Diminishing marginal returns for sensor networks in a water distribution system
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ABSTRACT
With increasing interest in the implementation/functionality of a contaminant warning system for water distribution systems, questions exist over the application to a real distribution system. A methodology is described to assess the impacts of changes in the numbers of sensors, on the time delay required to detect a contaminant intrusion event and to maximize sensor detection redundancy as protection against false positives. The methodology is used to explore the point of diminishing marginal return of detection likelihood, and the average time delay of detected intrusion events. Pareto front performance improvement with increasing numbers of sensors (from 2 through 50) is characterized through a case study application to the City of Guelph water distribution system (WDS). The results provide a methodology for utilities to employ for decisions on the number of sensors to use for a system. Within the two scenarios applied, five and four sensors are shown to be the point of diminishing marginal return for Guelph WDS in terms of the Pareto front performance improvement, detection likelihood, and the average time delay for the case study. Nevertheless, given that the timeframe to detect a contamination event may be lengthy, placing more sensors than the point of diminishing marginal return may be appropriate.

Key words | ingress, NSGA-II, number of sensors, Pareto optimality, time delay, water distribution

INTRODUCTION
The issue of contaminant intrusion into water distribution systems (WDS) is receiving increased attention for reasons including increased breakage rates with ageing infrastructure, and the fear of malevolent acts since water pressures may easily be overcome by pumps available at most home improvement stores (Kroll & King 2005). The role of security to protect a water distribution system is important as it is beyond other barriers (AWWA 2004), with the result that questions remain in light of significant municipal budgetary limitations. Much of the technical literature has focused on locating contaminant warning systems for very small water distribution systems for purposes of the development of methodologies; the true test of a contaminant detection system relates to the demonstration that the methodologies will be effective on a real WDS with all of its associated complexities. The budgetary implications for municipalities for scale-up are substantial because of the high cost of sensors in placement, and operations and maintenance (Ghazali & McBean 2009), and hence it is only practical to employ a small number in any WDS (Shen & McBean 2011).

A contaminant warning system is an option within which the number of sensors ‘needed’ for a specific WDS is an essential issue to be addressed before positioning the water quality sensors. Herein, the term ‘needed’ is defined by specific objectives in the framework of multiple objective optimization (MOO).

Two problems are examined in this paper: first, the investigation, for a specific WDS, to determine the merit of alternative numbers of sensors corresponding to the point of diminishing marginal return; second, when this number of sensors cannot meet an acceptable average time delay prior to detection of contamination, to determine
when a larger number of sensors may be required to reach an acceptable time delay.

**BACKGROUND**

There is considerable research which formulates sensor placement strategy as a MOO problem. The 8th Annual Water Distribution System Conference initiated a battle of sensor network (BWSN) (Ostfeld et al. 2006) in the framework of MOO. Prasad & Park (2004) applied a non-dominated sort genetic algorithm (NSGA) to examine a MOO: namely, the minimization of the network cost and maximization of a reliability measure; Prasad et al. (2004) used a multiple objective approach to investigate the minimization of the total disinfectant dose and maximization of the volumetric demand given specific residual limits.

A general framework for sensor placement in MOO can be formulated into three steps: (i) identify objectives for MOO; (ii) identify intrusion events for which protection is desired; and (iii) develop trade-offs between objectives.

Different groups of sensor placement objectives are found within sensor placement research. The BWSN applied four objectives: minimize the expected time of detection (z1); minimize expected population affected prior to detection (z2); minimize expected consumption of contaminated water prior to detection (z3); and maximize the detection likelihood (z4). Preis & Ostfeld (2008) applied z1, z4 and maximization of sensor detection redundancy. Austin et al. (2009) applied the same objectives as Preis & Ostfeld (2008). Preis et al. (2009) used z4 and maximization of sensor hydraulic sensitivity to variations in nodal demands. Ostfeld et al. (2008) indicated that z1, z2 and z3 are positively correlated with each other. Aral et al. (2010) formulated z1, z2, z3 and z4 into a single objective optimization problem, and solved it with a progressive genetic algorithm.

In MOO, objectives only make sense when competition exists between each pair, which suggests aggregation of z1, z2 and z3 may be necessary, or selecting a single one would be applicable. Shen & McBean (2011) further identified that, through minimizing z1, the sensor locations converge to a configuration of detecting none of the intrusion events (i.e. the sensor network detecting none of the intrusion events has the minimum z1 value), which suggests z1 may not be suitable as one of the objectives. Those authors subsequently proposed a new objective: namely, minimizing time delay before a water quality event is detected.

Before evaluating MOO objectives for sensor placement, it is essential to sample the intrusion events for sensors to detect. Different intrusion events may lead to quite different sensor locations. In BWSN, random injection events were selected from a prior density distribution (Ostfeld et al. 2006). As indicated by Krause et al. (2008), the simulation by EPANET (US EPA 2000) is time consuming, thus making it prohibitive to simulate large numbers of injection events, especially in large WDS. Weickgenannt et al. (2008) assigned importance values to each injection event and selected randomly, up to a designated number of injection events for evaluations of MOO objectives. The random selection procedure to identify injection events demonstrated difficulty in designing a sensor network, since going through the general framework for sensor placement for the second time, the intrusion events randomly selected are different, and hence, the objectives values will be different. The result is that the findings may end up with quite different sensor locations. Preis & Ostfeld (2008) described a procedure to select a few representative injection events in terms of spatial locations for the evaluation of the objectives, while not addressing the representation of injection time dimensions of the injection events. Shen & McBean (2011) relied upon a set of deterministic injection events using non-terminus nodes to investigate the alternatives for the locations of sensors.

NSGA-II, a MOO solver proposed by Deb et al. (2002), has been applied to a sensor placement field by Preis & Ostfeld (2008) and Weickgenannt et al. (2008). The MOO solver quantifies each individual design (i.e. alternative sensor locations) with rank (non-dominated individuals having rank zero, the individuals dominated by one individual having rank one, and likewise, each individual is assigned a rank) and crowding distance, and selects the preferred individual according to the lower rank and higher crowding distance. This methodology is applied herein to identify the Pareto front corresponding to a specific number of sensors. However, to date, there has been little research focused on determining the number of sensors required to adequately determine a contaminant warning system in the framework of MOO. Shen & McBean (2011)
used a metric, the ‘average normalized distance’ between two neighbours’ Pareto front curves, to characterize the sensor network performance improvement rate with increasing numbers of sensors, thereby developing a reasonable number of sensors (less than 10) for a WDS. The average normalized distance between Pareto front A and B proceeds in two steps: (i) calculate the normalized Euclidean distance, \(d_i\), from an individual \(A_i\) in A to every individual \(B_j\) in B, and identify the minimum one as the distance \(d\) from \(A_i\) to \(B_j\); and (ii) average over all \(d\) to obtain the average normalized distance from \(A\) to \(B\). This metric is again employed herein.

**OBJECTIVES DEFINITION**

To develop criteria by which the number of sensors is determined, two objectives are applied as per Shen & McBean (2011). They are: (i) minimize time delay; and (ii) maximize sensor detection redundancy. The definitions of the two objectives are as follows:

The first objective is minimization of the time delay of detected intrusion events on the premise of maximizing sensor detection likelihood of the sampled events. In other words, maximum coverage is guaranteed firstly to maximize the likelihood of detecting the events, and then sensor network designs are searched towards the minimum time delay of the detected events.

Minimize \(F_1 = \sum_{i=1}^{N_d} t_d(i) + N_{nd} t_{nd}\)

where \(F_1\) time delay, \(t_d(i)\) the time delay for the \(i\)th detected event, \(N_d\) the number of detected events, \(t_{nd}\) the time delay defined for each non-detected event, which is a large number to avoid interaction with \(\sum_{i=1}^{N_d} t_d(i)\), and \(N_{nd}\) the number of non-detected events.

It is important to acknowledge that water quality sensors are measuring water quality using surrogate parameters (such as total organic chlorine (TOC), chlorine, turbidity and others) instead of specific contaminants because of the unknown types of contaminant which may exist in real time in a WDS. Accordingly, the true time delay for a detected event should also address the measuring time for samples, and event detection algorithm run-time. By obtaining these two times, they can be added to the time delay. For example, from EPANET simulation, the time delay for an event is 30 min, and the sum of the measuring time and detection algorithm run-time is 5 min in total; then in real time, the time delay would be 35 min. However, these two times depend on the types of measurement equipment, and are specific to the event detection algorithm utilized. The study of these two times is beyond the scope of this paper, and thus are all set to zeros; in other words, the time delay for a detected event is directly from EPANET simulation.

The second objective is to maximize \(F_2\), the monitoring system redundancy, to reduce the false positive rates of sensor detection. Frequent false positive alarms will result in people ignoring the contaminant warning system and unnecessarily trigger costly emergency responses as well.

Maximize \(F_2 = \frac{1}{N_d} \sum_{i=1}^{N_d} r(i)\)

where \(F_2\) sensor detection redundancy, \(r(i)\) 1, if the event is detected by at least 2 sensors, and \(r(i)\) 0, otherwise.

**ANALYSIS OF \(F_1\)**

From the \(F_1\) value, the detection likelihood and the average time delay of detected events (termed ‘average time delay’ subsequently) can be quantified. By dividing \(F_1\) value by \(t_{nd}\), the integer part is the number of intrusion events not detected, the detection likelihood is parsed thereafter, and the modulus is the summation of time delay, and accordingly, the average time delay can be obtained. In addition, each \(F_1\) value corresponds to a particular sensor network configuration, from which each intrusion event detected can be identified from the populated database, and the statistics of time delay can be evaluated accordingly.

**EVALUATION OF OBJECTIVES FOR THE MOO FORMULATION**

Of interest herein is the application of the methodology to a real WDS, since the practical realities of scale-up to
a municipal system influence the number of sensors which can be utilized. The issue is more challenging also because, with a large number of WDS components, the computational effort increases dramatically. To avoid conducting simulations of duplicate injection events for evaluating optimization objectives, Aral et al. (2010) stored all the injection simulation data in computer memory. While this is an efficient procedure when the simulation data volume is feasible with respect to the computer memory, issues arise once the data volume surpasses computer capacity. For application to a real WDS, a modified methodology is necessary.

A modified methodology is accomplished by developing a database which is populated with arrays of injection events simulated by EPANET (US EPA 2000). Ideally, all combinations of injection nodes, injection time, injection duration, and mass rate should be simulated and stored in the database. However, this approach would be extremely time consuming, particularly for a large WDS (further explored below). However, to accomplish the investigation, only the arrays of intrusion events happening at every node, at the peak water demand hour (where there are potentially more people consuming water), single injection duration and mass rate are simulated and stored, the constructed database is referred to as the full database, which is also applied in contaminant source identification (Shen et al. 2009). Only through use of parallel computing is a more extensive simulation of more intrusion events feasible within a reasonable time period. $F_1$ and $F_2$ are quantified by querying a populated database derived from the full database (see Shen & McBean (2011) for details on query sentences).

In the following analysis, Scenario 1 is applied as a benchmark to evaluate the ramification of reducing the number of intrusion events (i.e. Scenario 2).

**Scenario 1**

For the situation of a deliberate or intentional attack on a WDS, there is little chance this will occur at terminus nodes, where there are few users that would be impacted. Firstly, terminus nodes can be easily located from a WDS map topology with minimal knowledge of the WDS. Secondly, if an intrusion event happens at a terminus node, it will not result in major consequences since there are limited numbers of water consumers downstream of terminus nodes and hence would not likely be a target for an intentional attack. Thus, in Scenario 1, only the intrusion events happening at non-terminus nodes are applied to evaluate the two objectives, $F_1$ and $F_2$. The procedure to identify non-terminus nodes described by Shen & McBean (2011) is applied herein, which views the WDS map as a graph, and then subsequently deletes nodes with degree 1 until there are no more to delete, with the result that there are only non-terminus nodes remaining. Database 1, containing only the records of injection events happening at these non-terminus nodes, is derived from the full database.

**Scenario 2**

Of interest is the reduction of the number of possible intrusion locations, to reduce MOO computational effort and, more importantly, the impact of the reduction on average time delay, detection likelihood and the point of diminishing marginal return. To accomplish this reduction, only the injection events that may impact at least a specified percentage of the WDS are selected from the full database to derive database 2 to evaluate $F_1$ and $F_2$.

**CASE STUDY**

The WDS of the City of Guelph (a community of approximately 110,000 people) is utilized in the case study. The WDS consists of 3,402 junctions, 15 reservoirs, 3 tanks, 4,253 pipes and 19 pumps. For confidentiality purposes, portions of the WDS components are deleted but the intent is to demonstrate the algorithms as they apply to a substantial WDS. All the EPANET simulations and MOO computations are completed in a desktop with 2.8 GHz CPU and 1 G RAM. The injection events happen at each node from 8.00 a.m. to 9.00 a.m. on the first day when the peak demand happens, and are simulated and stored in the full database for subsequent MOO computation. The EPANET simulation and full database construction take approximately 10 days, but only need to be completed once.
Scenario 1

With the procedure described above, 508 terminus nodes are identified from the 3,420 nodes, which leaves 2,912 non-terminus nodes. The NSGA-II runs required 9.8 days to obtain the Pareto fronts corresponding to sensor numbers from 2 through 50, which are presented in Figure 1. As apparent in Figure 1, Pareto fronts are arching from southwest to northeast, and to the northwest with increasing numbers of sensors, which indicates that both improved $F_1$ and $F_2$ are obtained as the number of sensors is increased. To quantify the Pareto front performance improvement, Figure 2 is developed, which indicates that the performance improvement is the largest from 4 to 5, and suggests that five sensors is the point of diminishing marginal return; that is, placing a fifth sensor is reasonable in the sense of performance improvement. By parsing the minimum $F_1$ value within each Pareto front, Figure 3 is developed. From the ‘detection likelihood’ and ‘Avg. time delay’ curves, the fifth sensor is again shown as the point of diminishing marginal return. It is observed for sensors numbered 5–10 that there is fluctuation in the ‘Avg. time delay’ curve. Comparing with sensor number 5, sensor number 6 decreases slightly in average time delay, compared with the findings from sensor numbers 2 through 5, which hence does not justify placement of the sixth sensor. Sensor number 7 indicates a significant increase in average time delay, which is not expected. This is due to the incremental number of sensors contributing all their power to search for the maximum detection likelihood, which gives slightly higher detection likelihood but longer time delay. Hence, utilization does not warrant the placement of the seventh sensor.

With the fifth sensor, the average time delay is 33.8 h, which is fairly high for water consumers in terms of public health, since this could cause a large percentage of water consumers of a WDS to consume the contaminated water prior to the sensor network detecting there is cause for alarm.

Scenario 2

Through querying the populated database with the condition ‘impact to at least 15% of the WDS nodes’, 510 important intrusion events out of 3,420 are selected, and are applied to evaluate $F_1$ and $F_2$. The resulting database 2 includes fewer records than database 1 in Scenario 1; thus, it takes less time to evaluate alternatives to each
sensor location by querying database 2. Not surprisingly, the NSGA-II requires about 5 days to obtain the Pareto front from the number of sensors 2 through 50, about 5 days less than the time required in Scenario 1. Figure 4 shows the Pareto fronts of each sensor. The ‘dots’ are zoomed to display clearly which numbers of sensors larger than four may not be necessary as the Pareto front performance does not improve significantly. Figure 5 is developed to illustrate the Pareto front performance improvement, which suggests that more than four sensors are unnecessary, or sensor number 4 is the point of diminishing marginal return. Figure 6 is developed to illustrate the minimum $F_1$ associated with each Pareto front. It is observed that: (i) all 510 intrusion events are detected with only two sensors, and thus no diminishing marginal return is observed; (ii) although not obvious, sensor number 4 can still be identified as the point of diminishing marginal return from the ‘Avg. time delay’ curve. Statistics of time delay in Scenarios 1 and 2 are listed in Table 1.

**DISCUSSION**

By reducing the number of intrusion events being investigated, as suggested in Scenarios 1 and 2, the points of diminishing marginal return change from five sensors to four sensors. Accordingly, the corresponding sensor locations change. Thus, it is essential to determine, firstly, the intrusion events the sensor network is going to detect. Comparing Scenarios 1 and 2, the average time delay of detected events is reduced from 33.8 h to 21.4 h as the number of intrusion events decreases from 2,912 to 510. This indicates that reducing the number of intrusion events for investigation may be an option to allow the sensor network design to detect intrusion events ‘earlier, on average’ (e.g. by identifying nodes of the greatest likelihood as target(s)). However, generalization of this finding to other WDS must be undertaken with caution.

Given fewer intrusion events of concern for detection, the average time delay may be further reduced. However, 510 out of 3,420 intrusion events already represents a small percentage of the total; hence, further reduction may

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**Table 1** Statistics of the time delay of detected intrusion events

<table>
<thead>
<tr>
<th># of injection events to detect</th>
<th>Average (h)</th>
<th>5% quantile (h)</th>
<th>Median (h)</th>
<th>95% quantile (h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario 1</td>
<td>2,912</td>
<td>33.8</td>
<td>5.5</td>
<td>27.0</td>
</tr>
<tr>
<td>Scenario 2</td>
<td>510</td>
<td>21.4</td>
<td>8.5</td>
<td>19.8</td>
</tr>
</tbody>
</table>
be inappropriate. This finding indicates that placement of more sensors than the point of diminishing marginal return is likely to be appropriate.

CONCLUSIONS

As concern with contaminant intrusion increases, contaminant warning systems as an option to prevent water consumers from being exposed to the risk represent an important action. The number of sensors required is an essential question to answer. This research reported the numbers of sensors required in the framework of MOO in two scenarios for a city with a population of approximately 110,000. By applying the normalized average distance between two neighbours as the metric, the Pareto front performance improvement curve with increasing number of sensors represents an effective strategy for identifying the point of diminishing marginal return. The average time delay and detection likelihood curves are also effective at demonstrating the point of diminishing marginal return for the number of sensors; however, the average time delay to detect contaminant incidence is excessive for public health, and hence may not be acceptable. Placement of more sensors than the point of diminishing marginal return will decrease the average time delay but, clearly, the high cost of sensors will limit the number of sensors which can actually be considered. Subsequently, it is shown that reducing the number of intrusion events of interest may decrease the average time delay, which provides an option while budgets are limited for sensor purchase and operation.

RECOMMENDATION

It is essential to determine the events to detect before investigating any sensor placement methodology. As suggested in the two scenarios developed herein, different sets of events to be detected will change the point of diminishing marginal return. Determining such a set of events involves conflict analyses among all stakeholders, such as schools, hospitals, governmental offices, especially when limited funds are available for sensor set-up, maintenance and operation to guarantee short time delay of detected events.

Aspects such as sensor accuracy/precision, event detection algorithms (i.e. determining whether variations of parameters suggest contamination as opposed to natural fluctuation) and emergency response should be addressed before recommending that a water utility locates sensors in its WDS.

REFERENCES


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