A three-dimensional non-linear model for a galactic dynamo

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ABSTRACT

We describe a new code to solve the non-linear mean field dynamo equation in three spatial dimensions, in a form suited to investigating dynamo action in the discs of spiral galaxies. The code is used to model the generation of non-axisymmetric magnetic fields during the tidal interaction between M81 and its companion galaxy NGC 3077. Additionally, it is shown that, in linear theory, a spiral wave perturbation to the alpha effect can enhance the growth rate of the bisymmetric dynamo wave by parametric resonance. Results are compared with those obtained previously using an essentially two-dimensional approximation.

Key words: magnetic fields - MHD - galaxies: individual: M81 - galaxies: interactions - galaxies: magnetic fields - galaxies: spiral.

1 INTRODUCTION

Some of the outstanding problems in galactic dynamo theory involve the generation of non-axisymmetric magnetic fields. These issues arise both from the problem of explaining the lack of axisymmetry of the magnetic fields observed in a number of galaxies, and in investigating the effects of non-axisymmetric streaming motions brought about by, for example, tidal interaction with a companion galaxy, or the effects of a central bar generating a non-axisymmetric gravitational potential (see Beck et al. 1996 for a fuller discussion).

Some progress in understanding non-axisymmetric magnetic field generation can be made in linear theory by use of a two-dimensional numerical code (e.g. Elstner, Meinel & Beck 1992), but for non-linear problems an essentially three-dimensional code is necessary. Moss et al. (1993) reported some results from using a non-linear three-dimensional dynamo code. However, this was an adaption of a code written in spherical coordinates to study stellar dynamos, and its geometry was ill-suited to studying dynamos in thin layers, surrounded by extensive haloes. Further, the azimuthal resolution available was quite limited. Moss (1995) developed a quasi-two-dimensional approximation applicable to thin discs, which retained the full azimuthal structure at the expense of losing the z-dependence (z being the coordinate perpendicular to the disc plane). This approach (the ‘NZ’ = no-z’ approximation) produced a number of interesting results, concerning, for example, excitation of bisymmetric structure by parametric resonance with the spiral arms (Moss 1996a) and the field structure arising from tidal interactions of galaxies (Moss 1996b). One important conclusion was that for certain problems high azimuthal resolution was essential (Moss 1996b; see also Sokoloff et al. 1997), and so it would be difficult to resolve some of these issues by using the general approach of Moss et al. (1993).

However, until results can be compared with those obtained from a fully three-dimensional code, some uncertainty must remain concerning the accuracy and general validity of the NZ approximation. [Note, however, that the results of Moss (1996b) are broadly consistent with those of Moss et al. (1993).] Furthermore, it is clear that some phenomena are intrinsically three-dimensional. Thus in this paper we describe a simple, fully three-dimensional, non-linear dynamo code, written in cylindrical polar coordinates and so naturally suited to describing disc geometries, and report some results that can be compared to those obtained previously with the NZ approximation.

2 THE MODEL

Moss (1995) describes the NZ approximation to solve the non-linear mean field dynamo equation

\[
\frac{dB}{dt} = \nabla \times (\alpha B + \alpha B) - \nabla \times (\epsilon \nabla \times B),
\]

(1)

where \(\alpha = \alpha(r, t) [1 + \alpha_0 B(r, t)^2]\), in a thin disc, using cylindrical polar coordinates \((r, \phi, z)\). The NZ code solves the equations for the field components \(B_r, B_\phi\). With this approximation, \(z\) does not appear and explicit reference to \(B_z\) has been removed by using the condition \(\nabla \cdot B = 0\). Setting \(\alpha_0 = 0\) gives a linear calculation, and \(\alpha_R = 1\) gives the standard \(\alpha\)-quenching non-linearity.

When solving the equations of the fully three-dimensional problem, it is necessary to calculate \(B_z\) explicitly everywhere. Further, the solution \(B(r, t)\) must satisfy \(\nabla \cdot B = 0\). One approach would be to solve directly for the vector potential \(A\), where \(B = \nabla \times A\). Another is to use a numerical scheme designed to preserve accurately the initial condition \(\nabla \cdot B = 0\) (cf. Moss et al., in preparation). Here we choose a rather less flexible alternative that is, however, simple to implement and capable of answering some of the questions raised in the Introduction. Our procedure is to integrate the equations for \(B_r\) and \(B_\phi\), and then to determine \(B_z\) by satisfying the condition \(\nabla \cdot B = 0\). We restrict ourselves to solutions of even parity with respect to the disc plane (as, indeed, does the NZ approximation).
approximation), so that \( B_z = 0 \) on \( z = 0 \). Thus, advancing the solution by one time-step involves integrating the partial differential equations for \( B_x \) and \( B_y \) and then solving \( \nabla \cdot B = 0 \) to determine \( B_z \). A modified Dufort–Frankel scheme is used for the integration procedure. The code allows \( \alpha \) and \( \beta \) to be functions of space and time, and \( \eta \) and the angular velocity \( \Omega \) to depend on \( z \). It is implemented on an \( N \times N \times NK \) grid, over \( 0 \leq r \leq r_{\text{max}} \), \( 0 \leq \phi \leq 2\pi \), \( 0 \leq z \leq Z_{\text{max}} \). Typically \( r_{\text{max}} = 15 \text{kpc} \), \( Z_{\text{max}} = 6 - 7.5 \text{kpc} \). The code allows a variety of boundary conditions to be applied: for example, on \( r = r_{\text{max}} \), \( B_x \) and \( B_y \) can be zero or an ‘open’ condition, \( \partial \phi (r^2 B_{\phi r}) = 0 \), where \( n \) is a small positive integer, can be used. On \( z = Z_{\text{max}} \), \( B_z = B_y = 0 \) or \( \partial \phi (r_{\phi y} B_{\phi y}) = 0 \), where \( r_{\phi y} = |r| = \sqrt{r^2 + z^2} \). In practice, results were found to be insensitive to the choice of boundary conditions. We note that when \( \eta = \eta(r) \), a ‘diamagnetic’ term, \( \nabla \times (\gamma \times \vec{B}) \), \( \gamma = -0.5 V \eta \), should appear on the right-hand side of equation (1) (e.g. Vainshtein & Zeldovich 1972). This was omitted for the investigations described here, but is included in a later, more general, version of the code. Limited experimentation indicated that the inclusion of this term would not significantly affect the results presented here.

The defining parameters of the models are the three magnetic Reynolds numbers, \( R_x = \alpha_\Omega r_{\text{max}} / \eta_0 \), \( R_y = \Omega_\Omega r_{\text{max}} / \eta_0 \), and \( R_m = \mu_\Omega r_{\text{max}} / \eta_0 \), where \( \alpha_\Omega \), \( \Omega_\Omega \), \( \mu_\Omega \) and \( \eta_0 \) are representative values of \( \alpha \), \( \Omega \), the non-axisymmetric velocities and \( \eta \) respectively. These arise from a non-dimensionalization such that length is measured in units of \( r_{\text{max}} \) and time in global diffusion times, \( r_{\text{max}}^2 / \eta_0 \). In the following we measure \( r \) and \( z \) in units of \( r_{\text{max}} \), without further comment.

3 TESTS

We tested the code by comparing results with those of two previously published linear calculations.

(i) Elstner et al. (1992) computed a number of non-axisymmetric eigenfunctions and corresponding eigenvalues, of the equation (1) with \( \alpha = \alpha(z) \), \( \eta = \eta(z) \). The code described in Section 2 reproduced quite closely both the marginal dynamo number and eigenfunction for their model III.

(ii) Moss & Brandenburg (1992), using a modified spherical dynamo code, studied both axisymmetric and non-axisymmetric dynamo modes in a thin disc. Although the present code could not implement precisely the boundary conditions of Moss & Brandenburg, it did reproduce the marginal \( R_s \) value for the S1 mode with \( z_\Omega = 0.15 \) to within about 10 per cent, and found a similar field structure. In particular, our solution gave a reasonably good reproduction of fig. 3 of Moss & Brandenburg. Given the differences between the codes, this was felt to be a satisfactory agreement.

Overall, these tests suggest that the present code performs satisfactorily in the linear regime.

4 M81 INTERACTION MODEL

The spiral galaxy M81 is believed to have undergone a recent tidal encounter with its companion galaxy NGC 3077. The resulting non-axisymmetric streaming motions can be expected to influence markedly any dynamo acting in M81. M81 shows evidence for a strong bisymmetric field component, and it is plausible to suggest that these phenomena are connected. This is the physical situation discussed in Moss (1996b), using the NZ model.

We attempted to reinvestigate the problem with the three-dimensional code. Typically we used a grid size of \( 61 \times 61 \times 51 \), with \( r_{\text{max}} = 15 \text{kpc} \), \( Z_{\text{max}} = 7.5 \text{kpc} \). At the disc plane, \( z = 0 \), we took \( \eta = 10^{26} \text{cm}^2 \text{s}^{-1} \). The disc was defined by setting \( \alpha = \alpha_\Omega \left(z_\Omega^2 - z^2\right) \), \( z < z_\Omega = 0.1 \), \( \alpha = 0 \), \( z \geq z_\Omega \), and putting \( \eta = 0.2 \eta_0 \) at \( z = 0 \), increasing smoothly to \( \eta_0 \) at \( z = z_\Omega = 0.2 \). We also experimented with a somewhat different \( z(\eta) \), with \( \eta = \eta_0 \text{ in } z < z_\Omega / 2 \), and \( \eta = \eta_0 \text{ smoothly in } z_\Omega / 2 < z \leq z_\Omega \), with \( 0.02 \lesssim f \lesssim 0.2 \). The general nature of the results was not very sensitive to such changes. The dynamo-active disc is rather thick (aspect ratio about 0.1), but is a compromise between a realistic representation of the true physical situation and acceptable speed of computation. We took a standard value \( R_s = 2 \times 10^4 \) giving a maximum value of \( \alpha \) of about 0.2 \text{km s}^{-1} \text{corresponding to } \alpha_{\text{max}} = 3.8 \times 10^{-4} \alpha_0 \). The velocities \( u(r, t) \) were supplied by Thomasson (private communication), and were based on the two-dimensional model of the M81–NGC 3077 interaction discussed by Thomasson & Donner (1993) — see Moss (1996b) for more details. As in the latter paper, the initial condition for the interaction calculation is the steady axisymmetric (S0) solution in the absence of the streaming velocities.

The evolution with time of the energies in the \( m = 0, 1, 2, 3 \) azimuthal Fourier modes is shown in Fig. 1, and projections of the fields on to the disc plane at approximately \( 5 \times 10^8 \) and \( 9 \times 10^8 \) \text{yr} after the beginning of the interaction are shown in Fig. 2. (The present epoch corresponds to a time of approximately \( 5.5 \times 10^8 \) \text{yr}.) The variations of \( B_y \) with \( \phi \) at two selected values of \( (r, \phi) \) are shown in Fig. 3. Sharp azimuthal gradients are present in the outer part of the disc. Increasing \( R_s \) to \( 5 \times 10^9 \) produced modest changes in the field structure (Fig. 4). These results are to be compared with those of Moss (1996b), in particular figs 5 and 10 of that paper. There are some differences, notably in the behaviour of the axisymmetric energy towards the end of the computation. Also, the field seems to form a more regular spiral pattern in the three-dimensional case. Nevertheless, there is a strong general correspondence between the NZ and three-dimensional solutions.
5 PARAMETRIC RESONANCE

5.1 Results

The idea motivating this investigation is that a parametric resonance with an $m = 2$ azimuthal modulation of $\alpha$, driven by the spiral density wave, may enhance the growth rate of the $m = 1$ dynamo mode. Classically (Landau & Lipschitz 1969), the resonance occurs at a 2:1 ratio of frequencies. In Moss (1996a), the alpha coefficient was modified to

$$\alpha = \alpha_0 [1 + s_2 \cos(2\phi - \omega_s t - Kr)],$$

(2)

where $\alpha_0$ is a constant with, fairly arbitrarily, $K = 20$ and $s_2 = 0.5$. In that paper it was shown that such a resonance occurs in the NZ approximation, over a comparatively wide range, $2 \leq \omega_s/\omega_0 \leq 4$, where $\omega_s$ is the driving frequency (equation 2) and $\omega_0$ the frequency of the undriven $m = 1$ mode. These results are discussed fully, and further references given, in Moss (1996a).

We performed some experiments similar to those of Moss (1996a) with the three-dimensional code. A resolution of $41 \times 21 \times 41$ was found to be adequate, and we set $Z_{\text{max}} = 6$ kpc, $R_{\text{max}} \approx 15$ kpc. We took

$$\alpha = \alpha^*(z) [1 + s_2 \cos(2\phi - \omega_s t - Kr)],$$

(3)

Figure 2. Projection of field at $z = 0.6$ kpc on to the plane $z = \text{constant}$ for a calculation with $R_s = 2 \times 10^4$ at (a) about $5 \times 10^8$ yr, and (b) $9 \times 10^8$ yr after the start of the interaction.

Figure 3. Azimuthal variation of $B_\phi$ at approximately $9 \times 10^8$ yr (the end of the simulation) at $r = 7.5$ kpc, $z = 0.3$ and $1.5$ kpc (continuous and long-dashed curves), and $r = 11$ kpc, $z = 0.3$ and $1.5$ kpc (medium- and short-dashed curves).

Figure 4. As Fig. 2(b), for a calculation with $R_s = 5 \times 10^4$. 


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5.2 Comparison with the NZ approximation results

For all the solutions discussed in Section 5.1, the frequency satisfies $\omega = \omega_0$, that is, the frequency of the azimuthal dynamo wave is unaffected by the driving term. This is in contrast to the solutions of Moss (1996a) in the NZ approximation, where it was found that frequency locking was present, with $\omega = \omega_0/2$ over the range where resonance was important. Further, in the NZ case, outside the range $2 \leq \omega_0/\omega_0 \leq 4$, it was found that $\Gamma \approx \Gamma_0$, that is, there was no enhancement of the growth rate at zero frequency. The current three-dimensional code finds $\Gamma$ to be significantly enhanced for $1 \leq \omega_0/\omega_0 \leq 3$, somewhat different from the range $2 \leq \omega_0/\omega_0 \leq 4$ of Moss (1996a). Also $\Gamma \neq \Gamma_0$ as $\omega_0 \to 0$, again in contrast to the NZ results.

The reason for the absence of frequency locking in the three-dimensional case is unclear. The variation of $\eta$ and $\alpha$ through the disc means that, if slices taken at different heights $z$ were to be considered independently of their neighbours, they would have different ‘local’ $m = 1$ unperturbed dynamo wave frequencies, $\omega_0(z)$. The frequency $\omega_0$ of the three-dimensional model would then be some sort of average over $\omega_0(z)$. It is possible that the result is a ‘smearing’ of the resonant response, and thus an absence of frequency locking. In contrast, $\omega_0$ is, of course, uniquely defined in the NZ model. The results presented above show that making $\eta$ independent of $z$ leaves $\omega_0 = \omega_0$ still. It is unrealistic to make $\alpha$ also independent of $z$, as the disc then is undefined.

For both the NZ solutions of Moss (1996a) and the three-dimensional solutions described above, the bisymmetric eigenmode is localized within a few kpc of the radius at which the $m = 1$ dynamo wave corotates with the gas, i.e. where $\omega = \Omega(r)$. For the model of Section 5.1, this condition is satisfied at fractional radius $r/R = 0.89$, and the maximum of the eigenmode occurs at $r/R \approx 0.88$. In the absence of frequency locking, this corotation radius does not change with $\omega_0$. In contrast, in the NZ case where $\omega = \omega_0/2$ in the resonant range, the corotation radius moves inwards as $\omega_0$ and so $\omega$ increases, and so does the maximum of the eigenmode. This effect is clearly seen in figs 2 and 3 of Moss (1996a): in fig. 2 where $\omega = \omega_0/2 = 3.0$, corotation occurs at $r/R = 0.64$, whereas in fig. 3 where $\omega = \omega_0/2 = 5.0$, $\omega = \Omega(r)$ at radius 0.35. Note that this is not corotation between the angular frequency of rotation and the spiral wave pattern frequency, but corotation with the dynamo wave frequency.

As pointed out by an anonymous referee, the period of the dynamo wave is $kh^2/\epsilon_d$, where $k = O(1)$ and $\epsilon_d$ is a representative value of the turbulent diffusivity. Thus the resonant condition implies that the decay time $h^2/\epsilon_d$ is approximately equal to the pattern period.

It thus appears that, when investigating the response of the model to a driving wave in $\alpha$, there are general similarities between the predictions of the NZ and three-dimensional codes, but also detailed differences, perhaps directly connected with the variation of physical properties perpendicular to the disc. Nevertheless the central result, that a resonant amplification of the linear growth rate of the bisymmetric mode occurs (and so a lowering of the corresponding marginal dynamo number), over a range of driving frequencies, is robust. Frequency locking is an interesting feature mathematically, but its relevance to the explanation and interpretation of observations is unclear.

Figure 5. Growth rate $\Gamma$ versus logarithm of driving frequency $\omega_0$. The value of the unperturbed growth rate $\Gamma_0$ is indicated by the broken line.

where

$$\alpha'(z) = \alpha_0(z) - z^2, \quad z < z_0,$$

$$\sigma'(z) = 0, \quad z = z_0,$$

with $\alpha_0 = 0.1$. For $z < z_0 = 0.2$, $\eta(z) = 0.2\eta_0$, and $\eta$ increases smoothly to $\eta_0$ at $z = 2z_0$. The angular velocity was given by a Brandt-type law,

$$\Omega = \frac{\Omega_0}{1 + \left(\frac{z}{z_0}\right)^{2/3}},$$

with $J_0 = 0.2$. Taking a disc plane value for $\eta$ of $10^{26}$ cm$^2$ s$^{-1}$, so that $\eta_0 = 5 \times 10^{26}$ cm$^2$ s$^{-1}$; we fixed, a little arbitrarily, the dynamo parameters as $R_0 = 2 \times 10^3$, $\gamma_0 = 10^4$, and followed the investigative procedure described in sections 2 and 3 of Moss (1996a).

With $\mathcal{S}_2 = 0$, the dimensionless undriven frequency is $\omega_0 = 222$, and the corresponding linear growth rate is $\Gamma_0 = 6.9$, i.e. $\Gamma_0 \propto \exp(\Gamma_0 + i\omega_0)t$. We then set $\mathcal{S}_2 = 0.5$, and allowed $\omega_0$ to vary, and determined the corresponding values of the frequency $\omega$ and growth rate $\Gamma$. The results are summarized in Fig. 5. There is a clear and broad resonant peak, centred at $\omega_0 = 2\omega_0$. For large $\omega_0$, $\Gamma \to \Gamma_0$. However, as $\omega_0 \to 0$, $\Gamma$ does not approach $\Gamma_0$. That is, even the imposition of a stationary spiral pattern in $\alpha$, without changing the value of $\int \alpha \, dV$ over the volume of the disc, enhances somewhat the $m = 1$ growth rate. Although perhaps a little surprising, somewhat similar behaviour was also reported by Moss, Brandenburg & Tuominen (1991), where the overall growth rate was found to be enhanced by a stationary $m = 1$ modulation of $\alpha$.

We also investigated, less thoroughly, a slightly supercritical model in which $\eta$ was independent of $z$. The results were quite similar to those described above: there was a broad resonant response over $1 \approx \omega_0/\omega_0 \approx 3$, with an enhancement of the growth rate by a factor of about 3 at $\omega_0 = 2\omega_0$.

We did not attempt to produce a model in which the enhanced growth rate of the $m = 1$ mode was greater than that of the fastest growing axisymmetric mode. This is only likely to occur for rather thinner discs than we can readily study, where the difference between the unperturbed $m = 0$ and $m = 1$ growth rates is correspondingly smaller.
6 CONCLUSIONS

We do not yet have a completely general, three-dimensional non-linear dynamo code, as we are limited to investigating solutions with even parity with respect to the disc plane. Nevertheless, conventional dynamo theory does suggest that even-parity modes are more readily excited, and so this restricted three-dimensional code does offer a first opportunity of making a truly three-dimensional non-linear calculation and, inter alia, of assessing the validity of the NZ approximation.

The results presented here suggest that the NZ approximation, as developed by Moss (1995, 1996a, b), does give a reasonable insight into the properties of non-linear dynamo solutions in relatively thin discs. In particular, the general features of the interaction between M81 and NGC 3077 do appear to be reasonably well represented by the NZ treatment. Some significant differences are found when investigating the possibility of parametric resonance enhancing the growth rate of the bisymmetric dynamo mode, in particular the absence of frequency locking, but still there are strong similarities between the results of the NZ and three-dimensional treatments. The NZ approach has the advantage of giving a very much faster code, with much smaller memory requirements, for given resolution in the disc plane.

Clearly the NZ approximation has severe limitations. By definition, information about field structure perpendicular to the disc plane is lost. Also, velocities perpendicular to the plane are excluded, it is only valid for very thin discs in the \( \omega_0 \)-limit, so central bulges and disc flaring cannot be included, and only even-parity fields can be investigated. The last deficiency is, of course, shared by the three-dimensional code described here, and interesting phenomena such as streaming velocities that are not symmetric with respect to the disc plane, symmetry breaking in general, and the effects of long-lived transients arising from the (unknown) initial conditions (e.g. Moss et al. 1993; Poezd, Shukurov & Sokoloff 1993) cannot be completely described. In order to represent more fully ‘real’ galaxies, a more general version of the code would be necessary. Nevertheless, the restricted version of this paper does allow light to be shed on certain problems.

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