Cepheids in NGC 1866: a test for pulsational models

Giuseppe Bono* and Marcella Marcone*

1 Trieste Astronomical Observatory, Via G. B. Tiepolo 11, 34131 Trieste, Italy
2 Department of Physics, University of Pisa, Piazza Torricelli 2, 56100 Pisa, Italy

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ABSTRACT
We present non-linear theoretical results concerning the predicted pulsational behaviour of stellar models suitable for the sample of Cepheids belonging to the Large Magellanic Cloud (LMC) star cluster NGC 1866. The blue and red edges of the instability strip, transformed into the observed plane \((V, B - V)\) by adopting current values for the distance modulus \((DM = 18.57 \text{ mag})\) according to Welch et al. and for the reddening \([E(B - V) = 0.06 \text{ mag}]\) according to Arp and Walker, are in agreement with the observed distribution of Cepheids in the quoted cluster. Moreover, the distribution of observational data in the Bailey diagram \((V\) amplitude versus period) appears to be in agreement with theoretical predictions derived by adopting the luminosity level predicted by canonical evolutionary models. The role played by the direct ab initio integration of the coupling between pulsation and convection and by the input physics on the smoothness of light and velocity curves is briefly discussed.

Key words: stars: oscillations – Cepheids – Magellanic Clouds – galaxies: star clusters.

1 INTRODUCTION
The LMC globular cluster NGC 1866 has already been used extensively for testing stellar evolutionary models and for shedding light on some problems currently debated in the literature (Chiosi et al. 1989; Brocato, Castellani & Piersimon 1994). This cluster is also expected to play a key role in the theory of stellar pulsation, since it provides a beautiful sample of 23 Cepheids with a common value for both distance modulus and reddening. Welch et al. (1991, hereafter WCFMM) already disclosed that this sample of radial pulsators shows a well-defined instability strip which is not revealed by current data on Galactic Cepheids, and represents a benchmark for both modal stabilities (fundamental and first-overtone) and pulsational amplitudes predicted by theoretical models.

As the first step of an extended and detailed investigation of Cepheid hydrodynamical models we use NGC 1866 to test both the accuracy and the internal consistency of the theoretical framework that we developed for evaluating the approach to limit cycle stability and the predicted pulsational amplitudes. The main aim of this paper is to discuss the comparison between a new set of limiting-amplitude, non-linear, non-local and time-dependent convective models and the observational constraints provided by this cluster. In Section 2 the theoretical results are briefly discussed, while Section 3 presents the comparison with the sample of Cepheids in NGC 1866 for which magnitudes, colours and pulsational amplitudes are currently available. Section 4 summarizes the main results of this investigation, and outlines future theoretical and observational perspectives.

2 THEORETICAL FRAMEWORK
According to current evolutionary predictions for Cepheids in NGC 1866, we adopted a fixed stellar mass \((M/M_\odot = 5.0)\). Even though no direct estimation of the metallicity of this cluster has yet been provided, we adopted the metal abundance obtained by Russell & Bessell (1989) and Russell & Dopita (1990) for young LMC stars, i.e., \(Z = 0.008\). We also adopted the helium constant \(Y = 0.25\) provided by Dufour (1984) on the basis of He abundance evaluations in H II regions. Similar physical parameters have also been assumed by Chiosi et al. (1992, 1993, and references therein) in their extensive investigation of the pulsational properties of Cepheid linear models. For proper evaluation
of the topology of the instability strip we computed four sequences of both linear and non-linear models located at different luminosity levels \((\log L/L_\odot = 2.80, 3.07, 3.3 \text{ and } 3.6)\) which cover a wide effective temperature range \((4800 \leq T_e \leq 6700 \text{ K})\). The location of the blue and red edges of fundamental and first-overtone pulsators has been evaluated by adopting, close to these boundaries, a temperature step of 100 K. The theoretical framework adopted for evaluating the pulsational properties of Cepheids has already been described in a series of previous papers (Bono & Stellingwerf 1993, 1994; Bono et al. 1997b, and references therein). In this section we discuss only the main differences from the quoted investigations concerning the physical assumptions adopted to approach Cepheid pulsational models.

For providing a good spatial resolution throughout the envelope model, the static structures have been computed by adopting an outer boundary optical depth \(\tau = 0.001\). At the same time, the mass ratio \((h)\) between consecutive zones has been assumed equal to 1.04 in the stellar layers located at temperatures lower than \(6.0 \times 10^4 \text{ K}\), whereas for higher temperatures we increased this mass ratio by \(\Delta h = 0.001\) per zone. The inner boundary has been fixed so that the base of the envelope was approximately located at a distance from the stellar centre of the order of 10 per cent of the photospheric radius (i.e., \(r_\odot = 0.1 \times R_\odot\)). Unlike envelope models of low-mass stars, these assumptions ensure that the mass included in a typical Cepheid model ranges from 40 to 50 per cent of the total stellar mass. Both linear and non-linear models have been computed by adopting the radiative opacity tables recently provided by Iglesias & Rogers (1996) for temperatures higher than 10 000 K, and the molecular opacities provided by Alexander & Ferguson (1994) for lower temperatures. The method adopted for handling the opacity tables is described in Bono, Incerci & Marconi (1996).

The non-linear analysis has been performed by perturbing the linear radial eigenfunction of the first two modes with a constant velocity amplitude of 10 km s\(^{-1}\). In contrast to non-linear RR Lyrae and Type II Cepheid models, which have been computed with time-steps fixed by the CFL condition for properly handling the development and propagation of the shock front along the pulsation cycle, the non-linear Cepheid models have been computed by adopting a number of time-steps per period which range from 300 for first-overtone models to 400 for fundamental pulsators. This choice is due to the evidence that along a full cycle Cepheid models develop only mild shocks in the hydrogen and helium ionization regions. As a test, few selected models computed by adopting the CFL conditions do not present any substantial difference in the pulsation amplitudes in comparison with the models computed by adopting a fixed time-step. The number of pulsation cycles needed to approach the asymptotic amplitudes depends on the location of the model inside the instability strip and ranges from 1000 to more than 10 000.

A thorough comparison of our models with both linear (Chiosi et al. 1993; Morgan & Welch 1996) and non-linear radiative Cepheid models (Sebo & Wood 1995; Simon & Kanbur 1995; Buchler et al. 1996), together with a detailed analysis of the physical structure of the envelope models, will be discussed in a forthcoming paper (Bono & Marconi 1997, in preparation). The top panel of Fig. 1 shows a selected sample of fundamental and first-overtone bolometric light curves located at \(\log L/L_\odot = 3.07\), i.e., at the luminosity level predicted by canonical evolutionary models. The first interesting result is that moving from the blue to the red edge of the instability strip the light curves show smooth changes along the pulsation cycle and do not present, at least in this period range, any peculiar bump and/or dip. The comparison with results available in the literature (Karp 1975; Carson & Stothers 1984) strengthens the key role played by the inclusion of a time-dependent treatment of convective transport which reduces the pulsational amplitudes and at the same time ensures a smooth excursion of physical variables across the ionization regions. The bottom panel of Fig. 1 shows the surface radial velocities of the same models as those shown in the top panel. The smoothness of these curves over the pulsation cycle is the result of a long \textit{ab initio} integration time, during which the radial motions approach their asymptotic amplitudes and the spurious high-order modes introduced by the initial perturbation settle down. A more detailed summary of the

![Figure 1](https://academic.oup.com/mnras/article-abstract/290/2/353/1035365/105965)
Table 1. Non-linear survey: observables.

<table>
<thead>
<tr>
<th>log (L/L_\odot)</th>
<th>(T_e)</th>
<th>(P)</th>
<th>(\Delta R/R_{ph})</th>
<th>(\Delta a)</th>
<th>(\Delta M_{bol})</th>
<th>(\Delta \log g)</th>
<th>(\Delta \log g_{eff})</th>
<th>(\Delta T)</th>
<th>(\Delta T_e)</th>
<th>(A_v)</th>
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<td>0.110</td>
<td>78.15</td>
<td>1.033</td>
<td>0.07</td>
<td>0.81</td>
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</table>

* Radial velocity amplitude (km s\(^{-1}\)). * Bolometric amplitude (mag). * Amplitude of logarithmic static gravity.
* Visual amplitude (mag).
whereas the Z-bump, at least for the assumed metallicity, barely affects the pulsation stability. On the other hand, the fundamental model is centrally located in the instability strip over a full cycle. The arrows mark the location of the main opacity bumps that appear in the outermost regions is worth mentioning. This feature, which has a negligible effect on the pulsation driving, could play a key role in the smoothness of both light and velocity curves. In fact, thanks to the inclusion of molecular opacities, the radial displacement of surface layers located above the hydrogen ionization region is governed by the physical excursion of temperature and density.

### 3 Comparison with Observations

In this section theoretical results are compared with the photometric data for NGC 1866 provided by WCFMM. These data refer to only nine out of the 23 Cepheids identified in NGC 1866, since photometry is not provided in the quoted paper for the remainder. In a subsequent paper Welch & Stetson (1993) applied their variable star detection technique to several frames centred on NGC 1866, and provided periods for all but one variable of the whole sample. However, both mean magnitudes and colours presented in that paper are affected by a poor photometric accuracy due to the crowding of central regions and thus are of no use.

Theoretical boundaries for both fundamental and first-overtone instability strips \((Z=0.008, Y=0.25\) and \(M=5\,M_\odot\)) have been transformed into the observed plane \([V-(B-V)]\) by using Kurucz's bolometric corrections and colour–temperature relations (Kurucz 1992). In Fig. 3 the transformed boundaries are shown, together with the observed Cepheids of NGC 1866 studied by WCFMM. Following the discussion by WCFMM, we have assumed for the cluster a reddening value \(E(B-V)\) = 0.06 mag given by Arp (1967) and Walker (1974, 1987) and a distance modulus \(DM=18.57\) mag given by Welch et al. (1991). Solid lines represent, from left to right, the first-overtone blue edge (FOBE), the fundamental blue edge (FBE), the first-overtone red edge (FORE) and the fundamental red edge (FRE). The FOB and the FBE intersect at \(logL/L_\odot\approx 3.4\), and therefore we expect that above this luminosity level the instability strip is populated only by fundamental pulsators. Indeed, at \(logL/L_\odot=3.6\) the non-linear models computed by adopting the linear first-overtone radial eigenfunction do not show a stable limit cycle. On the other hand, toward lower luminosities, the FBE and the FORE intersect at \(logL/L_\odot=2.8\), thus removing completely the ‘OR region’, i.e., the region of the instability strip where pulsators are characterized by a stable limit cycle in the fundamental and the first-overtone modes.

The agreement with observational data appears satisfactory, since most of the observed variables are located well inside the theoretical instability strip around a magnitude level corresponding roughly to \(logL/L_\odot=3.07\). In agreement with early results obtained by Bertelli et al. (1993), the comparison also suggests the occurrence of first-overtone pulsators among Cepheids in NGC 1866. The only deviant variable is HV 12204, already pointed out by WCFMM for its deviation from the theoretical instability strip.
its peculiar brightness and blueness. However, from the analysis of radial velocities these authors concluded that HV 12204 is not a cluster member. Two further features of the instability strip are worth mentioning.

(1) Moving from higher to lower luminosities, the instability strip becomes narrower; in the investigated luminosity range the colour width roughly decreases by a factor of 2 as the outer edges of the instability region present different slopes. These findings support the results obtained by Fernie (1990) for Galactic Cepheids, and the preliminary trend suggested by Olszewski (1995) for Cepheids in LMC clusters.

(2) The region of the instability strip in which only first overtones show a stable non-linear limit cycle becomes larger at lower luminosities. This outcome supplies a straightforward but qualitative explanation for the large amount of first-overtone pulsators - approximately 30 per cent of the total sample recently detected by microlensing experiments (Beaulieu et al. 1995; Cook et al. 1995) - and supports the hypothesis originally suggested by Böhm-Viteš (1988) concerning the occurrence of such variables among short-period Cepheids.

In Fig. 4 the theoretical relations between periods and $V$ amplitudes for both fundamental and first-overtone models and for the labelled assumptions on luminosity are compared with the observational data provided by WCFMM. Bolometric amplitudes have been transformed into the $V$ Johnson band by adopting Kurucz’s static atmosphere models. The $V$-magnitude curve can be used to derive either a magnitude-weighted amplitude or an intensity-weighted one, which is subsequently converted into magnitude units. The differences between amplitudes obtained from these two different approaches turned out to be negligible (less than 0.001 mag).

It is worth noting that while the comparison in the HR diagram (instability strips) could be affected by uncertainties in the bolometric corrections, in the colour–temperature relations, in the reddening, and in the distance modulus, as well as in the method adopted to average the observed colours and magnitudes over the pulsation cycle, the comparison in the $A_V$–log$P$ plane is independent of almost all these troublesome effects, and is therefore far more robust. As a consequence, we can infer that Cepheids in NGC 1866, at least those for which photometric data are given in the literature (WCFMM), are well reproduced in the Bailey diagram by a sequence of pulsational models with $Z=0.008$, $Y=0.25$, $M=5\,M_\odot$ and $\log L/L_\odot \approx 3.07$, in remarkable agreement with the adopted evolutionary predictions. Note that the sequences of models characterized...
Figure 4. Comparison in the Bailey diagram between predicted and observed $V$ amplitudes and periods. Symbols are referred to different luminosity levels (see labelled values). Fundamental and first-overtone amplitudes are plotted by adopting solid and dashed lines respectively. Asterisks show the observed $V$ amplitudes provided by WCFMM.

by the same values of stellar mass and chemical composition but by luminosity levels different from $\log L/L_\odot = 3.07$ are clearly unable to match observational data. This means that, in principle, the comparison in the period-amplitude diagram proves to be a powerful instrument for estimating the luminosity level of cluster Cepheids.

A further interesting aspect worth investigation in order to provide both a suitable test of the present theoretical scenario and a tight constraint on the pulsation characteristics of classical Cepheids is the comparison between the Fourier parameters of theoretical and observed light curves (Simon & Kanbur 1995). Even though such analysis is clearly beyond the scope of this paper, a comprehensive comparison with observational data and the impact of the pulsation/convection interaction on observables will be discussed in a forthcoming paper (Bono & Marconi 1997, in preparation).

4 CONCLUSIONS

The agreement between Cepheid theoretical models and the photometric data available in the literature provides sound support to the plausibility of the physical assumptions adopted in the development of this new pulsational scenario. In a homogeneous theoretical context we derived both the blue and the red edges of the instability strip. In particular, it is worth noting that the red edges have been evaluated without invoking any ad hoc assumption on the efficiency of the convective transport over the pulsation cycle. At the same time, present limiting-amplitude calculations strongly support the existence of first-overtone pulsators among Cepheids. In fact, we found that there is a well-defined region of the instability strip in which only first-overtone pulsators present a non-linear limit cycle stability. On the other hand, taking into account the distribution of pulsational amplitudes in the Bailey diagram, we find that the luminosity level of Cepheids along the blue loop is in agreement with the value predicted by canonical evolutionary models.

However, a deeper insight on the matter is prevented by the limited sample of Cepheids currently available. What is relatively surprising about photometric data of young LMC clusters (see table 2 in Welch, Mateo & Olszewski 1993, and references therein) with a good Cepheid sample is that even though they are the keystone for determining the P–L and the P–L–C relations on which rely the distance evaluations of the Local Group galaxies, and in turn the calibration of secondary distance indicators, we still lack a comprehensive observational scenario for these objects.

Moreover, although several approaches have been recently undertaken for evaluating the metal abundance of stellar clusters in the Magellanic Clouds (Geisler & Mateo
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1989; Olszewski et al. 1991), a detailed analysis of the chemical composition spread among Cepheids in Galactic and Magellanic Cloud clusters is even more urgent.

As far as future theoretical developments are concerned, we plan to extend the present non-linear pulsational scenario by taking into account different stellar masses and chemical compositions for properly disclosing the dependence of Cepheid behaviour on these astrophysical parameters.

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REFERENCES

Bono G., Caputo F., Castellani V., Marconi M., 1997b, A&AS, 121, 327