

An efficient method approach to large flowmeter calibration

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ABSTRACT

Flowrate is one of the most important process quantities measured in the water industry. To accurately measure flowrate, the flowmeters have to be calibrated regularly. An efficient method presented herein is used to calibrate flowmeters quickly for ensuring an accurate measurement of flowrate in the water distribution system. It is based on the regularity in open-channel flows that the ratio of the mean to maximum velocity in a given cross section is constant. The maximum velocity can be easily determined on a vertical, which tends to remain invariant with time and flowrate, and can be converted to the mean velocity of the cross section by this constant ratio. The most commonly used method, the velocity–area principle, which determines discharge as the cross-sectional area multiplied by the mean velocity is applied to estimate the flowrate. Then the flowrate is used to calibrate the coefficients of the large flowmeters. The acoustic Doppler velocimeter is used to determine the velocities for determining the flow patterns in the open channel. Those velocities are used to calculate the maximum velocities and flowrates for determining the constant ratio of the mean to maximum velocity. The available data from the Taipei Water Department is used to illustrate the proposed method. The results show it can efficiently, accurately and reliably calibrate the large flowmeters in water distribution systems.

Key words | Acoustic Doppler velocimeter, calibration, entropy, flowmeter, flowrate, maximum velocity

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INTRODUCTION

Accurate flowrate measurement is a basic parameter for evaluating the performance of water treatment and distribution systems. As the value of scarce water resources increases, so does the need for improved flowrate measurements. Large flowmeters are used in the water industry to measure the flowrate through piping. They usually need to be calibrated and higher accuracy levels for flowmeter calibration are increasingly required. Calibration is the heart of flowmeter accuracy (Hope 1994). It is the process used to check or adjust the output of an instrument in convenient units of gradations to provide agreement with the reference standard for ensuring the continual reliability of flowmeters. The flowmeter calibration must be carried out experimentally before it is delivered to a customer.

The coefficient of flowmeter, which compensates for head losses, depends mainly on the geometry of the particular device. As components of the flowmeter age and equipment undergoes changes (e.g. inner linings, voltage references, mechanical stress), critical performance gradually degrades. It will have an effect on the accuracy of the flowrate. Drift cannot be eliminated, but it can be detected and the effect can be minor through calibrating flowmeters regularly.

Flowmeter calibration is commonly referred to as primary flow standard and secondary flow standard (Hardy *et al.* 1999). The primary flow standard, which typically is the most accurate and complicated of the flow standards, refers to a flow-measuring system for estimating flowrate by the application of natural physical parameters

(e.g. mass, weight, time). The most common primary flow standard is a weight-versus-time system. Although the measurement concept of the primary flow standard is simple, it is usually time-consuming and labor-intensive. Such systems for calibrating large flowmeters are not available to flowmeter users and are usually maintained by large corporate or national calibration laboratories. The second flow standard refers to a flowmeter (e.g. Venturi flowmeter, thin-plate orifices) which is usually very simple and has been calibrated itself against a primary flow standard. In this paper, a large flowmeter calibrated by the proposed method which utilizes the primary flow standard measuring velocities and area to determining flowrate is described.

Owing to the characteristics of Taiwan's watersheds (erodible soil, uneven rainfall, steep slopes and high mountains), most of the water runs directly into the sea. Only a very small portion of water is available to be used in the Taipei area. Occasionally, experiences of drought and runoff from storms deplete the water supply. Water supply is always an important issue in Taiwan because of the continuous increase in water demand accompanying economic growth and opposition to the construction of new reservoirs by the heightened desire for environmental and ecological protection. The Taipei Water Department (TWD) provides about 2.6 million cubic meters of water to the Taipei Metropolitan area each day. TWD's water system is supplied by the Nanshi River and the Feitsui Reservoir. This water flow passes through the Chihtan Dam and the Chingtan Dam where 2.70 and 1.08 million m³/d of water, respectively, is diverted via tunnels to the Chihtan, Changhsing and Kungkuan Purification Plants for treatment. After purification the water is distributed to serve about 3.9 million people. Recent trends in industrial water demand have declined, but municipal water demand is continuously expanding. TWD can meet the needs of water demands. No project including modifying existing treatment facilities and constructing new plant is considered after 1996. The treatment capacity has remained the same; however, the water provided by the TWD keeps increasing. There is some evidence that the large electromagnetic flowmeter overestimates the flowrate and needs to be calibrated. The large electromagnetic flowmeter is widely used in the water industry. It is based upon Faraday's law

that any change in magnetic flux through a circuit will induce an electromotive force in the circuit. In the case of magnetic flowmeters, the conductor is the flowing water being measured:

$$E = cBLu \quad (1)$$

where E = the generated voltage proportional to the flowrate by a constant c , B = density of the magnetic field, L = path length and u = velocity of flow in pipe. Many methods are proposed to calibrate electromagnetic flowmeters (Hemp 2001; Michalski 2000); however, they cannot be used to calibrate TWD's electromagnetic flowmeter. Field calibrations are required for the large flowmeters, but it is a difficult task. To meet this calibration requirement, an efficient flowrate measurement method based on probability laws to describe the velocity distribution at a cross section in channels and pipes has been designed and built. It has as its basis the regularity of open-channel flows, i.e. that nature maintains a constant ratio of the mean to maximum velocities for a given cross section (Chen & Chiu 2002). The relation between the mean and maximum velocities is resilient and invariant with time and flowrate (Chen & Chiu 2004). In order to find the constant to efficiently estimate the flowrate, the acoustic Doppler velocimeter (ADV) is applied to measure the flow structure in the cross section. Then the observed flowrates are used to calibrate the large flowmeter.

REGULARITY IN OPEN-CHANNEL FLOW

The efficient method of flowrate measurement requires only a small number of velocity samples. The method attains its efficiency by observing and understanding the physical process and by taking advantage of regularities recognizable in the flow. The method is based on the constant ratio of the mean and maximum velocities (Chiu & Chen 1998). Basically, the flowrate is

$$Q = \bar{u}A = \phi u_{\max}A \quad (2)$$

where \bar{u} = mean velocity of the cross section; A = cross-sectional area and ϕ = constant ratio of the mean and maximum velocities. The mean velocity can be quickly

determined from the maximum velocity, and the maximum velocity is estimated from only one or a few velocity samples on the y axis. The y axis is the vertical on which the maximum velocity of the cross section occurs. The cross-sectional area can be easily estimated from the water depth. Therefore, the mean velocity and cross-sectional area determined by the present method are more efficient and accurate.

Typical velocity distribution equations were based upon the logarithmic law. They were inadequate to describe the flow pattern near either the water surface or channel bed. Moreover those velocity distribution equations cannot be used when the maximum velocity occurs below the water surface. A new velocity distribution equation (Chiu 1989) that can describe the maximum velocity occurring below the water surface is

$$u = \frac{u_{\max}}{M} \ln \left[1 + (e^M - 1) \frac{\xi}{\xi_{\max}} \right] \tag{3}$$

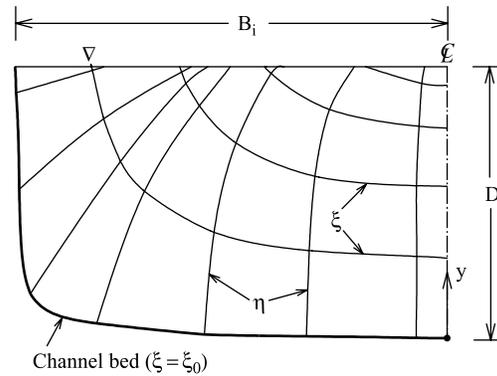
where u = velocity at ξ , ξ = constant value on an isovel as shown in Figure 1 (Chiu 1988), u_{\max} = maximum velocity in a channel section, M = parameter and ξ_{\max} = value of ξ when u is maximum. The η curves shown in Figure 1 are orthogonal trajectories of ξ curves:

$$\eta = \pm \frac{1}{z} \left(|1 - z|^{\beta_i \frac{(D + \delta_y - h)}{(B_i + \delta_i)}} \right) \times \exp \left[z + \beta_i \left(\frac{D + \delta_y - h}{B_i + \delta_i} \right)^2 \frac{y + \delta_y}{D + \delta_y - h} \right] \tag{4}$$

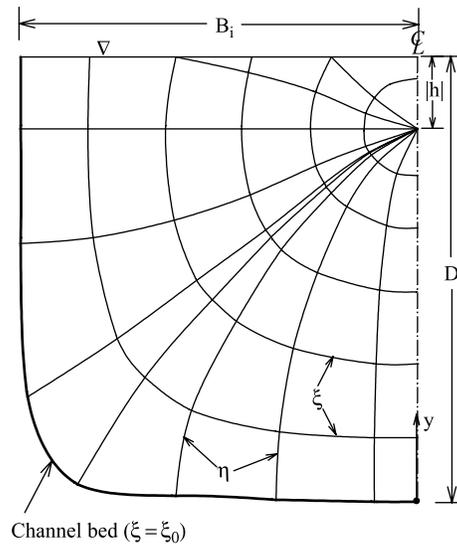
where D = water depth on the y axis, y = vertical distance from the channel bed, h = parameter that indicates the location of maximum velocity, B_i = distance from the y axis to the left or right bank of the channel cross section; z = coordinate in the transverse direction and δ_i , β_i and h are parameters characterizing the velocity distribution of the primary flow. The ξ - η coordinate can be used to model two-dimensional velocity distributions. ξ along the y axis can be expressed as the following function:

$$\xi = \frac{y}{D - h} \exp \left(1 - \frac{y}{D - h} \right) \tag{5}$$

where, if u_{\max} occurs on the water surface, $h \leq 0$. If u_{\max} occurs below the water surface, $h > 0$ and h is the actual



(a) Pattern I ($h \leq 0$)



(b) Pattern II ($h > 0$)

Figure 1 | ξ - η coordinate system: (a) $h \geq 0$; (b) $h < 0$.

depth of u_{\max} below the water surface. If $h \geq 0$, (3) can be rewritten as

$$u = \frac{u_{\max}}{M} \ln \left[1 + (e^M - 1) \frac{y}{D - h} \exp \left(1 - \frac{y}{D - h} \right) \right] \tag{6}$$

When $h < 0$, (3) becomes

$$u = \frac{u_{\max}}{M} \ln \left[1 + (e^M - 1) \frac{y}{D} \exp \left(\frac{D - y}{D - h} \right) \right] \tag{7}$$

Based on the concept of probability, ξ/ξ_{\max} in (3) is the probability of a velocity less than u divided by the total area. Thus

$$\frac{\xi}{\xi_{\max}} = \int_0^u p(u) du \tag{8}$$

where $p(u)$ is the density function of u that must satisfy the following two constraints:

$$\int_0^{u_{\max}} p(u) du = 1 \quad (9)$$

$$\int_0^{u_{\max}} up(u) du = \bar{u} = \frac{Q}{A}. \quad (10)$$

For the problem at hand, the average information content can be expressed by information entropy (Shannon, 1948) as

$$H(u) = - \int_0^{u_{\max}} p(u) \ln p(u) du. \quad (11)$$

To obtain the probability, (11) is maximized subject to (9) and (10) by the method of Lagrange:

$$p(u) = e^{(\lambda_1 - 1) + \lambda_2 u} \quad (12)$$

where λ_1 and λ_2 are coefficients. Substitution of (12) into (10) yields

$$e^{\lambda_1 - 1} = \frac{\lambda_2}{e^{\lambda_2 u_{\max}} - 1} \quad (13)$$

By defining $M = \lambda_2 u_{\max}$ and substituting (13) by (12), $p(u)$ becomes

$$p(u) = \frac{M}{u_{\max}(e^M - 1)} e^{\frac{M}{u_{\max}} u}. \quad (14)$$

Combining (14) and (11) the relation between the mean and maximum velocities is obtained as

$$\frac{\bar{u}}{u_{\max}} = \frac{e^M}{e^M - 1} - \frac{1}{M} = \phi. \quad (15)$$

The ratio of \bar{u} to u_{\max} for a cross section, ϕ , approaches a constant. It is a law of nature that a channel section maintains a constant ratio for the mean to maximum velocities (Chiu 1996). Different channel cross sections have different ratios. Moreover ϕ does not vary with time and flowrate (Chen 1998). The mean–maximum velocity relationship has been used successfully to estimate the discharges of tidal streams and natural irregular channels (Chen & Chiu 2002; Chiu & Chen 2003). Thus, \bar{u} can be

easily and quickly determined by the product of u_{\max} and ϕ in a rectangular channel.

DATA COLLECTION METHODS AND EQUIPMENT

The Chihtan Purification Plant has four purification plants. The large electromagnetic flowmeter needing to be calibrated was installed at the first purification plant, which was designed for an ultimate capacity of 0.5 million m³ per day, to measure the flowrate of raw water. The flowrate was measured in the open channel between the settling basin and the multi-media filter to calibrate the large electromagnetic flowmeter. Figure 2 shows the location of this flowrate measurement. From the sediment basin, the water flowing into eight small flumes was collected into the open channel. There was no place in the purification plant that could be used to measure the flowrate except the open channel.

The cross section of the open channel was 4 m wide and 4 m deep. The average water depths in the open channel were determined by a point gage to obtain the cross-sectional areas. The places to explore the flow pattern are limited. Even the fully developed turbulent flow cannot be found; the velocities were measured in the cross section located 11 m downstream of the eighth flume. However, the water depths were well controlled to change within a very small range to ensure the flow condition was stable when the velocity measurements were made. High accuracy of flowrate can be achieved depending on the velocity distributions in the cross section of an open channel. The uncertainty of the flowrate measurement decreases with increasing numbers of verticals used (Herschy 1999). The velocity distributions, therefore, were measured in eight verticals located at a distance of the wall equal to 0.25 m (v1 site), 0.75 m (v2), 1.25 m (v3), 1.75 m (v4), 2.25 m (v5), 2.75 m (v6), 3.25 m (v7) and 3.75 m (axial vertical, va). The velocities on the vertical were taken at 0.3 m increments from 0.1 m of the depth, as shown in Figure 3. Velocities and temperatures were obtained with a 5 cm downlooking acoustic doppler velocimeter (ADV) which was manufactured by SonTek Inc. (Kraus *et al.* 1994) and mounted on a coordinate meter, allowing the probe to move from one measurement point to another. ADV was based on a pulse-to-pulse coherent Doppler shift to instantaneously measure

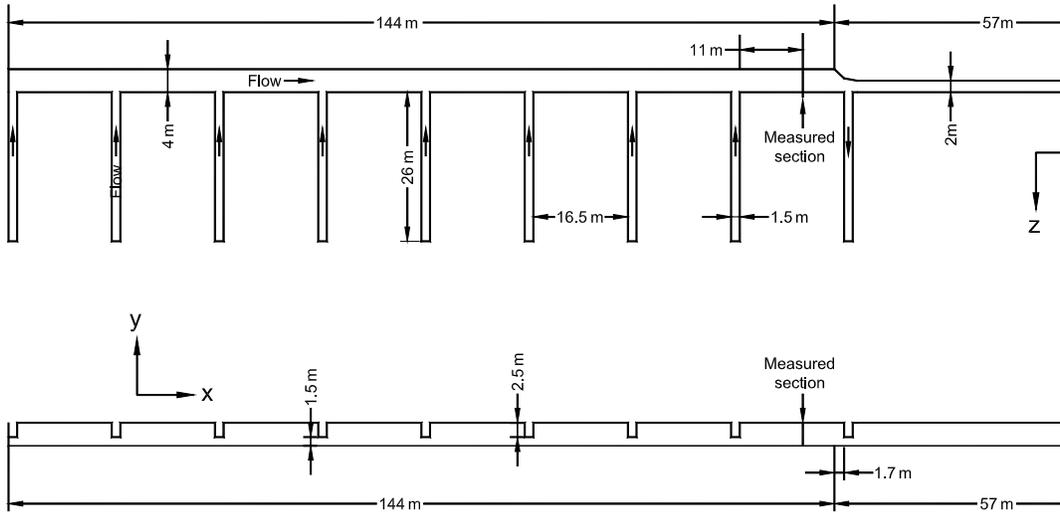


Figure 2 | Layout of study site.

three-dimensional velocity components (Nikora & Goring 1998). The sampling volume was located away from the measuring probe to provide undisturbed measurements.

In order to compute the flowrate in the open channel, the velocity–area principle (Rantz 1982) was used, which divides the cross section into segments by spacing verticals across the open channel. The flowrate is the summation of the products of the segment areas and their respective average velocities:

$$Q = \sum (a\bar{u}_v) \tag{16}$$

where a = an individual segment area and \bar{u}_v = the corresponding mean velocity of the flow normal to the segment.

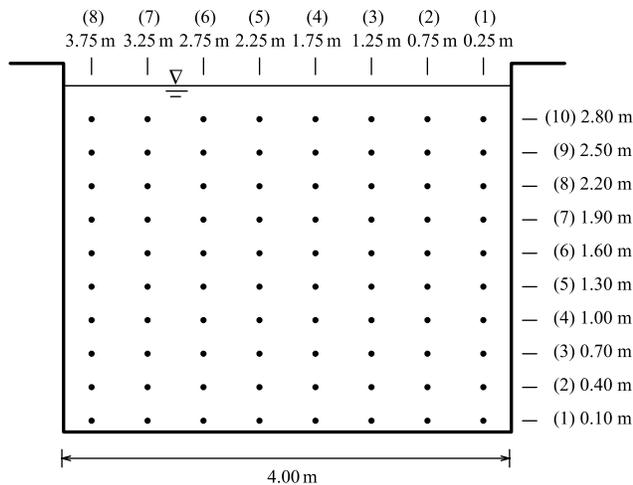


Figure 3 | Position of velocity sampling.

a was already known and \bar{u}_v could be obtained by integrating (6) or (7) with M , h and u_{\max} determined by the method of nonlinear regression.

It may not be easy to find the location of the maximum velocity in an open channel. Fortunately, the velocity data for determining the flowrate can be used to plot the isovels in the cross section, and these isovels reveal the location of the y axis. The y axis is very stable and invariant with time, flowrate and water depth if the channel bed does not change drastically (Chen 1998). Therefore u_{\max} can be easily and quickly determined on the y axis by taking a few velocity samples using new techniques such as acoustic Doppler current profilers (ADCP) using (6) or (7) and the method of nonlinear regression.

COMPARISON OF LARGE FLOWMETER RESULTS WITH ADV RESULTS

Flowrate data collected concurrently with both the large electromagnetic flowmeter and the ADV were compared directly one against the other. The process is repeated at several flowrates and the calibration covered a range from the ultimate capacity attainable to about 30% of the maximum. The flowmeter output was continuously recorded to average out small fluctuations. A total of twelve such matched sets of data existed that included nine sets of data used to find the y axis and ϕ . The other three sets of

data, which only measured the velocity distribution on the y axis, were used to estimate \bar{u} with ϕ and u_{\max} . The transducer is then adjusted based on analysis of the data points.

Figure 4 represents one set of vertical velocity distributions in the open channel. They show the capability of (3) to describe the velocity distribution over the entire depth,

regardless of whether u_{\max} occurs on or below the water surface. Figure 5 shows one of the isovels in the open channel. Unlike the velocity distributions in most open channels with a solid boundary having the maximum velocity occurring below water a distance of 0.05–0.25 of the depth (Streeter & Wylie 1979), the maximum velocity of the open channel occurs very close to the channel bed. It

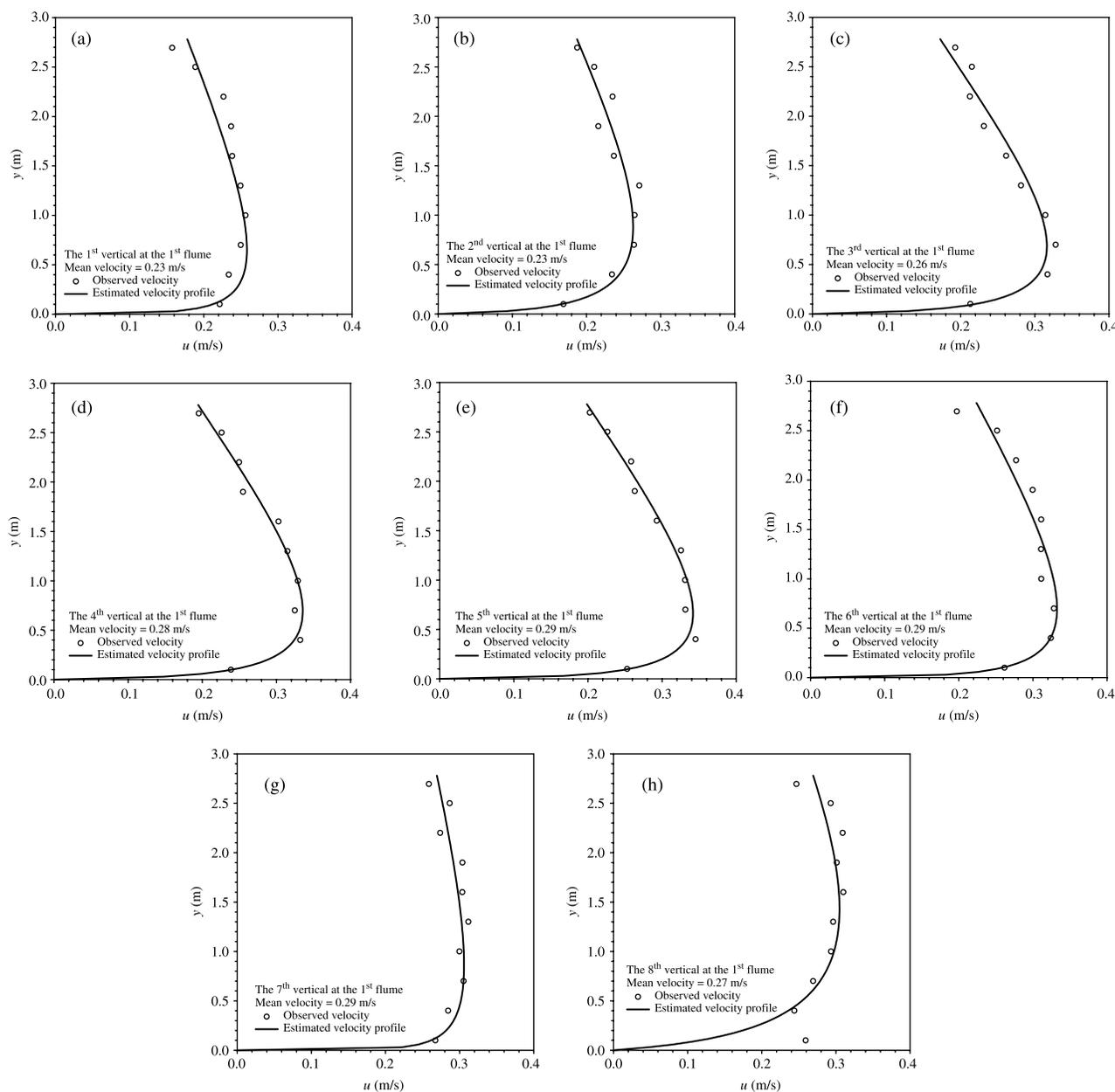


Figure 4 | Velocity distribution: (a) the first vertical; (b) the second vertical; (c) the third vertical; (d) the fourth vertical; (e) the fifth vertical; (f) the sixth vertical; (g) the seventh vertical; (h) the eighth vertical.

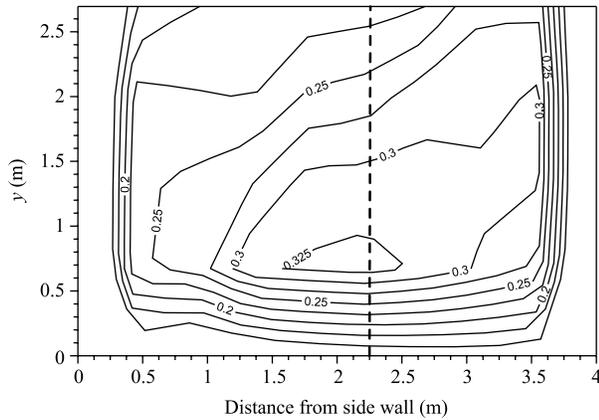


Figure 5 | Flume shape and isovels.

can also indicate the location of the y axis. The y axis, the dashed line in **Figure 5**, is located around the fifth vertical. ϕ and u_{\max} are not very sensitive to a possible error in locating the y axis (Chiu & Chen 2003); therefore, the y axis of the given cross section is about 2.25 m from the left wall. Owing to the effect of the side inflow, secondary flows as shown in **Figure 6** are found in the measurement section. Thus the velocity-dip phenomenon occurs in the narrow open channel. That is why the maximum velocities extend to the bottom area and the velocity distributions are not symmetric with respect to the center line. **Figure 7** shows velocity distributions along with those fitted by using (3) at five different flowrates. The occurrence of u_{\max} tends toward the channel bed when the flowrate increases. But h/D is between 0.6 and 0.7 and tends to remain stable. Such a tendency at a given section can be used as an aid in estimating u_{\max} .

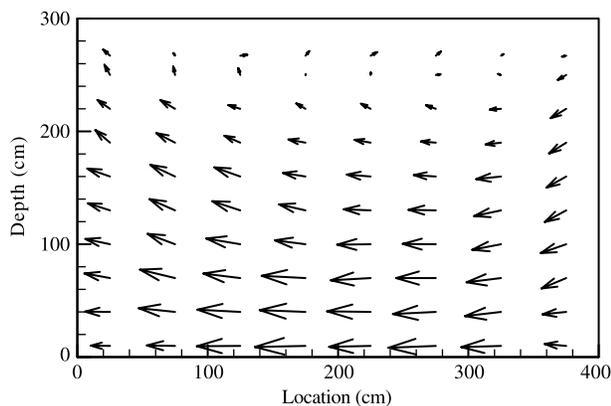


Figure 6 | Vector description of measured secondary currents in measurement section.

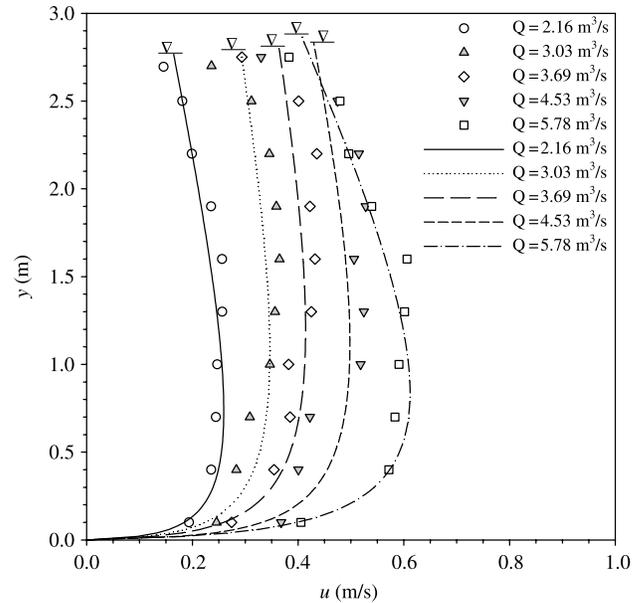


Figure 7 | Velocity distribution on the y axis.

Figure 8 shows the relation of \bar{u} to u_{\max} in the open channel at the measurement cross section. \bar{u} was obtained as Q/A and u_{\max} was determined along with h by regression using the velocities sampled on the y axis. The linear relation indicates that ϕ was stable and constant at the given section for a wide range of flowrates.

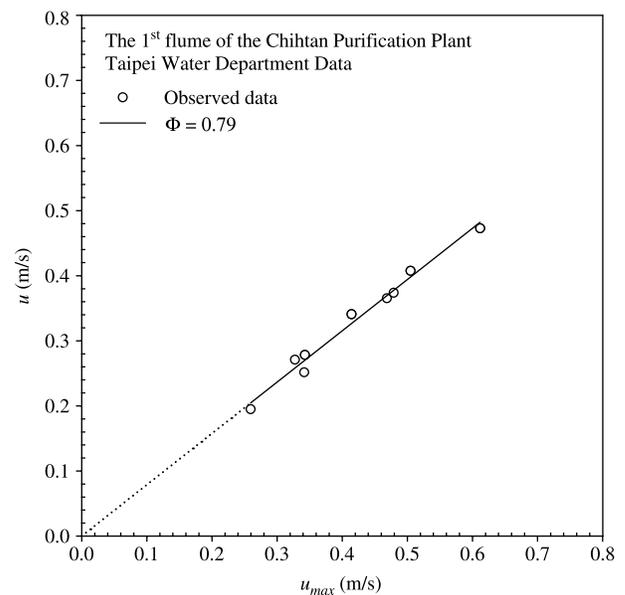


Figure 8 | $\bar{u} - u_{\max}$ relation.

ϕ is 0.79, corresponding to M equal to 4.53. The accuracy and reliability of this method is shown in Figure 9. The observed flowrate is obtained by the velocity–area principle with velocities measured by ADV. The estimated flowrate can then be determined by $0.79u_{\max}A$. Since the boundary of the open channel is rigid, A can be easily determined by D . The data points in the figure nicely fall around the agreement line. The flowrates determined by this method from ϕ and u_{\max} using several velocity samples on the y axis are almost as accurate as those determined by the velocity–area principle using velocity samples collected at more than 80 points spread over the entire cross section.

Comparison of the flowrate is made with the proposed method results expressed as a function of the large electromagnetic flowmeter results. The statistical procedure utilized is a least-squares linear regression. A graph of the data and statistical analysis is illustrated in Figure 10, where the flowrate (Q_e) obtained by the electromagnetic flowmeter is plotted versus the flowrate (Q_{est}) estimated by the proposed method. In this figure, circles and triangles denote the flowrates estimated by the velocity–area principle and u_{\max} on the y axis with ϕ , respectively. It shows the least-squares fit to the data and, superimposed, the line of perfect

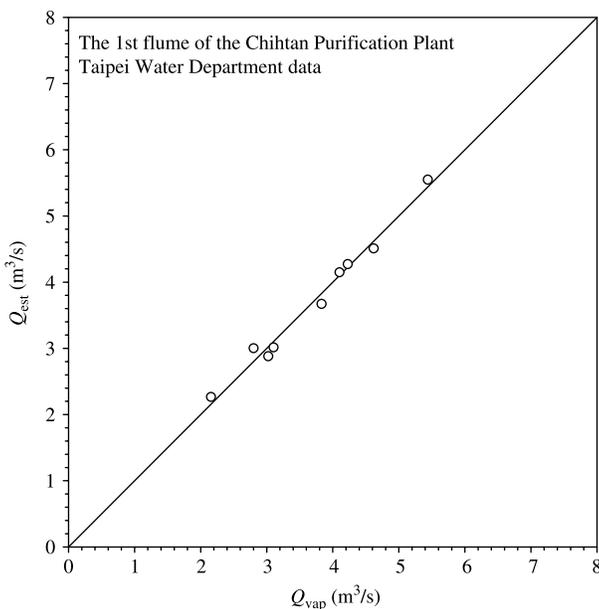


Figure 9 | Accuracy of estimated discharges.

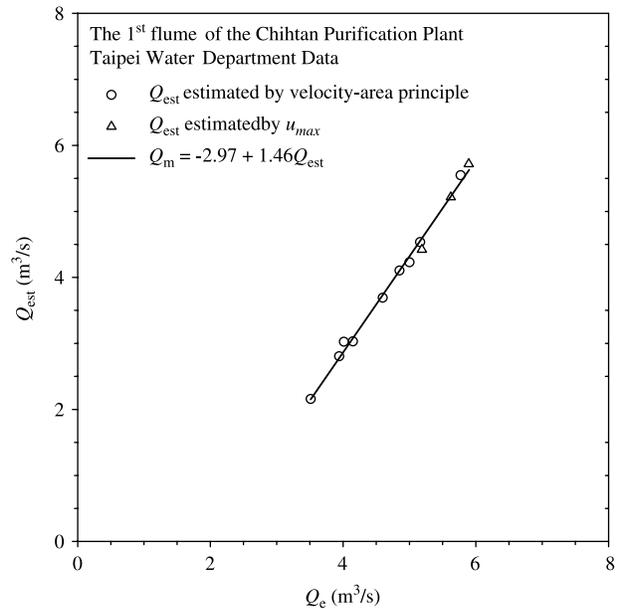


Figure 10 | Comparison of discharges measured by the magnetic flowmeter and the efficient method.

agreement. The relation determined is $Q_e = -2.97 + 1.46 Q_{\text{est}}$. It indicates that the flowrate estimated by the large electromagnetic flowmeter is underestimated. The flowrates of the large electromagnetic flowmeter can be easily estimated by the regression equation and the transducer can be adjusted to the calibrated values based on the regression equation. One of the advantages of the proposed method to calibrate large flowmeters is that the flowrate can be determined efficiently and accurately by u_{\max} with just a few velocity samples.

SUMMARY AND CONCLUSIONS

Flowrate is the key information necessary for the management of a water company. In light of the water shortage in Taiwan, flowrate is important for water allocation, usage and control. As a result, the accuracy of flowmeters becomes much more important and flowmeter calibration should accordingly become routine. However, calibration of a large flowmeter is time-consuming and labor-intensive. An efficient method, which does not require specialized equipment, is demonstrated here to calibrate large flowmeters. It is based upon the probability law that the velocity

distribution at a given cross section is resilient and invariant with time and flowrate. That is, ϕ is constant for a given cross section. The mean velocity of the cross section can be determined by the product of ϕ multiplied by the maximum velocity which is determined by a few velocities sampled on the y axis. Therefore the flowrate can be estimated efficiently and accurately by the product of the mean velocity multiplied by cross-sectional area. The calibration process involves establishing a steady flowrate through both the large electromagnetic flowmeter and the measurement cross section in the open channel. The flowrate of the large flowmeter is the average value of a series of samples and the flowrate used to calibrate the large flowmeter is estimated by the proposed method. The data of the Taipei Water Department is used to demonstrate the proposed method. Nine sets of velocity data spread over the entire cross section are measured by ADV to determine the y axis and ϕ and to estimate flowrates using the velocity–area principle. Three sets of velocity data measured on the y axis are used to estimate flowrates with ϕ . Then the comparison between flowrates obtained by the large flowmeter and the efficient method clearly shows their relationship. The large electromagnetic flowmeter underestimates flowrate. Therefore the transducer of the electromagnetic flowmeter can be adjusted based on this regression equation to provide a reliable and accurate flowrate. The results provide evidence that this efficient method gives us the ability to calibrate the large flowmeter very accurately and reliably.

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