Standardized mortality ratio and life expectancy: a comparative study of Chinese mortality

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Background
Various models have been proposed for rapid conversion of the standardized mortality ratio (SMR) to life expectancy using data from developed countries.

Methods
We compared two methods for converting the SMR to life expectancy using mortality data from the largest developing country, China.

Results
The first model, using the Gompertz function, does not provide a good fit to the life expectancy and SMR of China. The regression lines derived from the second, a log-linear model using parameters estimated from the US white population are not a good fit to Chinese males and older females. However, if the parameters in the log-linear model are estimated using Chinese mortality data, the resultant regression lines fit the data reasonably well.

Conclusion
The relationship between life expectancy and SMR based on mortality data from developed countries may not be valid for developing countries. Based on our empirical study, separate estimates of the coefficients of the model are required for developing countries.

Keywords
Chinese mortality, Gompertz function, life expectancy, regression, standardized mortality ratio

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Methods and Materials
China contains more than one-fifth of the world population. As in most developing countries, the vital registration system in China is neither comprehensive nor complete. However, China has conducted a population census every 10 years to collect relatively comprehensive demographic data. In its latest population census in 1990, detailed mortality information on the entire population was collected. The life expectancy of each province was published by the State Statistical Bureau of China. Based on the age-specific mortality data, we calculated the SMR for each province using the whole Chinese population as the standard population.

We studied two types of relationship between life expectancy and SMR. One was the log-linear model in Lai et al. and the other was based on the Gompertz function in Haybittle. From Lai et al. we have:

\[ \log \frac{e_x}{e_s} = \alpha + \beta (1 - SMR) \] (1)

where \( e_x \) and \( e_s \) are the life expectancy at age \( x \) for the study population and the standard population respectively, and \( \alpha \) and \( \beta \) are parameters which can be estimated from the observed values of the life expectancies and SMR from subregions.

In the second approach by Haybittle, we have:

\[ e_x^* \approx e_s - \frac{\log(\text{SMR})}{k} \] (2)

Life expectancy is widely used by health professionals and the general public as an indicator for summarizing the mortality experience of a population. Epidemiologists and biostatisticians also use the standardized mortality ratio (SMR) which measures the level of excess or deficit in mortality in a study population compared with the expected level based on the mortality of a standard population. The SMR is computed as the ratio of the observed deaths to the expected deaths in the study population. The expected deaths are obtained by multiplying the age-specific mortality rate of a chosen standard population by the age-specific population size of the study population and then adding terms to get the total expected number of deaths. Since the SMR is not well understood or used by the general public, several relationships have been established to convert SMR values into change in life expectancy. These studies were based on developed countries because of the ready availability of data.

In this study, using data from China, the largest developing country, we compared the methods proposed by Lai et al. and Haybittle.
where $e_x^*$ and $e_x$ are the same as in equation (1) and $k$ is a constant estimated from the observed age-specific mortality rate based on the Gompertz function. Equation (2) is an approximated model of a general model (equation (2) and equation (3) given in Haybittle). We conducted a comparative study using model (1), model (2) and the general model, equation (2) and (3) of Haybittle. Since the general model did not achieve better results than those of model (2) above, we only report the comparative results of model (1) and model (2).

Let $\mu_x$ be the age-specific mortality rate of the standard population. (In our case, the standard population was the whole Chinese population.) The Gompertz function is defined as:

$$\mu_x = Be^{kx}$$

Equivalently, we can have two types of relationship between the changes in life expectancy and the SMR:

$$e_x^* - e_x = (e^{a+\beta}(1\text{-SMR}) - 1)e_x$$

and

$$e_x^* - e_x = -\log(\text{SMR})/k$$

where the SMR is computed from the whole range of age groups of the Chinese population.

### Results

Using mortality data from the 1990 Chinese population census, we plotted the logarithm of the age-specific mortality rate versus age (Figure 1).

The general feature of the plot is similar to the plots presented in Haybittle for the data for England and Wales in 1992. To estimate the constant ($k$) in the model, we applied the method of least squares to the Gompertz function after transforming to log scale:

$$\log \mu_x = \log B + kx$$

The regression analyses were performed for the age groups 30–34 upwards as suggested in Haybittle for the Gompertz function. The estimates of $k$ for males and females were 0.0890 and 0.0904, respectively. The value of $k$ in equation (2) is assumed to be the same constant for all age groups.

The parameters in equation (1) were also estimated using the least squares method for males and females as well as for the age groups 25, 45 and 65 years (Table 1). For comparative purposes, estimates of the parameters ($\alpha$, $\beta$) based on the data from the US population in Lai et al. are also presented in Table 1.

For a better visual comparison, we plotted the data and the regression lines for various gender-age groups in Figure 2.

Figure 2 shows the estimated equations for each method, the solid lines (———) represent equation (1) with the parameters estimated from the Chinese data and the dash/dot lines (–·–·–) represent equation (1) using parameters estimated from the US data. The dotted lines (………) represent equation (2) with $k$ estimated based on the Gompertz function.

For the 25- and 45-year age groups in Figure 2, the regression lines of equation (2) and equation (1) with parameters estimated from the US white population are very close to each other in the neighbourhood of SMR = 1. However, these regression lines have noticeable differences when compared to the regression line of equation (1) with parameters estimated from the Chinese population. For females aged 25, the regression lines of all equations are very close to each other as shown in Figure 2(d). Generally, the slopes in the absolute value of the regression lines of equation (2) and equation (1) with parameters estimated from the US white population are greater than those of the regression lines of equation (1) with parameters estimated from the Chinese population.

For a better cross plot comparison, we selected a common range (0.5–1.7) of SMR for all plots and used 20-year intervals for the vertical axis of Figure 2(a) and Figure 2(d), 15-year intervals for Figure 2(b) and Figure 2(e), and 10-year intervals for Figure 2(c) and Figure 2(f).

Figure 3 compares the changes in life expectancy of the study population associated with various values of SMR based on equations (4) and (5).

The representation of the lines in Figure 3 is the same as in Figure 2. The patterns of the changes in life expectancy are the same as the regression lines in Figure 2 since the changes in life expectancy and the regression lines differ only by a constant, the life expectancy of the standard population. However, due to the scale difference, the plots may look different. Numerically, an increase of 0.5 in SMR was converted to a decrease of 2.76.

### Table 1 Estimated parameters in the regression equations by sex and age for the Chinese population in 1990 and the US white population in 1980

<table>
<thead>
<tr>
<th>Age (x)</th>
<th>$e_x$ (China)</th>
<th>China</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>Male</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>46.13</td>
<td>0.0043</td>
<td>0.1318</td>
</tr>
<tr>
<td>45</td>
<td>27.83</td>
<td>0.0011</td>
<td>0.1587</td>
</tr>
<tr>
<td>65</td>
<td>12.47</td>
<td>0.0056</td>
<td>0.1489</td>
</tr>
<tr>
<td>Female</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>49.76</td>
<td>0.0003</td>
<td>0.1918</td>
</tr>
<tr>
<td>45</td>
<td>31.17</td>
<td>0.0085</td>
<td>0.1566</td>
</tr>
<tr>
<td>65</td>
<td>14.88</td>
<td>-0.0038</td>
<td>0.2391</td>
</tr>
</tbody>
</table>

a There are 30 data points (provinces) for the Chinese data set and 50 data points (states) for the data set of the US. The results for the US white population were extracted from Lai et al.
2.09 and 0.83 years in life expectancy for Chinese males at ages 25, 45 and 65 years based on equation (4), respectively. From equation (5), the change was 4.56 years in life expectancy for all age groups. For the Chinese females at ages 25, 45 and 65 years, based on equation (4), an increase of 0.5 in SMR was converted to a decrease of 4.54, 2.10 and 1.73 years in life expectancy, respectively. Using equation (5), the change was 4.49 years in life expectancy for all age groups (Figure 3).

Discussion

An easy and quick conversion from SMR to changes in life expectancy is a convenient, useful way to provide information to the general public. Previous studies on the relationship between life expectancy and SMR were all based on data from developed countries. In our study, we compared two simple models for a quick conversion based on Chinese mortality data.

In a developed country or in a small country, the range of SMR is usually small due to the relatively homogeneous population. The regression lines established based on these regional data may not be suitable for more heterogeneous populations. For example, the SMR of the 25-year age group of the white population in the US are between 0.8 and 1.1. However, in a large developing country, such as China, mortality patterns are not so homogeneous across regions. The SMR of the Chinese population range from 0.7 to 1.7.

To establish the regression lines of equation (1), one needs to have observations from each region of the country. For the models (equation 2) proposed in Haybittle4 using the Gompertz function, information from regions is not needed. However, the model does not provide a good fit to the Chinese mortality data. The regression lines generated from equation (2) are very sensitive to the estimates of \( k \). Very different regression lines can be produced by including younger (<30) age groups for estimating the value of \( k \). Once the value of \( k \) is obtained, the same value applies to different age groups (25, 45 and 65 years) studied.

Haybittle4 showed that model (2) did not fit the older age group well, based on data from England and Wales. Equations (2) and (3) of Haybittle4 were recommended for the older age group. We tried this approach on the Chinese data set but did not find a better fit than that of model (1) in this article. The results are not presented here but are available upon request. The values of the correlation coefficient \( r \) of \( \log(c_i/c_e) \) and \( (1 - \text{SMR}) \) for the Chinese population at age groups 25, 45 and 65 years were 0.8303, 0.7056 and 0.4875 for the males and
0.8857, 0.7798 and 0.6624 for the females, respectively. Equation (2) is derived from the Gompertz function and equation (1) of the US white population is not based on regression on the Chinese data. Therefore, we could not compute the values of $r^2$ for equation (2) and equation (1) of the US white population from the Chinese data. In general, equation (1) provided a reasonable fit for all age groups studied based on the Chinese data set. However, no global conclusions can be drawn from this single application. The approach appears useful in a developing country (China) and in a developed country (US).

Acknowledgement

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References