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A shallow-water ocean acoustics inverse problem

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Abstract: A shallow-water ocean acoustics experiment is proposed from which it is possible to determine in principle the scattering data necessary to recover the sound-speed profile in the ocean bottom. These data are a "reflection coefficient," which is not the usual one. The reflection coefficient is amplitude of the outgoing wave at infinite depth due to a unit amplitude plane wave incident from infinite depth. The unusual fact is that this reflection coefficient can be recovered from a measurement in the ocean. The essential assumptions are that the sound-speed profile is known in the ocean but unknown in the bottom, and that the measurements are made in the ocean layer.

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1. Introduction

A knowledge of the acoustic properties of the ocean bottom is essential for the detection, localization, and classification of objects in the ocean itself. An experiment carried out in the ocean is proposed from which, in principle, the bottom sound-speed profile can be recovered. However, the usefulness of the method it remains to be determined through numerical examples and actual experiments. For this shallow-water ocean acoustics inverse problem, the following assumptions are made:

(1) The acoustic sound-speed profile in the ocean and in the bottom depend only on the depth coordinate z, $c=c(z)$, for $0 \leqslant z < \infty$.
(2) The density is a constant for all z. If the density is not a constant then the differential equation for the pressure field can be changed to a constant density case. In this case a function of the sound speed and the density is recovered.
(3) The ocean occupies the region $0 \leqslant z \leqslant L$ where L is known, and the sound-speed profile in the ocean is assumed known. In an earlier paper by Stickler and Deift,[1] the sound speed in the ocean was also assumed unknown. This greatly complicates the solution of the inverse problem.
(4) The ocean bottom occupies the region $L < z < \infty$ and the sound speed profile there is unknown and is to be determined.
(5) The sound speed approaches a constant c_1 sufficiently rapidly as z approaches infinity and the constant c_1 is known.
(6) The acoustic pressure vanishes at the ocean surface.

The object is to determine $c(z)$ for $z > L$ from measurements in the ocean $0 \leqslant z \leqslant L$. A sketch of the environment is shown in Fig. 1.

2. The oceanographic experiment

An experiment for determining the data necessary to recover $c(z)$ is described below. This experiment has also been described in Refs. 2–4.

(1) At a depth z_0 with $0 \leqslant z_0 \leqslant L$ in the ocean, an isotropic, low-frequency [ω is its angular frequency and the time convention is $\exp(-i\omega t)$] point source projects acoustic energy. The frequency is chosen to be low enough so that no eigenmodes are excited. This means that only the continuous spectrum is excited. If there are eigenmodes, the experimental mea-

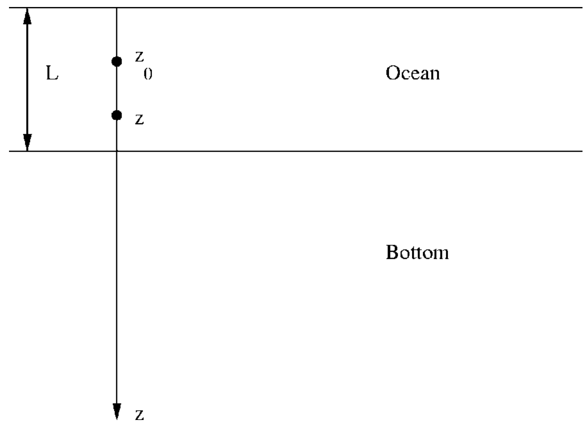

Fig. 1. The geometry of the experiment.

surement is the same, only the technique for the recovery of $c(z)$ needs to be modified. When eigenmodes are excited, the additional data necessary is described in a later section.

(2) The pressure field is measured at a depth z again in the ocean for all range points, that is, for $0 \leqslant r < \infty$. The measurement depth and the source depth can be the same.

The continuous spectrum decays one power of r faster than the eigenmodes. Thus, if only the continuous spectrum is present the range measurement is much easier to realize.

The Hankel transform with respect to r of the pressure field yields a one-dimensional Green's function (the Hankel transform variable is denoted by μ)

$$\frac{d^2 g(z,z_0,k)}{dz^2} + [k^2 - q(z)]g(z,z_0,k) = \delta(z - z_0), \tag{1}$$

where, since the ocean surface is a pressure release surface

$$g(0,z_0,k) = 0, \tag{2}$$

and the radial wave number is replaced by the vertical wave number, k

$$k^2 = (\omega/c_1)^2 - \mu^2, \quad \text{with } \mathrm{Im}k \geqslant 0, \tag{3}$$

and the sound-speed profile is replaced by $q(z)$

$$q(z) = \omega^2[1/c_1^2 - 1/c^2(z)]. \tag{4}$$

This one-dimensional Green's function is called the measured data below.

3. Some properties of the Green's function and the reflection coefficient

Let $U(z,k)$, $S(z,k)$, and $C(z,k)$ satisfy the homogeneous form of Eq. (1) with

$$U(z,k) \sim e^{ikz} \text{ as } z \to \infty. \tag{5}$$

The function $U(z,k)$ represents a unit amplitude outgoing wave at infinity. In addition, $S(z,k)$ and $C(z,k)$ satisfy the initial conditions $S(0,k)=0$, $S'(0,k)=1$, $C(0,k)=1$, and $C'(0,k)=0$, where the prime denotes differentiation with respect to z. For real k these three functions are pairwise linearly independent solutions of the homogeneous equation with $S(z,k)$ and $C(z,k)$ real. $U(z,k)$ is complex. For real k the complex conjugate of $U(z,k)$, denoted by $\bar{U}(z,k)$, is $U(z,-k)$, and $U(z,-k)$ represents an incoming wave at infinity.

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For real k, $U(z,k)$, $C(z,k)$, and $S(z,k)$ are related by

$$U(z,k) = U(0,k)C(z,k) + U'(0,k)S(z,k). \tag{6}$$

In addition, for real k, not only are $U(z,k)$ and $U(z,-k)$ complex conjugates, but they are independent solutions as well. Their Wronskian is given by

$$W[U(z,k),U(z,-k)] = -2ik = -2i\mathrm{Im}[\bar{U}(0,k)U'(0,k)], \tag{7}$$

or

$$\mathrm{Im}[U(0,-k)U'(0,k)] = k. \tag{8}$$

The measured data needed to recover $q(z)$ for $L<z<\infty$ are[5,6]

$$S_c(k) = U(0,-k)/U(0,k), \quad \text{for } 0 \leqslant k < \infty. \tag{9}$$

This is the reflection coefficient except for a multiplicative factor of -1 for a unit amplitude plane wave incident from $z=\infty$, and reflected from the pressure release surface at the ocean-air interface. For k real, $U(0,-k)=\bar{U}(0,k)$ and, therefore, the modulus of $S_c(k)$ is unity.

The analysis below shows how in principle $S_c(k)$ can be recovered from a measurement in the oceanic layer.

Two techniques exist for the recovery of $q(z)$. One requires the solution of a Fredholm integral equation due to Marchenko,[5] and the other requires solving an initial value problem for an ordinary differential equation. The latter is called the trace method, and is due to Deift and Trubowitz.[6] They show how both results can be derived from a single starting point. The data needed for both methods are $S_c(k)$.

The Green's function, that is, the measured data, is given by

$$g(z,z_0,k) = -S(z_<,k)U(z_>,k)/U(0,k), \tag{10}$$

where $z_<$ is the smaller of z and z_0, and $z_>$ is the larger. The measurement in the oceanic layer yields $g(z,z_0,k)$. Since the sound-speed profile is known in the ocean, both $S(z,k)$ and $C(z,k)$ can be calculated in that layer.

In Eq. (10), substitute for $U(z,k)$ from Eq. (6). This yields

$$g(z,z_0,k) = -S(z_<,k)[U(0,k)C(z_>,k) + U'(0,k)S(z_>,k)]/U(0,k). \tag{11}$$

In the oceanic layer the Green's function has been measured; $S(z,k)$ and $C(z,k)$ can be calculated. Thus, from Eq. (11) the impedance at $z=0$, $U'(0,k)/U(0,k)$ can be recovered.

Next, take the imaginary part of Eq. (11). This yields

$$\mathrm{Im}g(z,z_0,k) = -\mathfrak{I}[U'(0,k)/U(0,k)]S(z_>,k)S(z_<,k). \tag{12}$$

This may be rewritten as

$$\mathrm{Im}g(z,z_0,k) = -\mathrm{Im}[U(0,-k)U'(0,k)]S(z_<,k)S(z_>,k)/|U(0,k)|^2. \tag{13}$$

Next, substitute from Eq. (8). This yields

$$\mathrm{Im}g(z,z_0,k) = -kS(z_>,k)S(z_<,k)/|U(0,k)|^2. \tag{14}$$

Thus, from experimental data, Eq. (14) also yields the modulus of $U(0,k)$. The phase can be determined as follows:

$$U(0,k) = |U(0,k)|\exp[i\Theta(k)]. \tag{15}$$

However, $\Theta(k)$ and $\ln(|U(0,k)|)$ are conjugate harmonic functions. In addition, the modulus of $U(0,k)$ is $1+O(k^{-1})$ as $k\to\infty$ and analytic in the upper-half plane. Hence, $\Theta(k)$ can be recovered from the modulus of $U(0,k)$

$$S_c(k) = \exp[-2i\Theta(k)]. \tag{16}$$

This phase, which is approximated in Ref. 7, is identified as the critical data to be recovered. The experiment described above shows how it can be recovered. Finally, if there are eigenmodes, they are finite in number and are located at $k = 1\beta_j$ for $j = 1, 2, \ldots, N$ with β_j real. The values of β_j, as well as a set of N constants which are a measure of the energy in each mode, are required.

4. Conclusions

An experiment has been described which, in principle, can be used to determine a reflection coefficient, $S_c(k)$, needed in the recovery of the sound-speed profile in the ocean bottom. The reflection coefficient is not the usual reflection coefficient, but rather the reflection coefficient of a unit amplitude plane wave incident from infinity. What is remarkable is that it can be recovered from a measurement in the ocean. Once recovered, this coefficient can be used in either a Marchenko-type equation or a trace formula equation to recover $c(z)$, the sound-speed profile. If there are eigenmodes some additional information is needed. If both $U'(0,k)$ and $U(0,k)$ are recovered, then the sound-speed profile can be recovered without knowledge of the eigenmode mode data.

References and links

[1]D. C. Stickler and P. Deift, "Inverse scattering for a stratified ocean and bottom," J. Acoust. Soc. Am. **7**, 1723–1727 (1981).
[2]J. F. Lynch, S. D. Rajan, and G. V. Frisk, "A comparison of broadband and narrow-band modal inversions for bottom properties at a site near Corpus Christi, Texas," J. Acoust. Soc. Am. **89**, 648–665 (1991).
[3]K. M. Becker, S. D. Rajan, and G. V. Frisk, "Results from the Geoacoustic Inversion Techniques Workshop using modal inverse methods," IEEE J. Ocean. Eng. **28**(3), 338–341 (2003).
[4]K. Ohta and G. V. Frisk, "Modal evolution and inversion for seabed geoacoustic properties in weakly range-dependent shallow-water waveguides," IEEE J. Ocean. Eng. **22**(3), 501–521 (1997).
[5]Z. S. Agranovic and V. A. Marchenko, *Inverse Scattering Theory* (Gordon and Breach, New York, 1963).
[6]P. Deift and E. Trubowitz, "Inverse scattering on the line," Commun. Pure Appl. Math. **32**, 121–251 (1979).
[7]P. Joseph, "Complex reflection phase gradient as an inversion parameter for the prediction of shallow-water propagation and the characterization of sea-bottoms," J. Acoust. Soc. Am. **113**(2), 758–768 (2003).