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DISCUSSION

D. J. Burns[2]

In his introduction Mr. Haslam states that the fatigue strength of a cylinder is severely reduced as a result of the penetration of the high pressure fluid into the fatigue crack. Does Mr. Haslam have any experimental evidence to show what size the cracks would have to be before they would admit sufficient oil to cause pressure on their faces? Also has he tried to reconcile his equations (1) and (2) with the "oil penetration" analyses used in earlier papers by Jones and Tomkins [11] and Frost and Burns [12]?

There appears to be a typographical error in the preprint in the section "Correlation of Fatigue Strength." If the maximum shear stress, τ, is the Lamé shear stress, it should be $PR^2/R^2 - 1$ not $P(R^2 - 1)/R^2$. In any case I do not understand why Fig. 1 is plotted in terms of τ. The author uses stress equation (1) and the Gough elliptical fatigue criterion to predict a fatigue limit pressure, P, c.f., equation (2). This predicted pressure should be compared directly with the experimental observations of fatigue limit pressure.

Previous authors presented their results in terms of τ because they believed that the parameter controlling fatigue life was the maximum range of shear stress. Since Mr. Haslam is not using the "maximum range of shear stress" fatigue criterion it is unnecessary and confusing to plot Fig. 1 in terms of τ.

In the section on influence of mean pressure no reference is made to the experimental data published by Burns and Parry [13] on the influence of mean pressure on thick-walled cylinders. Their results are far more comprehensive than the data shown in Fig. 2 which is only for thin-walled tubes. Has the author tried to analyse the aforementioned thick-walled data using the Gerber parabola law and equation (1)?

To predict the fatigue crack propagation life of a cylinder the author uses equation (8), which in its more general form $a \cdot \sigma_a^3 > C$ implies that the threshold between propagating and nonpropagating cracks is only a function of crack length and stress amplitude. Have Frost, et al. shown that C is independent of mean stress? If a specimen loading makes the stresses always compressive, will the crack propagate?

Equations (4) and (6) have been used previously by Crosby, Burns and Benham [14] to correlate fatigue data for thin-walled Al-Cu alloy tubes subjected to repeated internal pressure. They compared the behavior of two cylinders containing subcritical flows of initial depth, a_0, subjected to repeated normal stresses σ_1 and σ_2. They concluded that if the initial stress intensity factors, K_{11} and K_{12}, are small relative to the fracture toughness, K_{1c}, then equations (4) and (6) give

$$\frac{N_1}{N_2} = \left(\frac{\sigma_2}{\sigma_1}\right)^m$$

where N_1, N_2 are the total number of cycles to failure. This approach can be used to analyze the data given in the present paper providing one has a reliable value for m and fatigue results for at least one stress (pressure) amplitude.

Finally I would like to know why Py has been calculated using the maximum principal stress criterion rather than the well established maximum shear stress or octahedral shear stress criteria [15].

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Authors' Closure

The author is indebted to Professor Burns for his obvious interest in the paper. No direct comparison has been made between the equations (1) and (2) and the analyses presented by Jones and Tomkins [11] and by Frost and Burns [12], but in all approaches it has been assumed that the pressurized oil has a deleterious effect on embryo cracks. Jones and Tomkins state that such cracks would be of the order of a few microns; in Table 1 of the present paper the initial crack sizes have values from 1.65 to 2.88 microns. Although experimental work has shown conclusively that the pressurizing medium decreases the fatigue strength of the cylinders when in intimate contact with the bore surface, no evidence has been presented to define the physical action which causes this effect; the possibility of oil penetration into the fatigue crack was originally suggested by Morrison, Crossland, and Parry [16]. The paper does not attempt to define the physical action of the pressurizing fluid, but to assess, quantitatively, the influence of this action on the fatigue strength of the cylinders.

The expression for maximum shear stress is a typographical error and should be corrected as suggested. As Professor Burns has observed, previous authors have presented their results in terms of maximum shear stress, and, since there was not always sufficient data available in the literature to enable a conversion from shear stress to internal pressure, Fig. 1 was plotted in terms of shear stress to facilitate a direct comparison between the proposed theory and experimental results.

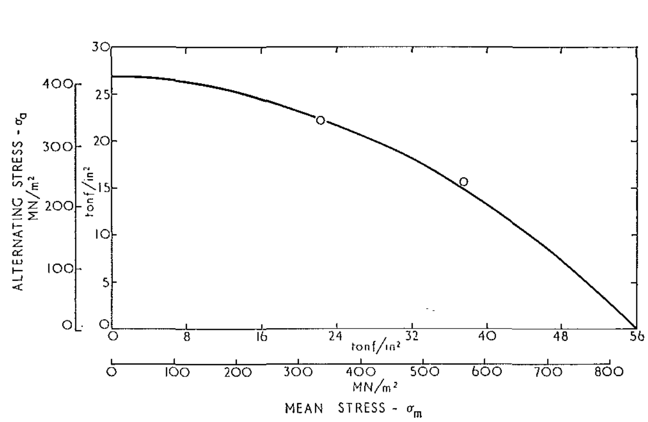

Fig. 8

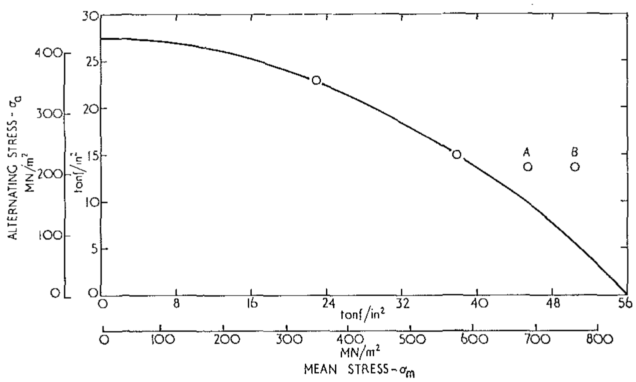

Fig. 9

The results from high mean pressure tests on En25 steel produced by Burns and Parry may be analysed in terms of the local hoop stress as shown in Figs. 8 and 9. The results agree with the Gerber law except in cases where the maximum effective local stress ($\sigma_a + \sigma_m$) exceeds the yield stress of the material, as in points A and B of Fig. 9. In the paper it is suggested that this may be expected due to autofrettage, however, in the tests from which points A and B were obtained, the nominal (Lamé) hoop stress did not exceed the yield stress, therefore general autofrettage did not occur in the cylinders. The local hoop stress at the crack did exceed the yield stress and, therefore, local reversed yielding would occur. Under these conditions an embryo crack may be prevented from propagating at the stress amplitude expected, thus causing points A and B to be well above the Gerber parabola.

With respect to the material crack propagation characteristic C, Professor Burns will be interested to know that further work has been carried out to investigate the dependence of this property on the tensile mean stress [17]. Frost and Greenan showed that the property C was inversely proportional to the tensile mean stress; the value of C used in the present paper is that for repeated tensile stress conditions.

Since fatigue crack propagation depends upon the crack opening and closing, a crack will not propagate in a specimen loaded in compression in all three axes. However, if a specimen is loaded in compression in only one or two directions, a tensile strain is created along the third axis and it is conceivable that a fatigue crack may propagate under these conditions.

The author was not aware of the work described in reference [14] and is grateful to Professor Burns for drawing his attention to it. First impressions of this work are that it offers a convenient method of estimating the fatigue life of a component under defined conditions, but there is an obvious danger in basing a life estimation on one single fatigue result.

The maximum principal stress criterion was adopted for calculating P_y because it gave the least conservative value; since this value was still below all the experimental results it was considered safe to use in practice.

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