Information theory–based measures of similarity for imaging shallow-mantle discontinuities

C. Bapanayya, P.A. Raju, S. Das Sharma, and D.S. Ramesh
NATIONAL GEOPHYSICAL RESEARCH INSTITUTE (COUNCIL OF SCIENTIFIC AND INDUSTRIAL RESEARCH), HYDERABAD 500007, INDIA

ABSTRACT

Mode conversions, such as the P-to-s (Ps) converted waves, are now employed in a routine manner to image the velocity boundaries in Earth's interior. However, there exists an ambiguity in establishing shallow-mantle velocity discontinuities in the depth range 100–300 km through the Ps receiver function approach. This primarily stems from overlap in the time windows of arrivals of direct converted phases from the target depth boundaries (~100–300 km depth) and reverberations originating from shallow structures (e.g., from crust and/or shallow lithosphere layers). Attempts have been made to address this problem. Classically, limited success has been achieved through methods such as the move-out, which are essentially performed in the measurement space. Recognizing this, we explore generic space-related information theory measures of similarity to extract diagnostics of discrimination between the primary converted waves and the multiply reflected arrivals in Ps receiver functions. Various measures of similarity, such as the mutual information (MI), and associated normalized distance measures, like the normalized variation of information (NVI) and normalized information distance (NID), are successfully applied to receiver function data sampling regions of different tectonic regimes of wide antiquity. Our seismological stations are located in the Archean-Proterozoic craton–mobile belt regions of SE India, Canada, and Phanerozoic United States. Significantly, at several locations in SE India, we interpret the unambiguous presence of midlithospheric discontinuities for the first time. We also either confirm or negate the presence of midlithospheric discontinuities beneath stations FFC (Flin Flon) and HRV (Harvard) located in North America. This study reinforces the presence of significant velocity contrast features related to the Lehmann discontinuity depth beneath the Precambrian cratonic stations of India (Hyderabad and Cuddapah) and the Phanerozoic Pasadena station in the United States. Further, multiple deep discontinuities of opposing velocity contrasts are delineated at depths of ~275 km and ~320 km beneath station PAS (Pasadena). Our results therefore show tremendous potential to unambiguously detect and distinguish between direct converted seismological phases and multiple reverberations. These information theory approaches discriminate the seismic phases unequivocally. This new approach thus complements the S-to-p (Sp) receiver function technique, which is suitable for detection of shallow-mantle discontinuities (midlithospheric discontinuities, lithosphere–asthenosphere boundary, Lehmann, etc.).

INTRODUCTION

A stratified Earth model as revealed by seismological observations based on P-to-s (Ps) receiver function (Vinnik, 1977) results seems to be a good proxy to the real planet Earth. The Mohorovicic discontinuity (Moho), which separates Earth’s crust from the underlying mantle, and the 410 km and 660 km interfaces, which bound the mantle transition zone, are the three major velocity depth discontinuities inside Earth’s upper mantle. Their detection remains unequivocal mainly due to their global presence. As a consequence, the presence of these three major boundaries (the Moho, 410 km, and 660 km) is robustly manifested in a wide variety of observations/data, both geophysical and geochemical, that represent and sample deep layers in Earth. However, issues related to the unambiguous detection of the possible presence of further stratification, especially, at shallow-mantle depths (~150–300 km), particularly using Ps data, remain contentious (e.g., Sheehan et al., 2000; Li et al., 2002; Ramesh et al., 2010a). Essentially, complexities associated with registration and recognition of the direct converted signals originating from shallow–mantle target boundaries such as the Lehmann (Lehmann, 1959) and X–discontinuities (Revenaugh and Jordan, 1991) residing in the depth range of 150–300 km render them equivocal. In addition, recent recognition of the presence of intralithospheric layering in the form of midlithospheric discontinuities, especially beneath the cratons (Abt et al., 2010; Fischer et al., 2010; Ford et al., 2010; Yuan and Romanowicz, 2010), at ≤100 km depth further complicates this issue. The main problem in the Ps receiver functions is that the time windows of arrival of the primary conversions originating from the ~100–300 km depth interfaces overlap with the multiply reflected waves arising from shallower structures. Though undesirable, this interference by the reverberations related to shallow structures with primary conversions is inescapable and imparts ambiguity in detection of the desired target depth signals. Because the presence of shallow-mantle layers and their possible association with chemical changes and/or mineralogical phase transitions have profound significance for mantle composition, dynamics, and convection, in addition to regulation of plume activity, their unambiguous detection in Earth’s interior assumes importance.

A schematic ray-path diagram presented in Figure 1 shows the various receiver function phases that form the focus of our present study. Attempts have been made previously to distinguish between these wave types based on certain criteria. For example, Yuan et al. (1997) recognized that the direct converted waves (Moho: Pms; Lehmann: PLs; 410 km: P410s, etc.; see Fig. 1) and reverberation phases (crustal multiples: Pps, Pss, etc.; see Fig. 1) have opposite move-out as a function of ray parameter. Several researchers have therefore explored this characteristic to discriminate between these arrivals when they tend to arrive in overlapping time windows (Sheehan et al., 2000; Li et al., 2002). However, owing to the small move-out of the direct conversions from boundaries in the depth range
In this research article, we further expand our earlier studies and explore information-based similarities between the primary converted phases and the multiples employing a Gaussian distribution as a datum. The mutual information (MI) and associated normalized metric distance measures emerge as the prime candidates for seismological phase discrimination when applied to both synthetic data and observations accrued from diverse tectonic regimes of the globe of wide antiquity. Our results show that information theory–based similarity measures can indeed distinctly and unambiguously discriminate between the primary converted phases and the reverberations. This study reinforces the presence of significant velocity contrast features related to the Lehmann discontinuity depth beneath the Precambrian cratonic stations of India (Hyderabad and Cuddapah) and the Phanerozoic Pasadena station in the United States, as already reported by Ramesh et al. (2010a). In addition, this method is tested to confirm or negate the presence of midlithospheric discontinuities beneath stations Fin Flon and Harvard, North America. Finally, the efficacy of this technique in recovering information about the presence of multiple deep discontinuities is also demonstrated. Therefore, we feel justified in claiming that this new approach complements the S-to-p (Sp) receiver function technique that is now often employed for imaging shallow-mantle discontinuities (midlithospheric discontinuities, lithosphere-asthenosphere boundary, Lehmann, etc.).

THEORETICAL BACKGROUND AND METHOD

In this section, we first enumerate some specific observations on the characteristics of conversion coefficients of different Ps receiver function phases. This enables us to address the seismological problem of identification of shallow-mantle layers using receiver functions by application of measures of similarity based on information theory.

Conversion coefficients (open circles) of direct conversions and reverberation phases are computed as a function of angle of incidence following Aki and Richards (1980), and they are presented in Figure 2. A cursory observation of Figure 2 suggests that the direct and multiple waves seem to follow distinctly different distributions. Taking a cue from this, these data (open circles) were fitted using an optimal (minimum) number of Gaussian distributions. At least two Gaussian distributions were found to be necessary in each case to obtain a good fit ($R^2 > 0.994$). The respective constituent peaks are shown as dashed and thin curves, while the cumulative fits are shown by thick curves. However, the constituent peaks in each fit differ both in terms of their amplitude and full width at half maximum (FWHM). It is important to note that for the direct phases, viz. Pms and PLs, the first constituent peak (dashed curve) is the major contributor, while in case of the multiples, viz. Pps and Pss, the second peak (thin curve) assumes significance (see Fig. 2 and its caption). It is essentially the relative dominance of the constituent peaks that incisively discriminates between the direct phases and multiples. Although differences in the observed distributions were mentioned in our earlier publication (see Ramesh et al., 2010a), these are now demonstrated on evidently stronger theoretical foundations as products of superposition of two different attendant constitutive Gaussian peaks. To sum up, the like phases, either the primary conversions or the multiples, share mutually consistent characteristics among themselves irrespective of their genre. Therefore, it is expected that the distributions of the actual phases in time domain also differ considerably. It is in this context that information theory can assume a vital role in seismological problems too, aside from its success in economics, biology, communication theory, etc.

The concept of entropy as familiar to physicists can be traced to the troika of Clausius, Boltzmann, and Gibbs starting from the 1850s. Entropy was given a probabilistic interpretation in information theory by Claude Shannon in his seminal paper in 1948 (Shannon, 1948), heralding its extensive practical applications in various branches of engineering and basic sciences. Since entropy statistics (measures) provide important tools to indicate variety in distributions at particular moments in time and to analyze evolutionary processes over time, they are perhaps best suited for time-series analysis. Therefore, entropy statistics can play a vital role in discriminant analyses of time signals such as primary converted phases...
Detection of shallow-mantle layers using measures of similarity

originating from depth discontinuities inside Earth and the interfering multiples, which have a different origin.

Information theory–based similarity and/or distance measures could represent specific approaches to compare data clusters or objects in general. These form a fundamental class of measures for comparing clusters. A reasonable beginning based on information would be to determine the similarity between two clusters or distributions. One such measure of similarity is the mutual information (MI). It is the most basic measure of similarity between two distributions. MI between two variables x and y on discrete spaces X and Y is defined as

\[ MI(x, y) = -\sum P(x, y) \log_2 \frac{P(x, y)}{P(x) \cdot P(y)} \]  

(1)

where, \( P(x) \) and \( P(y) \) are the respective probability distributions of \( x \) and \( y \), and \( P(x, y) \) is the joint probability distribution. MI therefore (1) characterizes the mutual dependence (similarity) between the two variables. (2) can also trace nonlinear similarities, and (3) is a non-normalized, dimensionless non-negative quantity. However, for comparing two features, a quantity that is metric and normalized is a more intuitive measure (Li et al., 2004; Vinh et al., 2010), though, for some particular applications MI can still be useful as a robust measure. Recently, information theory–based distance measures such as normalized variation of information (NVI) and normalized information distance (NID) have come into vogue and are shown to satisfy both the normalized and metric properties (Vinh et al., 2010). These are possibly the best candidate indices of discrimination in our case, where the objective is to unambiguously identify direct converted phases and reverberations.

A metric distance measure should satisfy the following three properties:

1. Positive definiteness,
   \( d(x, y) \geq 0 \) (non-negativity),
   \( d(x, y) = 0 \) if \( x = y \) (identity);
2. Symmetry,
   \( d(x, y) = d(y, x) \);
3. Triangle inequality,
   \( d(x, z) \leq d(x, y) + d(y, z) \).

From the properties of metric, it can be emphasized that a distance measure that is normalized and metric is a more intuitive measure for cluster comparisons.

Figure 2. Gaussian fits to conversion coefficients (open circles), showing fits to primary conversions; Pms, PLs, and multiply reflected phases; Pps and Pss. Superposition of two Gaussian phases is found adequate to obtain a good fit to the conversion coefficient curves. In each panel, dashed and thin solid curves represent the constituent peaks, while thick solid curve is the cumulative fit. An observation relevant to the entire range of angles of incidence is that peak 1 plays a dominant role to explain the nature of primary conversions (Pms and PLs), whereas peak 2 contributes significantly to elucidate reverberations (Pps and Pss). Note that the Ps receiver function distance range roughly corresponds to 20°–40° angle of incidence. In this range, peak 1 is the dominant contributor to direct conversions [Pms and PLs], while peak 2 has an insignificant role. In contrast, for the multiply reflected phases [Pps and Pss], both peaks contribute rather significantly.
The distance measures NVI and NID are related to the normalized mutual information. In the former, the normalization factor is the joint entropy, \( H(x, y) \), while the latter uses the maximum of the entropies, \( \max \{ H(x), H(y) \} \), of the two variables \( x \) and \( y \). These are formally defined as:

\[
\text{NVI}(x, y) = 1 - \frac{\text{MI}(x, y)}{H(x, y)}
\]

and

\[
\text{NID}(x, y) = 1 - \frac{\text{MI}(x, y)}{\max \{ H(x), H(y) \}}
\]

The Venn diagrams presented in Figure 3 bring out the inverse relationship between joint entropy and mutual information. The distance measures NVI and NID and their relation to mutual information are also shown in Figure 3. The ordinates on either side of the distance measures are the information scale in arbitrary units. It is evident from Figure 3 that the upper bound of NVI is variable, while that of NID is rigid. This difference between the respective upper bounds can directly be traced to their normalization by distinct entropies. For these reasons, each incremental value between 0 and 1 of NVI cannot be obtained as a measure of comparison because of its varying upper bound (joint entropy). In contrast, NID yields all values in view of its rigid upper bound (maximum entropy). Of these two distance measures, NID always measures the smallest distance. In fact, NID is the smallest among all possible distance measures (Li et al., 2004). This property of NID is often referred to as the universality property. Hence, NID is treated as a general purpose measure for comparison of clusters (Vinh et al., 2010). It is interesting to note that the distance measures NVI and NID attain the same value in two contrasting situations: (1) When the two variables are completely dependent and their information content is equal, i.e., perfectly similar distributions, both the distance measures become zero (not shown in Fig. 3); and (2) when the two variables are mutually exclusive, the two distance measures attain the maximum value of unity.

Mutual information, the most basic similarity measure determines the similarity between two distributions. In contrast, NVI and NID determine how deviant one distribution is from the other. In this context, it is important to realize that the incident P wave is the most generic representative of all seismic phases on a Ps receiver function. Hence, we first determined the distribution of this reference phase utilizing a synthetic receiver function and found it to be perfectly Gaussian (Fig. 4). The Gaussian fit (solid...
Detection of shallow-mantle layers using measures of similarity | RESEARCH

The methodology developed here essentially quantifies the measures of similarity of the seismological phases (such as the mode conversions from various interfaces and reverberations) with that of the datum (reference) P pulse without involving the measurement space. It is important to obtain clear observations of Ps conversions and multiples, as recorded in our data from SE India. This method would work irrespective of the geometrical disposition of the target interfaces as long as the seismological phases are well recorded and remain correlatable. Therefore, this method is quite robust, as demonstrated later herein.

APPLICATION TO DATA AND RESULTS

We applied the concept of information theory–based distance measures first to the synthetic receiver functions, followed by application to well-recognized converted phases like the Pms and P410s and crustal multiples such as the Pps and Ps registered at various locations around the globe. Data related to the PLs phase identified in our previous study (Ramesh et al., 2010a) are explored using this new approach to establish the veracity of this usually ambiguous seismic phase. Further, the utility of our present technique in classifying a receiver function phase is clearly brought out by its application to a few prominently registered unknown phases and background noise across various time windows at several locations. All computations were done using MATLAB 7.6.

Synthetic receiver functions (Fig. 5) were generated using the model described in Ramesh et al. (2010a), with a shallow-mantle discontinuity at 220 km depth adopting standard methods (Haskell, 1962; Langston, 1977, 1979). For the computation of MI, NVI, and NID, a joint frequency distribution table corresponding to each seismic phase (e.g., Pms, Pps, PLs, etc.) and the reference Gaussian phase (shown in Fig. 4) was constructed following a standard procedure (e.g., Vinh et al., 2010). The class-width (h) used to construct such a two-dimensional contingency table was determined by Scott’s law (Scott, 1979), which is defined as

\[ h = 3.5 \times s \times n^{-1/3}, \]

where, \( s \) stands for standard deviation of the series, and \( n \) represents the length of the series.

![Figure 4. Reference P wave as a perfect Gaussian distribution. A synthetic P wave (open circles) with a Gaussian fit (solid line) is displayed. A full width at half maximum (FWHM) value of 0.642 s Gaussian distribution with an amplitude of 0.39 best describes the generated P-wave (R² = 1). This P wave is used as the reference distribution with respect to which similarity and distance measures of observed data (Pms, PLs, P410s, Pps, and Pss) are computed.](https://pubs.geoscienceworld.org/gsa/lithosphere/article-pdf/3/4/289/3038738/289.pdf)

![Figure 5. Synthetic Ps receiver functions. Ps receiver functions generated using the model described in Ramesh et al. (2010a) show the prominent converted and reverberation phases. The PLs phase can also be seen in the figure. The synthetic receiver functions are arranged in order of increasing slowness (decreasing distance). The slowness intervals are in steps of 0.002 s/km. The measures of similarity MI, NVI, and NID were first applied to these synthetic seismogram phases prior to their application to the observed data. Note that the Ps receiver function synthetics are displayed in ZRT coordinate system.](https://pubs.geoscienceworld.org/gsa/lithosphere/article-pdf/3/4/289/3038738/289.pdf)
A sample contingency table involving two arbitrary series $x$ and $y$ divided into four and five classes, respectively, is shown in Table 1. Summation along rows of the contingency table yields marginal frequency distribution of $x$, and that along columns provides marginal frequency distribution of $y$. Dividing each cell of the contingency table by the total length ($n$) of the series $x$ yields the joint probability distribution $P(x, y)$ of $x$ and $y$ (e.g., $f(Ix, Ily)/n = P(Ix, Ily)$; see Table 1). Summation of joint probabilities along the rows produces the marginal probability distribution of $x$, and those along columns yield the marginal probability distribution of $y$. Entropies $H(x)$ and $H(y)$ are computed using the marginal probability distributions of respective variables. MI, NVI, and NID are computed from the joint probability distribution and marginal probability distribution described by Equations 1–3. We applied this procedure to both synthetic as well as observed data across various time windows.

MI, NVI, and NID of synthetic seismic phases were measured relative to the reference Gaussian phase. Figure 6 shows the variation of MI, NVI, and NID of the Pms, PLs, Pps, and Pss phases related to synthetic receiver functions with respect to epicentral distance. A linear fit to the data reveals that MI of the direct phase Pms has a negative slope, while those of multiples (Pps and Pss) have positive slopes. The other distance measures, NVI and NID, by their definition, would yield slopes contrary to MI. Therefore, positive and negative slopes in MI would have an opposite sense in NVI and NID. This is indeed clearly brought out in Figure 6. Therefore, there is a clear discrimination between direct

<table>
<thead>
<tr>
<th>$x$</th>
<th>Iy</th>
<th>IIy</th>
<th>IIIy</th>
<th>IVy</th>
<th>Vy</th>
<th>Marginal frequency</th>
<th>Marginal probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ix</td>
<td>$f(Ix, Ily)$</td>
<td>$f(Ix, IIly)$</td>
<td>...</td>
<td>...</td>
<td>$f(Ix, IVy)$</td>
<td>$f(Ix)$</td>
<td>$f(Ix)/n = P(Ix)$</td>
</tr>
<tr>
<td>IIx</td>
<td>$f(IIx, Ily)$</td>
<td>$f(IIx, IIly)$</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>$f(IIx)$</td>
<td>$P(IIx)$</td>
</tr>
<tr>
<td>IIIx</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>$f(IIIx)$</td>
<td>...</td>
</tr>
<tr>
<td>IVx</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>$f(IVx, IVy)$</td>
<td>$f(IVx)$</td>
</tr>
<tr>
<td>Marginal frequency</td>
<td>$f(Iy)$</td>
<td>$f(IIy)$</td>
<td>$f(IIly)$</td>
<td>$f(IVy)$</td>
<td>$f(Vy)$</td>
<td>$n$</td>
<td></td>
</tr>
<tr>
<td>Marginal probability</td>
<td>$P(Iy)$</td>
<td>$P(IIy)$</td>
<td>...</td>
<td>$P(IVy)$</td>
<td>...</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Note: $Ix, IIx, ..., Ily, IIly, ..., etc., represent class intervals of $x$ and $y$, respectively. The row elements $f(Ix, Ily), f(Ix, IIly), ..., etc., indicate joint frequency of $Ix$ for different classes of $y$. Likewise, the column elements $f(Ix, Ily), f(IIx, Ily), ..., etc., yield joint frequency of $Iy$ for different classes of $x$. Other details are described in the text.

Figure 6. Application of similarity and distance measures to synthetic data. Direct converted phases Pms and PLs have opposite characteristics to those of multiples in the plots of MI (bits), NVI, and NID. Note that the unit describing the vertical axis is valid only for MI. This suggests that the primary conversions and the multiples can be distinguished based on the above three measures of similarity. It is interesting to note that NID always yields the minimum distance, as discussed in the text.
converted and multiply reflected phases in these plots. Further, the phase identified as PLs in our synthetic seismograms mimics the characteristics shown by the primary converted wave Pms in all the three measures. Thus, this enables us to clearly discriminate between primary conversions and reverberations of varied origin in a stratified Earth, especially in detection of shallow-mantle layers. NID has a smaller value among the two distance measures NVI and NID (see Fig. 6), establishing that NID always measures the minimum distance between two distributions. Also, the diagnostic of contrasting slopes of the primary and multiply reflected phases in all three measures is quite pronounced and is valid irrespective of whether the measure is a metric or not.

Data from six seismic stations, Hyderabad (HYB) and Cudappah (CUD) of India, Pasadena (PAS) and Harvard (HRV) of the United States, Flin Flon (FFC) and Fabhiser Bay (FRB) from Canada, are used in the present study. The choice of these six stations in our analyses was governed by their mapping of different geological/tectonic regimes of wide antiquity, besides enabling us to either reconfirm or further our earlier findings presented in Ramesh et al. (2010a). Ps move-out–corrected receiver function stacks corresponding to all the stations are reproduced in Figure 7A and Figure 7B. In Figure 7, several well-recognized phases, such as the Pms, P410s, Pps, and Pss, are indicated, respectively, as P1, P2, M1, and M2. The PLs phase, marked as P2, is based on our earlier results (Ramesh et al., 2010a). In addition, a few unclassified phases that are registered prominently in the receiver functions are indicated as Up1 and Up2. Besides these labeled arrivals, we also explored the nature of background noise across various time windows (not shown in Figs. 7A and 7B) employing the aforementioned measures of similarity.

The measure of similarity MI and distance measures NVI and NID at all six stations was calculated adopting the procedures detailed previously, using the same data presented in Ramesh et al. (2010a). Individual values of MI, NVI, and NID related to each seismic phase were sorted in increasing order of epicentral distance of the receiver function. Scott’s law (Scott, 1979) was applied to arrive at the optimal number of classes for grouping/binning the individual measures distance-wise. The number of classes turns out to be six in our case. The mean similarity measure value of each class was calculated and then plotted as a function of mean epicentral distance of the class. To offset the effect of different class frequencies, a weighted least-square fit to the data was employed. Figure 8 shows mean MI of different seismic phases as a function of mean epicentral distance. At all six stations, the MI of well-recognized direct phases (Pms and P410s) consistently yielded a negative slope. On the other hand, the fit to multiple phases (Pps and Pss) showed distinct positive slopes. The mean values of NVI and NID, as a function of epicentral distance, are shown in Figures 9 and 10, respectively. In these diagrams, the primary converted waves Pms and P410s exhibit positive slopes, while multiples show negative slopes in consonance with their definitions (see Eqs. 2–3). Therefore, in all these three measures, MI, NVI and NID, the direct conversions and multiples show remarkably distinct character by way of their opposing slopes. It is pertinent to note that the slopes do vary, ranging from gentle to steep in these plots at each location (Figs. 8–10), perhaps related to temporal variations in the ambient noise beneath a seismic station. The final trends in all the three measures, however, retain the positive and negative attitudes. The observed contrasting behavior of primary conversions and multiply reflected phases in data is also consistent with that from synthetic data. With this pronounced discrimination of seismic phases, it is therefore possible to unambiguously detect the manifestation of velocity discontinuities in the depth range 150–300 km, even when the related direct conversions from these target boundaries and crustal multiples are likely to arrive in overlapping time windows.

In an earlier study, at stations HYB, CUD, and PAS, prominent arrivals primarily in the time window 20–23 s were identified as PLs phase based on cluster entropy and information dimension (Ramesh et al., 2010a). In this study, we investigated arrivals in the same time window applying the measure of similarity and distance measures. The similarity and distance measures of this identified PLs phase yield negative slopes of MI and positive slopes of NVI and NID, in concert with those of the primary conversions Pms and P410s (Figs. 8–10). Hence, this similarity reconfirms that this candidate seismic phase is indeed a primary conversion originating from a shallow-mantle boundary popularly known as the Lehmann discontinuity.

With this assurance, we focus our attention on a few conspicuous arrivals of unknown origin labeled as Up1 and Up2 at various time windows (see Fig. 7A) in the receiver functions at stations PAS (26–29 s and 30–34 s), FRB (11–13 s), and CUD (8–12 s) in order to characterize them. The MI of these unclassified phases at these stations is shown in Figure 11. For the purpose of comparison, MI values of well-established Pms and Pps phases from stations CUD and PAS are also shown. It is clear from this figure that Up1 of FRB and CUD shows characteristics similar to those of reverberation phases. Hence, these arrivals can be treated as scattered waves and not as genuine conversions. Further, MI slopes of the two unknown phases Up1 and Up2 of PAS share similar characteristics as the primary converted phases. Interestingly, this Up1 is a positive-polarity phase, while Up2 has a negative polarity (Fig. 7A). The NVI and NID characteristics of all these phases (not shown in Fig. 11) affirm the above. However, further confirmation of these primary converted phases through other geophysical methods, due to presence of appropriate velocity contrast interfaces beneath PAS, is warranted.

Similar analyses were carried out at stations FFC and HRV for the unknown phases Up1 and Up2 (Fig. 7B) with negative polarity. The Up1 phase recorded at FFC (8.5–12 s) is characterized by slopes in conformity with a converted phase, while both the phases (Up1 and Up2) under investigation at station HRV in the time intervals 5.5–8 s and 9–12 s turn out to be reverberations (Fig. 11).

IMPLICATIONS OF THE RESULTS

The Lithosphere-Asthenosphere Boundary, Midlithospheric Discontinuity, and Cratonic Architecture

The discovery of a high-velocity lid over a pronounced low-velocity zone at an average depth of 80–100 km beneath the oceanic regions (Gutenberg, 1926) is viewed as the marker boundary between the mechanically stronger lithosphere resting on a yielding, viscous asthenosphere to signify the depth to the lithosphere-asthenosphere boundary. Therefore, observation of low seismic velocities in the upper-mantle regions is often interpreted as indicative of asthenosphere. However, the anticipated presence of a low-velocity zone is unfortunately not so well documented beneath the cratons compared to the oceans. This therefore has naturally added to the elusive nature of detection of lithosphere-asthenosphere boundary beneath the continents/cratons.

Recent advancements in receiver function analyses have enabled us to detect presence of both positive- and negative-velocity contrasts in the mid-lithospheric depths beneath the continents. The positive-velocity contrasts are interpreted depending on the tectonic settings of the region. However, the interpretation related to the observed negative-velocity contrasts at these depths remains contentious in several cases. Generally, two prominent negative-velocity contrasts within the shallow mantle are reported globally. Of these two, one is prominent in the general depth range 50–130 km, while the other is confined to 130 km ≤ depth ≤ 300 km (see Fischer et al.,
Figure 7. (Continued on following page).
An important question arises: What do these negative-velocity contrast interfaces beneath the continents represent?

In particular, reports of a shallow lithosphere-asthenosphere boundary (~80–100 km) in the cratonic regions of the globe cannot be reconciled with the surface-wave tomography results at the same locations that clearly document a fast lithospheric layer at depths of 150 km or beyond (Yuan and Romanowicz, 2010; Abt et al., 2010; Fischer et al., 2010). Further, geochemical analyses and thermobarometric estimates on cratonic xenoliths reveal a cold, depleted mantle until at least 150 km depth, which remained coupled to the overlying crust for few billion years (Griffin et al., 1999; Lee, 2006). These two lines of evidences strongly argue that the shallower negative discontinuity (~80–100 km) within the cratonic lithosphere cannot be interpreted as the lithosphere-asthenosphere boundary.

Although the origin of this negative discontinuity at ~100 km depth beneath cratons is not fully understood, there are several mechanisms in vogue to explain this feature. These include: (1) thin, low-velocity stacking of strata that represent partial melting and dehydration (Mierdel et al., 2007); (2) formation of an low-velocity zone in response to ascent of kimberlite magmas, where a strong and chemically distinct Archean layer acts as a barrier (Sleep, 2009); (3) presence of water (Karato and Jung, 19998); (4) availability of partial melts (Hammond and Humphreys, 2000); and (5) presence of hydrated minerals such as phlogopite, chlorite, and talc (Hacker et al., 2005; Mainprice et al., 2008).

Documentation and interpretation of the negative phases corresponding to 130–300 km depth range are also ambiguous. Data from the same locations in the cratonic regions of Australia, South Africa, Canada, etc., yield conflicting results, both in terms of observation of the corresponding seismic phases related to the lithosphere-asthenosphere boundary and its interpretation (Bostock, 1998; Sacks et al., 1979; Heit et al., 2007; Wittlinger and Farra, 2007; Snyder, 2008; Savage and Silver, 2008; Hansen et al., 2009; Vinnik et al., 2009; Ford et al., 2010). For example, based on rigorous Sp-wave modeling, Ford et al. (2010) concluded that the cratonic lithosphere-asthenosphere velocity gradient is not a sharp boundary and is distributed over more than 50–70 km in depth beneath central and western Australia. This result is in contrast to earlier findings from the same region (see Ford et al., 2010, and references therein).
To summarize, four independent criteria can be invoked to guide our interpretation of the observed negative phases in receiver functions as related to lithosphere-asthenosphere boundary or midlithospheric discontinuities beneath cratons based on: (1) presence or absence of a fast lithospheric layer (lid) from surface-wave tomography at ~150 km or more, (2) nature of the gradational velocity contrast across the target interface from Sp-wave imaging (e.g., in case of lithosphere-asthenosphere boundary, velocity gradient is distributed in excess of 50 km), (3) character of the heat-flow regime (e.g., low heat flow of ~45 mWm⁻² in locales with a deep lithosphere-asthenosphere boundary), and (4) geochemical and thermobarometric constraints on xenoliths.

Observations from SE India—Eastern Dharwar craton

Midlithospheric Discontinuities

In the backdrop of this discussion, we mention that a negative phase at ~10 s (~80 km depth), marked as low-velocity zone, in the SE India Sp receiver function stack sections (see Figures 2 and 5 in Ramesh et al., 2010b) has been identified. This negative phase was neither equated with the lithosphere-asthenosphere boundary signal nor explicitly named as a midlithospheric discontinuity. However, based on our final interpretation of a thick lithosphere beneath SE India (see Ramesh et al., 2010b), we implied that this negative phase observed at ~10 s in the data is indeed not related to the lithosphere-asthenosphere boundary. Also, in the current study, the majority of the designated independent interpretation criteria are in concert to support the interpretation that this ~10 s signal arises from a midlithospheric discontinuity placed at ~80 km depth within the SE India lithosphere of the Eastern Dharwar craton. Therefore, the observed low-velocity zone at ~10 s beneath stations of SE India (HYB, CUD, and KDM) in the Eastern Dharwar craton obviously represents a midlithospheric discontinuity, similar to that found beneath cratons of North America and western Australia. This is the first report of the presence of midlithospheric discontinuities from the Indian Shield. Though the origin of midlithospheric discontinuities remains uncertain, given the Archean antiquity of the Eastern Dharwar craton, it could be viewed as a relic of cratonic mantle formation, i.e., a “birth mark” related to the formation of the craton.
Detection of shallow-mantle layers using measures of similarity

**RESEARCH**

*Signals Related to the Lehmann Discontinuity Depths*

Reaffirmation of the presence of mode-converted signals related to the Lehmann discontinuity (L-discontinuity) depth region through the present approach together with the results presented in Ramesh et al. (2010b) provide several broad implications for the tectonics, evolution, and lithospheric configuration of the SE India. Recent Ps and Sp images of the Eastern Dharwar craton–Eastern Ghats belt region reveal a prominent westerly dipping velocity feature embedded at 160–220 km depth that was interpreted as a relict subducted remnant believed to be a product of a pre-Grenvillian collision episode (Ramesh et al., 2010b). This interpretation of operation of Wilson cycle during the Proterozoic receives support from ca. 1.85 Ga sensitive high-resolution ion micro-probe (SHRIMP) U-Pb ages on zircon (Vijaya Kumar et al., 2010) separated from the Kandra ophiolite complex along the SE margin of India. Further, documentation of a suite of rock occurrences in a particular sequence along with their age constraints (Vijaya Kumar et al., 2011) is reminiscent of our earlier proposed predominance of westward subduction in SE India (Ramesh et al., 2010b). Preservation of such a subducted relict feature, at least since the Mesoproterozoic at the L-discontinuity depth region, testifies as to its resistance to plate-recycling processes. This would become possible in a thick lithospheric root configuration. These findings together with presence of a midlithospheric discontinuity at ~80 km depth therefore collectively reinforce that the relict feature preserved at the L-discontinuity depth region is indeed a direct consequence of the presence of a thick lithospheric root beneath SE India, reiterating our earlier negation of a thin Indian lithosphere.

*Observations on North America Results*

**Station FFC**

The Ps receiver functions at station FFC reveal the presence of a negative phase that arrives at ~10.5 s (marked as Up in Fig. 7B). The information theory measures (Fig. 11) of this seismic arrival suggest that it corresponds to a negative-velocity contrast target interface at ~90 km. This boundary was interpreted as the lithosphere-asthenosphere boundary by Rychert and Shearer (2009). However, the seismic depth profile beneath
station FFC based on joint inversion of long-period seismic waveforms and SKS wave splitting data unambiguously argues that this negative phase, also observed by Rychert and Shearer (2009) in Ps receiver function stack, is a midlithospheric discontinuity rather than the lithosphere-asthenosphere boundary (see Yuan and Romanowicz, 2010). Further, the previously stated four independent criteria are in consonance with the interpretation of this negative phase (centered on ~10.5 s) as a midlithospheric discontinuity. Also, such an interpretation seems reasonable considering the Paleoproterozoic affinity of the rock exposures in the Flin Flon area of Canada on which station FFC is sited. The lithosphere-asthenosphere boundary beneath this location is shown in excess of 200 km (Yuan and Romanowicz, 2010).

Station HRV

The recent Ps (Rychert and Shearer, 2009; Abt et al., 2010) and Sp (Abt et al., 2010) results at station HRV indicate presence of two negative-velocity contrast boundaries at depths ~60 and ~100 km. While the former study concedes that the Ps signal corresponding to the shallower layer (~60 km) is ambiguous, in the latter, this signal is hardly registered in the Sp data enough to warrant interpretation. However, the conversion (both Ps and Sp) from the deeper interface ~100 km depth was interpreted as being caused by the lithosphere-asthenosphere boundary in both the studies. The Sp phase related to the lithosphere-asthenosphere boundary in Abt et al. (2010) is relatively weak (with an amplitude of ~0.045 fraction of parent S phase). However, it qualifies as a genuine Sp converted phase based on a predefined threshold difference of 0.5% slower Vs relative to any other local minimum below the Moho. Interestingly, our mutual information versus mean epicentral distance plots (Fig. 11) for these two arrivals (Fig. 7B) designated as Up1 (centered on ~7 s) and Up2 (centered on 10.5 s) yield positive slopes akin to scattered waves (multiples) rather than true conversions. This suggests that our results at HRV do not correlate these arrivals either with midlithospheric discontinuities or the lithosphere-asthenosphere boundary. This issue, however, remains to be resolved.

Station PAS

Measures of similarity results from station PAS in California present an interesting scenario. Aside from the reconfirmed presence of the

Figure 10. Normalized information distance (NID) between different seismic phases and the reference phase. NID is another normalized measure of similarity that uses the maximum of the marginal entropies as the normalizing factor. In addition to all other properties being same as NVI, NID invariably measures the minimum distance between the two distributions. This additional property of NID, often referred to as the universality property, imparts robustness, thereby making it a general purpose measurement. The diagnostics observed in MI and NVI plots for distinguishing various seismic phases are reflected in this plot.
Detection of shallow-mantle layers using measures of similarity

Lithosphere 3(4) 2012: 289–303

RESEARCH

L-discontinuity (at ~220 km depth), our analysis reveals additional stratification of the mantle beneath station PAS by way of the presence of two more layers of opposing impedance contrasts at deeper depths. A positive contrast boundary is located at ~275 km (phase Up1 in Fig. 7A), while the negative impedance layer is placed at a depth of ~320 km (phase Up2 in Fig. 7A). The presence of multiple positive impedance discontinuities in Ps receiver functions reminiscent of our observations has been reported in the contact regions of lithospheres of diverse antiquity (e.g., Li et al., 2002), ancient suture zones (e.g., Ramesh et al., 2010b), and several Phanerozoic orogenic zones (e.g., Dueker and Sheehan, 1997; Yuan et al., 1997). As station PAS typifies Phanerozoic terrain, the related interpretations for the origin of multiple positive contrast mantle discontinuities based on the tectonic setting assume relevance. We, however, elaborate on the negative impedance contrast boundary observed at ~320 km depth beneath PAS corresponding to the low-velocity layer waveform in the time window centered on 32 s (see Fig. 7A), above the depressed P410s arrival. Invoking arguments along the lines of Song et al. (2004) and Jasbinsek and Dueker, (2007), we believe that this deepening of the 410 km boundary by ~2 s at PAS is not caused by any velocity effect because the 660 km discontinuity is not correspondingly depressed. Hence, we speculate that a compositional anomaly related to a partial-melt layer linked to Farallon plate subduction and related backarc extension processes seem to be the most plausible explanation for the observed low-velocity layer atop the 410 km boundary (Song et al., 2004; Jasbinsek and Dueker, 2007). A similar interpretation was preferred to explain the negative impedance contrast at 330 km depth beneath the Sea of Japan, Yellow Sea, and easternmost Asia as recorded by seismic stations located in eastern China (Revenaugh and Sipkin, 1994).

CONCLUSIONS

Information theory measures successfully reveal robust discriminating diagnostics that can be used to characterize seismological phases of varied
ACKNOWLEDGMENTS

We are grateful to Vinod K. Gaur (Indian Institute of Astrophysics, Bangalore, India) for keen interest and critical preview of our work. The quality of this presentation has improved through discussion with him. R.K. Chadha (National Geophysical Research Institute) is acknowledged for sharing data from Indian stations. Two anonymous reviewers are thanked for their useful comments that were helpful in reorienting the original manuscript. Excellent handling of our manuscript by Raymond M. Russo, science editor of Lithosphere, is deeply appreciated. This work received support from the Council of Scientific & Industrial Research (New Delhi) in the form of Supra Institutional Project (SIP-0012-28) of the National Geophysical Research Institute and an in-house major laboratory project MLP-6509-28 (Das Sharma).
Detection of shallow-mantle layers using measures of similarity


