tained analytically for finite length bearings and by uncorrected short bearing theory. Again, the finite-length correction factor compares very favorably with the analytic solution over a wide range of eccentricity and L/D ratios. For L/D ≤ 0.25, the results for both steady-state and radial squeeze motion using the three methods are nearly identical, as is expected, and are not shown here. The finite length correction factor described herein does not result in any appreciable increase in the computational time required for short bearing analysis. The effect of increased L/D ratios in plain journal bearings, seals, and squeeze film damper bearings can, therefore, be efficiently treated in rotor-dynamic transient analysis.

Acknowledgments

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DISCUSSION

M. Rahmanzadeh-Asl and C. M. M. Ettles

The authors have presented a valuable addition to the literature of fluid film lubrication. In the analysis, they have used the Sommerfeld conditions to derive a simple expression for the functional form of G. The disadvantage of the method presented is that it is only applicable to the plain (circular) journal bearing without any grooves.

Alternative fast methods of solution are available and some of them can be applied to all types of bearings with Reynolds’ condition of cavitation. That was pointed out by P. Shelly and C. Ettles [16] for medium-length journal bearings, a characteristic found by Ocvirk [17], that the pressure profile varies parabolically in the axial direction:

\[ P(\theta, z) = P_c \left( 1 - \frac{2z}{L} \right) \]

where \( P_c \) is the pressure on the center-line and \( z \) is the nondimensional axial coordinate, being zero on the center-line and unity at the edge. This simple assumption was found to be reasonably accurate for medium-length bearings at eccentricities less than \( \varepsilon = 0.8 \). The useful range was extended by using a more general pressure distribution in the form:

\[ P = P_c \left( 1 - \frac{2z}{L} \right)^{\phi} \]

A mean value of \( \phi = \phi_m \) was derived from the expression:

\[ \int P dz = \int P_c \left( 1 + \frac{2z}{L} \right)^{\phi} \times dz = \int \left[ 1 - \left( \frac{2z}{L} \right)^{\phi} \right] \times dz \int P_c dz \]

The values of \( \phi_m \) were analysed for rotation and squeeze to produce the general expressions:

\[ \phi_m = f(\varepsilon, L/D) \text{ pure rotation} \]

\[ \phi_m = g(\varepsilon, L/D) \text{ pure squeeze} \]

and an overall general expression:

\[ \phi_m = H(\varepsilon, L/D) \text{ combined motion} \]

The dimensionless Reynolds’ equation using equation (1) will be of the form:

\[ \frac{\partial}{\partial \theta} \left[ \alpha_0 \frac{\partial P}{\partial \theta} \right] = \frac{1}{1 - \left( \frac{2z}{L} \right)^{\phi_m}} + \alpha \left( \frac{\partial^2 P}{\partial z^2} \right) \left[ 1 - \left( \frac{2z}{L} \right)^{\phi_m} \right] = \frac{\partial h}{\partial \theta} \]

This gives an infinite number of equations according to the value of \( z \). Setting \( z = 0 \) gives reasonable accuracy, but this is improved by using integrals (averages) of these functions across the film. For dynamically loaded bearings the governing equation will be of the form:

References

\[ \frac{d}{d\theta} \left( \frac{1}{h^2} \frac{dP}{d\theta} - \beta a h^3 \frac{\partial P}{\partial h} \right) = - \epsilon (0.5 - \psi) \sin \theta + i \cos \theta \]

where \( \beta = 2 \) if \( \tilde{P} = P_c \) (centre-line pressure) and \( \beta = 3 \) if \( \tilde{P} = P_a \) (mean axial pressure). This equation cannot be integrated directly. Let the solution of the equation be of the form:

\[ \tilde{P} = K P_L(\theta) \quad (2) \]

where \( P_L \) is the long bearing solution, and \( K \) is a side leakage correction factor. By substituting equation (2) into the long bearing solution, a residual \( R \) may be found which is given by:

\[ R = \beta K a h^3 P_L + (K - 1) \epsilon (0.5 - \psi) \sin \theta - i \cos \theta \]

Following the references [18] and [19], the simplest form of the side leakage factor will be the form:

\[ K = 1.0 \left[ 1.0 + \frac{\beta a h^3 P_L(\theta_m)}{\epsilon (0.5 - \psi) \sin \theta_m - i \cos \theta_m} \right] \]

where \( \theta_m \) is the value of \( \theta \) at maximum pressure, which can be calculated by the long bearing approximation. Cavitation was allowed for by imposing the usual boundary conditions:

\[ P_L(\theta_m) = \frac{dP_L}{d\theta} \bigg|_{\theta_m} = 0 \]

Investigations show that the method has sufficient accuracy for journal bearing stability calculations.

T. Lloyd, et al. [20] presented another fast method which involved numerical calculation methods. Basically, they applied a linear combination of the wedge and squeeze induced pressures by assuming no oil film disruption. That is, if \( P' \) and \( P'' \) represent the pressure fields due to unit wedge and unit squeeze, respectively, then these can be added as follows:

\[ P = S(bP' + b'P'') + \bar{P}' \]

where \( S \) is the speed modifying factor resulting in their case from considering connecting rod bearings, \( S \) is an influence function, and \( P' \) the feed pressure.

The bearing was assumed circular and to be fed from a full circumferential groove. The value of \( b \) and \( b' \) are defined as follows:

\[ b = 1 - 2 \frac{d\phi}{dt}/w, \quad b' = \frac{dc}{dt}/w \]

where \( w \) is the instantaneous angular velocity of the journal relative to the bearing. The marching out of the shaft centre position to produce a locus used prestored solutions of \( P' \) and \( P'' \) for various values of \( \epsilon \). The use of half Sommerfeld conditions is probably valid for engine bearings (as considered by Lloyd) since these are, in general, very heavily loaded.

Additional References


S. B. Malanoski

The authors present an apparently, efficient method for calculating the dimensionless hydrodynamic forces for finite-length fluid films with cavitation. In addition, they present a selected collection of these forces as a function of \( \epsilon \) and \( L/D \)-comparing various theories.

Especially for those who are new to this field, it would be well to also become familiar with additional references [21 and 22], in which the main subject deals with the use of these forces in a rotor-bearing system.

It should be noted that it was concluded in [21], pp. 35-36, that the effect of the \( L/D \) ratio is nominal when comparison is based on the same \( \epsilon \) value, and the bearing analysis, based on \( L/D = 0 \), yields both force and force derivatives which give a realistic representation of the constraints imposed by the fluid film on the journal motion.

When the investigation in [21] was carried out, it was necessary to prepare tables of numerically calculated forces for \( L/D > 0 \). These data are given in Table 1 of [21] as a function of \( L/D, \epsilon \), and \( \epsilon \). In Table 2 of [21], the relationships of the Taylor stiffness and damping coefficients and modified Sommerfeld number are given as a function of \( L/D \) and \( \epsilon \). These results corroborate the numerical data of the present paper.

(Not, that the ordinate of Figs. 2, 3, 4 of the present paper is equal to the modified Sommerfeld number multiplied by the constant \( 4/\pi \). Also, note that at a given \( \epsilon \), the ordinate of Fig. 2 multiplied by the ordinate of Fig. 6 is typically, approximately, equal regardless of the theory used. This is also true for Figs. 3 and 7, and 4 and 8.)

In the Authors’ Closure, the discussor would like the authors to address and discuss the comments and observations above and in addition complete the nomenclature list for symbols such as \( N' \) and \( F \) (which have implied definitions from the figures) and all of the unidentified symbols in Fig. 1.

Additonal References


D. L. Taylor and J. F. Booker

The authors present a correction which can be applied to the short bearing solution to better approximate forces for finite length bear-

2 Mechanical Technology Inc., Latham, N.Y.

3 Assistant Professor and Professor, respectively, Sibley School of Mechanical and Aerospace Engineering, Cornell University, Ithaca, N.Y. 14853.
ings. A major consideration with techniques such as this is the computational time required, since the normal goal is incorporation in some numerical application such as time transient response. Two steps are involved in such a procedure: 1) obtaining the pressure distribution; and 2) obtaining the resultant forces for use in the equations of motion. The first step can be completed either numerically (e.g., a finite element solution of the 2-D Reynolds equation) or analytically (e.g., the Ocvirk short bearing solution). The second step requires an integration of the pressure distribution (however obtained), which again can be accomplished either numerically or analytically. For a comparison of execution times, it is important to know how both steps were accomplished. Specifically concerning this paper, how were the four integrals in equations (25), (26) evaluated?

Relative execution times have been reported by Childs et al. [8] for three cases. Combining the analytical Ocvirk short bearing pressure distribution with numerical evaluation of the resulting pressure integrals gives a relative time of 10. Analytical Ocvirk short bearing pressure distribution with analytical evaluation of the pressure integrals, as summarised by Booker [23], gives a relative time of 1. (The 10:1 ratio agrees with our experience.) The finite length impedance procedure described by Childs, (based on a weighted average of short and long bearing analytical solutions), gives a time of 1.67.

Since the thrust of the present paper is a correction on the analytical short bearing solution, execution times would be expected to be somewhat above the value of 1. The time cannot be shorter than this factor since these basic calculations are required so that they can be corrected. (Specifically, the bare minimum is the evaluation of the integrals in equations (25), (26).) The discussors expect that the evaluation of $R_1, R_2, R_3$, and $R_4$ from equations (15), (16), (23), (24) and the calculation of the correction terms in equations (25), (26) are likely to be of the same computational complexity as the analytical solution and so a relative time of nearly 2 would be expected. The stated speed improvement factor of 2 to 3 over the finite length impedance is puzzling, since this would imply relative times of 0.56 to 0.83. Could the authors clarify the situation by comparing execution times for their corrected short bearing solution with the underlying basic short bearing solution?

Also, the discussors would like to point out that the terminology of an impedance formulation describes completely the method presented by this paper. The impedance method [24] simply relates forces to generalized velocities and displacements and is thus a perfect vehicle for comparisons of this sort. Confusion results only if careful distinction is not made between the various means of obtaining impedance data. The only substantive questions involving the accuracy and speed with which impedance data are determined—analytically, numerically, or even experimentally.

We note that the authors have not addressed the question of accuracy for the general case which combines the two specific cases of rotation-precision and radial-squeeze motions. Their basic gambit of combining solutions for (linear) full-film problems prior to imposition of (nonlinear) cavitation boundary conditions is an old and accepted one [25]; though legitimate for exact solutions, it may lead to difficulty with numerical approximations.

Additionally, it would seem that the authors might wish to explain and justify their (apparent) choice of cavitation boundary conditions.

In conclusion, the discussers feel the correction factor presented should prove most useful in the modelling and analysis of specific rotor equipment. The additional computational time required for such corrections is well invested to achieve quantitatively meaningful results. As a point of interest, could the authors please comment on whether this correction could be expected to yield dynamic response which is in any sense qualitatively different from the short bearing cavitated model?

Additional References


D. F. Li

This paper represents a welcomed effort for obtaining fast computation of the hydrodynamic pressure in a finite length journal bearing. While the Reynolds equation cannot be solved successfully by the method of separation of variables because of the boundary conditions, authors resort to an approximate solution consisting of the product of a circumferential pressure and an axial pressure. In this case, the authors seek a product solution that approaches the short bearing and the long bearing in the limits, with the hope that it will also be valid for bearings having intermediate length to diameter ratios.

The hyperbolic cosine axial pressure form in this paper ensures that the pressure in Eq. (5) reduces to the short bearing solution regardless of the value of $G$. The required form of $G^2$ is found (Eq. (9)) by forcing the pressure to also approach the long bearing solution at large $L/D$ ratios. Note that a substitution of $G^2$ into Eq. (5) will yield the more familiar form of the "side-flow leakage" corrected long bearing solution:

$$p(\theta, z) = p(\theta) \left[ 1 - \frac{\cosh(Gz/R)}{\cosh(GL/D)} \right]$$ (27)

The subtle difference in the authors' undertaking is, therefore, that (in lieu of Eq. (27)) Eq. (5) with an average value of $G^2$ is used.

Excellent accuracy of the approximation using the hyperbolic form in the axial pressure is self-evident in the results presented in this paper and also in the paper by Warner. This is in part due to the parameter $G$ which allows the axial pressure profile to vary with journal eccentricity and bearing operating condition.

Since the product $P(\theta)P(z)$ does not exactly satisfy the Reynolds equation, one would expect some limitations in its range of accuracy. Table 1 shows the result of a study carried out by the discusser to

Table 1 Results for a centrally loaded squeeze film journal bearing ($x$)

<table>
<thead>
<tr>
<th>$L/D$</th>
<th>$G$</th>
<th>$\bar{w}$ (exact)</th>
<th>$\bar{w}$ (product)</th>
</tr>
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<tbody>
<tr>
<td>0.1</td>
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<td>0.3609</td>
<td>0.3609</td>
</tr>
<tr>
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<td></td>
<td>13.59</td>
<td>19.57</td>
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<tr>
<td>0.95</td>
<td></td>
<td>104.1</td>
<td>104.1</td>
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<tr>
<td>0.99</td>
<td></td>
<td>3949.2</td>
<td>3949.2</td>
</tr>
<tr>
<td>0.5</td>
<td>0.5</td>
<td>7.310</td>
<td>7.310</td>
</tr>
<tr>
<td>0.9</td>
<td></td>
<td>209.6</td>
<td>209.4</td>
</tr>
<tr>
<td>0.95</td>
<td></td>
<td>745.5</td>
<td>747.0</td>
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<td></td>
<td>12137.4</td>
<td>10957.5</td>
</tr>
<tr>
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<td>18.71</td>
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<tr>
<td>0.99</td>
<td></td>
<td>13982.4</td>
<td>12788.0</td>
</tr>
</tbody>
</table>

4 General Motors Research Laboratories, Mechanical Research Department, Warren, Mich. 48090.

compare the “best” possible product solution to the exact solution for a π bearing in squeeze motion. The “best” solution is obtained by minimizing the functional of the Reynolds equation for the pressure

\[ P(\theta, z) = P_B(\theta) \left[ 1 - \frac{\cosh(Gz/R)}{\cosh(GL/D)} \right] \]  

(28)

It can be seen that, for all practical purposes (up to an eccentricity ratio of 0.95), the assumed product form does indeed accurately compute the load capacity of a journal bearing.

Authors’ Closure

The authors wish to thank the discussers for their comments, and for pointing out some additional methods for approximating finite length solutions for bearing films. Certainly the use of simpler methods for solving the two-dimensional Reynolds equation is not new. Present interest in such techniques appears to be motivated by the desire to incorporate nonlinear finite length bearing forces into more general rotor-bearing system calculations.

In essence, most approximations to the two dimensional problem involve selecting a functional form of the solution and adapting it to give a “best” solution. Often this technique involves the assumption of an axial pressure profile as mentioned by the discussors. The use of weighted short and long bearing solutions can also be optimized with respect to the weighting functions to yield a “best” solution. As noted in the discussion the choice of the approximate solution used in any particular application depends on the accuracy and computational speed desired (as well as personal preferences).

The authors find the comparison of the exact solution for pure radial squeeze motion to the “best” solution obtainable for an assumed product solution using a hyperbolic axial pressure profile most interesting. It should be noted, however, that this “best” solution involves optimization of both \( G \) and \( p_0 \).

The accuracy in the use of the corrected pressures in calculating the forces for general dynamic conditions depends primarily upon the accuracy of the pressure calculations. The pressures within the bearing are superposable. The accuracy of the corrected pressures can be judged from Figs. 2-8. The corrected pressures for radial squeeze motion are quite accurate for the \( L/D \) ratios considered. The pressures for rotational-precessional motion are somewhat less accurate. This is partly due to the change in circumferential variation in the axial centerline pressure with increasing \( L/D \) that is not entirely accounted for by the use of radial and tangential components for \( g^2 \). The limits of integration, \( \theta_1 \) and \( \theta_2 \) in equations (25) and (26) are determined from the short bearing centerline pressure, \( p_{oc1} \) and \( p_{oc2} \). For the half Sommerfeld cavitation boundary conditions usually employed in nonlinear transient dynamics, these angles correspond to the values that render \( B \) in equation (1) zero. These angles are unaffected by the pressure correction factors. Equations (25) and (26) were evaluated analytically.

The half Sommerfeld cavitation boundary condition was chosen because it is easily applied to the short bearing analytic solution and because it appears to be a reasonable boundary condition for large scale nonlinear dynamic motion for which the present analysis is intended.

For the circular geometry considered, the results for any \( L/D \) ratios are qualitatively the same as shown by numerical solutions of the two dimensional Reynolds equation. As mentioned in the paper and discussion the purpose of the present analysis is to improve the quantitative results without substantially increasing the computation time.

Since the authors have not implemented the finite length impedance formulation on a computer, the estimate of a reduction in computation time of 2 to 3 for the present analysis compared to the impedance method was based on comparisons to the short bearing formulation with numerical integration of the circumferential pressure distribution. This latter method has been used by the authors. If this method is used as a basis, the execution time depends on the integration method and the number of intervals in circumferential direction. It would appear that the different base programs used by the authors and discussors required substantially different execution times. Using the uncorrected short bearing solution as a basis, the evaluation of the corrected integrals in equations (25) and (26) requires a relative time of 1.2 to 1.3.

The symbols not defined in the nomenclature are

\[ F = \text{total force}, \sqrt{F_x^2 + F_y^2} \]  

\[ N^1 = \text{shaft rotational speed}, \text{rev}\cdot s^{-1} \]  

\[ N_{x,y} = x \text{ and } y \text{ components of } W, F \]