On the ephemeris of the pulsating hydrogen-deficient star V652 Her

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ABSTRACT
Some new timings of maxima of the pulsating hydrogen-deficient star V652 Her (BD + 13° 3224) obtained during 1991–95 are reported. The data show that the ephemeris for times of maxima can no longer be represented by a cubic polynomial, and that a quartic term is necessary. The star could have brightened by up to 0.008 mag over the last decade, although this is not firmly established (the apparent change could also be due to small zero-point effects, for example). The shape of the light curve appears essentially unaltered over the same time interval; any changes are less than ~ 0.003 mag in size.

Attempts have been made to understand the changing ephemeris in terms of changing stellar radius (R), mass (M) and pulsation constant (Q). A satisfactory match to the observations is obtained by a function including the first three derivatives of radius, \( \dot{R} \), \( \ddot{R} \) and \( \dddot{R} \). The addition of a small mass-loss term does not significantly improve the fit. With the available data we cannot rule out (or distinguish between) small values of \( M \) or \( Q \), but the values derived for the radius derivatives should stringently constrain pulsation models for V652 Her, and suggest extremely rapid evolution not yet accounted for in non-linear radial pulsation calculations.

Key words: stars: evolution – stars: individual: V652 Her – stars: oscillations – stars: variables: other.

1 INTRODUCTION
The blue star BD + 13° 3224 (= V652 Her) was discovered to be hydrogen-deficient by Berger & Greenstein (1963) and shown by Landolt (1975) to be variable with a very short period near 0.108 d. An atmospheric analysis by Hill et al. (1981) derived \( \log L / L_\odot = 3.1 \pm 0.2 \), \( \log R / R_\odot = 1.6 \pm 0.2 \), \( T_{\text{eff}} = 26500 \pm 2000 \) K and a hydrogen abundance of \( \sim 1-2 \) per cent by number, very hydrogen-deficient but not in the 'extreme Helium' (eHe) star class (see also Jeffery, Heber & Hill 1986). Lynas-Gray et al. (1984) determined a mass of \( 0.7 M_\odot \), and radial velocity studies combined with photometric data show that the star is a radial pulsator (Hill et al. 1981; Jeffery & Hill 1986).

Fundamental developments in our understanding of V652 Her have resulted from the extensive new opacity calculations. Saio (1993, 1994) showed that new opacities (Rogers & Iglesias 1992) allow driving of the pulsations via the kappa-mechanism by the iron opacity peak near \( 2 \times 10^5 \) K. (Saio also predicted that the same mechanism should produce pulsations in LSS 3184 – this was recently verified by Kilkenny & Koen 1995.) Fadeyev & Lynas-Gray (1995, 1996) have used opacities from the Opacity Project (Seaton et al. 1994) in non-linear pulsation calculations to model the radial velocity and luminosity variations – results which are in remarkably good agreement with the observations. They find that the iron peak ('Z bump') opacities are driving fundamental radial pulsations in a \( 0.72 \pm 0.23 M_\odot \) star.

What makes V652 Her particularly interesting, to our knowledge uniquely so, is the very fast rate of change observed in the pulsation period. In his paper describing the discovery of light variations, Landolt (1975) used 10 timings of maxima distributed over 214 cycles to determine a period of \( 0.107995 \pm 0.000001 \) d. Kilkenny & Lynas-Gray (1982) were able to tie the discovery observations to later observations, and to demonstrate that the period was decreasing at a rate of about \( 46 \times 10^{-10} \) d per cycle. Kilkenny & Lynas-Gray (1984) used 45 maxima distributed over 30 000 cycles to suggest that the period decrease rate was itself decreas-
ing; the necessity for a cubic ephemeris was confirmed, and the ephemeris was improved in subsequent papers by Lynas-Gray & Kilkenny (1986), Kilkenny (1988) and Kilkenny & Marang (1991). The last paper used 98 timings of maxima, including the Landolt discovery data, distributed over more than 57 000 cycles.

In this paper, we present measurements of maxima of V652 Her made since the Kilkenny & Marang (1991) note, show that a cubic ephemeris is no longer sufficient to represent the pulsational maxima of this peculiar star, and attempt to interpret the ephemeris variation in terms of secular changes in the mean stellar radius.

2 OBSERVATIONS

As with previously reported SAAO data (see the Introduction for references), all observations were made using photoelectric photometers on the 0.5- and 1.0-m telescopes at the Sutherland site of the South African Astronomical Observatory (SAAO). All data were acquired using 30-s integrations through a single filter, usually a Johnson B filter, but occasionally a V filter. Local comparisons were not used for most measurements of maxima, because a good sampling rate was necessary to define each maximum; usually no attempt was made to reduce the data to a standard system, since only the timing of the maximum in each case was required. Where there was any indication that conditions were less than good, the data were rejected. The new results are listed in Table 1, which gives the year of observation, HJD of maximum, assigned cycle number (based on the ephemeris given by Kilkenny & Marang 1991), and telescope and filter used for the observations.

3 THE EPHEMERIS

When the quadratic term in the ephemeris of V652 Her was discovered (Kilkenny & Lynas-Gray 1982), an error of one cycle was made bridging the ~17 500 cycles between the Landolt (1975) data and the earliest SAAO results (see Kilkenny & Lynas-Gray 1984). Since then, a cubic ephemeris has been sufficient to predict maxima of V652 Her; more data have enabled a more precise ephemeris to be derived, but the cycle numbers have not needed to be changed and we are confident that these are correct. Continuing observations, particularly during the 1994 and 1995 seasons, have begun to show a deviation from a cubic fit, and a quartic fit is now necessary to obtain residuals comparable with the observational errors. A least-squares fit to the data using the expression

$$T_{\text{max}} = T_0 + nP_0 + n^2k_1 + n^3k_2 + n^4k_3 \quad (1)$$

(where \(n\) is the cycle number) gives

$$T_0 = 2216.804\,05 \pm 0.000\,34 \, d,$$

$$P_0 = 0.107\,993\,19 \pm 0.000\,000\,08 \, d,$$

$$k_1 = (-44.953 \pm 0.056) \times 10^{-10} \, d,$$

$$k_2 = (+4.04 \pm 0.13) \times 10^{-15} \, d,$$

$$k_3 = (-0.908 \pm 0.091) \times 10^{-20} \, d.$$

Fig. 1 shows the residuals from least-squares linear, quadratic and cubic fits to the Landolt and SAAO timings of maxima. For the first time, it is clear that a cubic fit is insufficient, and a higher order polynomial representation is necessary. Polynomials of fifth and sixth order do not give much better fits than the quartic representation given above; further discussion of the residuals from the quartic fit will be given in Section 5.

The quadratic \((k_1)\) rate of change of period is remarkably fast — so fast that we can check the quadratic term by deriving an 'instantaneous' period from observations in each season. Fig. 2 shows a plot of this period against cycle number; the gradient of this plot gives a value \(dP/dn\) for

<table>
<thead>
<tr>
<th>Year</th>
<th>HJD</th>
<th>cycle no.</th>
<th>Tel.</th>
<th>Filter</th>
</tr>
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<tr>
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<td>9189.304</td>
<td>64730</td>
<td>1.0m</td>
<td>V</td>
</tr>
<tr>
<td>1994</td>
<td>9476.496</td>
<td>67403</td>
<td>1.0m</td>
<td>B</td>
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<td>B</td>
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<tr>
<td></td>
<td>9480.577</td>
<td>67441</td>
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<tr>
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<td>B</td>
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<tr>
<td>1995</td>
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<tr>
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<tr>
<td></td>
<td>9947.289</td>
<td>71786</td>
<td>0.5m</td>
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Figure 1. \((O - C)\) residuals for V652 Her for linear, quadratic and cubic least-squares fits to the timings of maxima. The points near zero cycle number are the Landolt (1975) data. Note the difference in abscissae scales.
The pulsating hydrogen-deficient star V652 Her

4 THE LIGHT CURVE

As the period of V652 Her is changing so quickly, it would be interesting to know whether the light curve shows any change; this is not so easy to determine, because we have not generally observed local standards or transformed to a standard system. Lynas-Gray & Kilkenny (1986) have published a very (internally) precise $B$ light curve for the star, obtained in 1985 and differentially corrected to one of Landolt's (1975) comparison stars (BD +13° 3220) using $B=9.056$ from $UBV$ photometry by Hill et al. (1981). The observations were made with 60-s continuous integrations and only occasional breaks to measure the local comparison. A similar light curve was obtained in 1994, but using 30-s integrations and corrected to the other Landolt comparison star, BD +13° 3223 (only one comparison star was used in each case to maximize observing time on V652 Her; unintentionally, these were not the same for each run).

These data were reduced in the same way as the earlier data, averaged in pairs to give 60-s data points, and then corrected to the brightness at maximum is the same (see text for discussion). Both light curves have been phased using the appropriate 'instantaneous' period and represent a complete ~3-h run (i.e. no data point is repeated).

\[ \frac{dP}{dn} = -8.31 \times 10^{-13} \, \text{d}, \]

which is in very good agreement with the value for $2k_1$ obtained from a quadratic least-squares fit to the timings of maxima, namely $-8.3 \times 10^{-10} \, \text{d}$. Since the quadratic term is well established in this way, and since the differences in the residuals from the cubic solution between successive years are only a small fraction of the period, we can have some confidence that the solution is correct (in the sense that we have probably not lost or gained any cycles in our numbering – see also Kilkenny 1988). In fact, in Fig. 2 it is just possible to see the curvature which is evidence for the cubic term; the earliest and latest data lie significantly above the line representing the quadratic term. Grouping the data from Fig. 2 in various ways (the scatter in the results is quite high) and calculating 'instantaneous' values for $\frac{dP}{dn}$ results in values for $d^2P/dn^2$ around $1.7 \times 10^{-14}$ (with about 10 per cent error) or $k_2=1/6(d^2P/dn^2)=(2.8 \pm 0.3) \times 10^{-15}$, which agrees well with the value $k_2=(2.77 \pm 0.02) \times 10^{-15}$ obtained from a cubic fit to the timings of maxima (i.e., ignoring the quartic term). The ephemeris solution thus seems to be well established, in that the allocated cycle numbers are correct despite the fact that there are gaps in the data which are large compared to the pulsation period.
which occurs as the model is evolved can also reproduce the
discernible. Parameter values obtained were

\[ T(0) = 2193.693 \pm 0.000 \, \text{d}, \]
\[ \Pi(0) = 0.107955 \pm 0.0000081 \, \text{d}, \]
\[ (R/R)_0 = - (5.5498 \pm 0.0058) \times 10^{-8} \, \text{cycle}^{-1}, \]
\[ (R/R)_0 = + (1.463 \pm 0.037) \times 10^{-13} \, \text{cycle}^{-2}, \]
\[ (R/R)_0 = - (1.29 \pm 0.10) \times 10^{-18} \, \text{cycle}^{-3}. \]

Uncertainties cited above are formal least-squares standard deviations. It is interesting to note that the functional form

\[ T(n) = T(0) + \Pi(0) \sum_{i=1}^{n} \left[ 1 + i \left( \frac{R}{R_0} \right) + i^2 \left( \frac{R}{R_0} \right) + i^3 \left( \frac{R}{R_0} \right) \right]^{3/2}, \]

yields a least-squares solution

\[ \left( \frac{M}{M_0} \right) = - (1.88 \pm 0.14) \times 10^{-11} \, \text{cycle}^{-1} \]

when all other parameters are fixed at the above values. Although Fadeyev & Lynas-Gray (1996) have suggested a small mass-loss rate for V652 Her, equation (7) indicates a rate of \( 6 \times 10^{-6} \) solar masses per year, which is much higher than the IUE observations would allow. In addition, applying the mass-loss rate indicated by equation (7) gives no appreciable improvement to the fit obtained using (5). All three fits (to equations 1, 5 and 6) give an rms error very close to 0.0011 d, and can therefore be regarded as giving an equally good representation of the observations and predictions for maxima. It must also be realized that the trend in residuals which the derived \( (M/M)_0 \) tries to represent could equally well be interpreted as a non-zero \( (Q/Q)_0 \), it is not possible to distinguish between these cases with the data presently available.

Fadeyev & Lynas-Gray (1996) used their non-linear radial pulsation calculations, together with ephemeris coefficients by Kilkenny & Marang (1991), to estimate \( (R/R)_0, (R/R)_0, (Q/Q)_0 \), and \( (Q/Q)_0 \); they assumed constant stellar structure parameters (the coefficients in equation 2) for the period 1974 to 1991 over which \( T(n) \) observations were previously published. Data analysed in the present
paper suggest \( \frac{Q}{Q} \) to be at least one order of magnitude smaller than Fadeyev & Lynas-Gray's result of \( \frac{Q}{Q} = 4.8 \times 10^{-9} \) cycle\(^{-1} \); their suggestion, that non-linear radial pulsation calculations be generalized to include stellar evolution, is therefore endorsed because calculated values of \( \frac{Q}{Q} \) would guide the interpretation of the presently available \( T(n) \) observations.

Values of \( \frac{R}{R} \) and \( \frac{R}{R} \) deduced in the present paper were consistent with Fadeyev & Lynas-Gray's (1996) estimates, once differences in \( \frac{Q}{Q} \) and \( \frac{Q}{Q} \) were taken into account. A substantially improved fit to the observations is obtained when \( \frac{R}{R} \) is included in (5), and its value has been estimated here for the first time. As discussed below, an exacting test of new theoretical models of V652 Her could be provided by values of \( \frac{R}{R} \) (\( \frac{R}{R} \) and \( \frac{R}{R} \) taken with their standard deviations) reported here.

Transformation of (5) into the same form as (1) is tedious but straightforward; after some algebra, it is found that, to a (numerical) accuracy of 1 part in 10\(^7\) or better,

\[
P_0 = \Pi(0),
\]

\[
k_1 = 3\Pi(0) \frac{(\dot{R}/R)_0}{4},
\]

\[
k_2 = \Pi(0) \left[ (\dot{R}/R)_0^2 + 2(\dot{R}/R)_0 \right]/8,
\]

\[
k_3 = \Pi(0) \left[ - (\dot{R}/R)_0 + 6(\dot{R}/R)_0 (\ddot{R}/R)_0 + 4(\dddot{R}/R)_0 \right]/64.
\]

For ephemeris coefficients given by (8) to be directly comparable with those obtained by the linear least-squares solution of (1), the original epoch was recovered by returning to standard cycle numbers; this meant that the first nine \( T(n) \) by Landolt (1975) had to be discarded before repeating the non-linear least-squares solution of (5). Substituting results of the second (99-observation) non-linear least-squares solution in (8) gives \( P_0 = 0.107993 \) d, \( k_1 = -44.914 \times 10^{-10} \) d, \( k_2 = 3.97 \times 10^{-15} \) d and \( k_3 = -0.870 \times 10^{-20} \) d; these are consistent, to within formal linear least-squares error limits, with the ephemeris solution presented in Section 3.

Residuals obtained by comparing fit (1) with the observed times of maxima are plotted in Fig. 4. The errors in the timings of maxima have been estimated to be about \( \pm 0.001 \) d, and indeed the scatter for a given season has a typical total range \( \sim 0.002 \) d for all the data later than about 1980 (after which most of the data were obtained with the 1-m rather than 0.5-m telescope). It can be seen in Fig. 4 that the residuals from the quartic solution are comparable with these errors, although fifth- and sixth-order polynomials do make a small improvement, slightly 'flattening' the residuals \( \sim 65000-72000 \) cycles, but not significantly affecting the other residuals. Another possible interpretation is that the residuals appear to exhibit a sinusoidal modulation. The referee of the Kilkenny (1988) paper pointed out that the residuals to the (cubic) solution presented in that paper did not appear completely random, but it was felt that the data available at that time did not merit more detailed treatment. This might still be the case but, whilst acknowledging the somewhat speculative nature of what follows, it is none the less interesting to follow through the possibility of a small sinusoidal term in the residuals.

A linear least-squares fit, to residuals from the fit to (1), of the form

\[
r = r_0 \sin \left[ \frac{2\pi (n + \phi)}{P} \right]
\]

(9)

gives

\[
r_0 = 0.00043 \pm 0.00010 \text{ d},
\]

\[
\phi = -300 \pm 3000 \text{ cycle},
\]

\[
P = 27400 \pm 1800 \text{ cycle}.
\]

The amplitude \( r_0 \) is of the same order as the observational error in \( T(n) \), and so the significance of any sinusoidal trend in the residuals is not high, but, if real, a sinusoidal modulation could be understood in terms of the time delay which

![Figure 4](https://academic.oup.com/mnras/article-abstract/283/4/1349/1071192)
would arise if V652 Her were a component of a binary system and consequently in orbit about the system barycentre. It must be stressed that extensive photometric and spectroscopic monitoring of V652 Her over two decades has not revealed any evidence of binarity (although Kilkenny & Lynas-Gray 1984 did consider binarity as one possible explanation of the ephemeris cubic term); any companion would therefore be expected to have a low mass and luminosity.

The well-studied blue radial pulsator was assumed to be the primary of a binary system and the other component to be an object having both a low mass and a low luminosity; both were considered to have a fixed separation \( a \) by virtue of their orbits (assumed to be circular with radii \( a_1 \) and \( a_2 \)) about the barycentre. An orbital period of 27 400 pulsation cycles (8.1 yr) was obtained from the linear least-squares fit to (9); Kepler’s third law then implies \( a = a_1 + a_2 = 778 \, \text{R}_\odot \) if the secondary mass can be neglected. An amplitude derived from the least-squares fit to (9) implies \( a, \sin i = 0.00043 \) light-day = 16 \( \text{R}_\odot \), where \( i \) is the inclination of the orbits to a plane orthogonal to the line of sight. If the orbital plane lies in the line of sight, then \( \sin i = 1 \) and \( a_2 = 762 \, \text{R}_\odot \), which implies a secondary mass of 0.015 \( \text{M}_\odot \) if Fadeyev & Lynas-Gray’s (1996) mass of 0.72 \( \text{M}_\odot \) is adopted for the radial pulsator. Smaller \( \sin i \) values result in correspondingly larger secondary masses; for example, if \( \sin i = 0.1 \), then \( a_1 = 160 \, \text{R}_\odot \) and the secondary mass would be 0.19 \( \text{M}_\odot \).

An orbital period of 8.1 yr and an orbital radius of 16 \( \text{R}_\odot \) correspond to a radial velocity modulation amplitude, for the blue radially pulsating primary, of 0.03 km s\(^{-1}\); this would not have been detected in any radial velocity monitoring carried out to date. It is therefore clear that the sinusoidal distribution of residuals, seen in Fig. 4, could indicate that V652 Her is a binary; the blue radial pulsator would have to be separated from its low-mass companion by roughly 780 \( \text{R}_\odot \). At least one more cycle of the sinusoidal modulation in residuals needs to be observed before much weight can be attached to the binary hypothesis suggested here.

6 CONCLUDING REMARKS

New timings of maximum light for V652 Her presented in this paper show a departure from the cubic ephemeris, first proposed by Kilkenny & Lynas-Gray (1984), and necessitate the introduction of a quartic term; a new linear least-squares solution then gives an excellent agreement with observation. A non-linear least-squares fit to available timings of maximum light provides estimates of \( (\bar{R}/R)_0 \), \( (\bar{R}/R)_\alpha \) and \( (\bar{R}/R)_\beta \) which successfully account for the ephemeris coefficients obtained by linear least-squares. Further timings of maximum light for V652 Her will be needed for many years (if not decades) to come, so that the ephemeris interpretation presented here can be developed and checked.

Values of \( (\bar{R}/R)_0 \), \( (\bar{R}/R)_\alpha \), and \( (\bar{R}/R)_\beta \) obtained here (together with their standard deviations) will, especially if extended by future observations, provide a stringent test of any theoretical models for V652 Her; they also suggest extremely rapid evolution not yet accounted for in non-linear radial pulsation calculations. Jeffery’s (1984) evolutionary sequences for V652 Her were necessarily computed with opacities now considered to be obsolete (Seaton et al. 1994); a revision is suggested by the successful application of new opacities to modelling pulsation and the less than perfect agreement between observed and predicted values of \( (\bar{R}/R)_0 \) and \( (\bar{R}/R)_\beta \) as already noted (Jeffery 1984; Fadeyev & Lynas-Gray 1996). Non-linear radial pulsation models for V652 Her, which take rapid evolution effects into account, are also needed; these could be carried out using techniques described by Dorfi & Feuchtinger (1991), and should provide \( (\bar{R}/R)_0 \), \( (\bar{R}/R)_\alpha \), and \( (\bar{R}/R)_\beta \) (as well as corresponding derivatives of mass and pulsation constant) for direct comparison with observation.

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