains greater than 100, which would be the absolute maximum in the absence of scattering. A small amount of diffusion...
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Figure 3. Logarithmic histogram of the spectrum of CR injected at 4R_{shock}. The horizontal axis is the ratio E/E_0 of the final (E) to the initial (E_0) CR energy. The height of each box in the histogram is the fraction of CR with final energy E multiplied by (1.6E/E_0)^2. The histogram would be flat if the differential CR spectrum were proportional to E^{-3}.

Figure 4. The trajectory of a CR injected at 4R_{shock} and projected on to one quadrant in the (R, \theta) plane. \omega_\tau = 38. The energy increases by a factor of 109.6. The pole is vertical.

5 INJECTION FROM WITHIN THE NEBULA

In Paper I, and in this paper, the CR have been injected from outside the inner shock. It may alternatively be possible for CR to be injected from inside the inner shock, possibly originating at the surface of the neutron star or from slowly moving debris from the supernova or pre-supernova. A CR source within the inner shock might be more efficient, because it would avoid the difficulty of the CR
penetrating deep into the nebula against the outward plasma flow. However, other difficulties arise. If the neutron star itself were to be the CR source, the CR would be cooled adiabatically as the plasma expands away from the star. In the absence of heating mechanisms, the CR would arrive at the inner shock with a Lorentz factor equal to that of the wind and travelling with the wind radially away from the star. The Lorentz factor of the wind $\gamma_{\text{wind}}$ might be $10^5$ (Wilson & Rees 1978) or even $10^6$ (Kennel & Coroniti 1984b), in which case a proton would have an energy of $10^{20}$ or $10^{21}$ eV, but further acceleration would be unlikely, since the CR is travelling radially through the inner shock and a return to the shock is improbable.

Alternatively, a less anisotropic (not radially beamed) source of protons closer to the inner shock could be accelerated to high energy by the shock. They would be picked up by the wind and accelerated by the fields to a Lorentz factor of the order of $\gamma_{\text{wind}}$, but adiabatic cooling would not have time to impose a purely radial motion. The most favourable likely source would emit CR isotropically in the nebular rest-frame (i.e., not moving with the wind). If the CR are emitted with a Lorentz factor $\gamma_{\text{source}}$ pick-up by the wind or an encounter with the inner shock could then increase the energy by a further factor $\gamma_{\text{wind}}$, yielding CR which already have very high energies. Further diffusive acceleration by the shock would be a possibility.

If there is a plentiful source of ions inside the inner shock (e.g., from the neutron star) such that the ions modify the wind structure when they encounter the inner shock, then our calculation becomes invalid. A self-consistent kinetic description, such as that used by Arons & Tavani (1994) and Gallant & Arons (1994), is then necessary. Arons & Tavani show that ions participate in the shock dissipation process. A partially thermalized population of ions is formed with a characteristic Lorentz factor of the order of $\gamma_{\text{wind}}$. This may provide an ideal seed of energetic particles for acceleration by the process discussed in this paper.

We experimented numerically with various CR sources within the inner shock, and found that the spectrum after acceleration was strongly dependent on the position and isotropy of the source. Acceleration of CR injected in the heart of the nebula has the potential to produce very high-energy CR, but a credible theory depends upon the identification of a suitable CR source.

6 CONCLUSIONS

We find that strong scattering at a level notionally equivalent to the Bohm rate inhibits efficient acceleration of CR injected from outside the inner shock. However, if the scattering frequency is a few times less than this, then CR are efficiently accelerated. Furthermore, they can reach a higher energy than that possible in the absence of scattering. Efficient acceleration to very high energies of CR injected from within the inner shock may be possible, but it depends heavily upon the existence of a CR source with appropriate characteristics.

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