A word should be added about the half-Sommerfeld condition used for the cavitating flow solution. Its accuracy was questioned in the past and some better approximation of the cavitating region was suggested [16]. The half-Sommerfeld condition, although not fully consistent with the conservation of mass flow at the trailing end of the lubricating film, gives very good results for short journal bearings performance [17]. Radial face seals are in many aspects similar to short bearings. For this reason, and because of the great deal of simplicity it adds to the mathematical treatment, the half-Sommerfeld condition is used in this paper for the limiting case of pure hydrodynamic pressures. The reader should be aware of this approximation when the exact cavitation boundaries are desired.

## Concluding Remarks

Hydrodynamic effects in a flat seal having an angular misalignment were analyzed, taking into account the radial variation in seal clearance. An analytical solution for axial force, restoring moment and transverse moment was presented for the whole range from zero to full angular misalignment.

A strong coupling was found between angular misalignment and the transverse moment in both low and high pressure seals. Such coupling is a possible source of dynamic instability and can result in a wobbling of the primary seal ring. Hence, it can be concluded that the hydrodynamic transverse moment, which was overlooked in the past, is certainly playing an important role in radial face seal operation. It can affect mainly the dynamic stability of radial face seals and hence, has to be considered in any dynamic analysis of these seals.

## Acknowledgment

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# DISCUSSION

## A. O. Lebeck[1]

The author raises the interesting possibility of the transverse moment acting as a source of wobble in face seals. A complete dynamic solution is needed to evaluate this possibility and the author's results can be used as a part of this solution. However, squeeze effects must be included as a part of a dynamic solution.

Does the author have any evidence of this type of dynamic instability occurring in actual flat face seals? Since a flat face seal is inherently hydrostatically unstable, contact would be expected. Could this type of dynamic instability occur if the seal were in partial contact?

Seal faces are generally not flat as is assumed in the author's analysis. In particular, under conditions of misalignment, the non-uniform pressure distribution will cause an out-of-flat deflection of a significant magnitude relative to the seal gap for all but the most rigid of seal rings.

Would the author please comment on these points.

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## R. A. Burton[2]

The author has examined the simplest possible model of seal hydro-dynamics, taking account of tilting of the stator but assuming the rotor has no run out and that the stator and rotor are both dead flat. The most interesting finding is that for small displacements the restoring moment disappears and a transverse moment appears. If the ring is flexibly mounted it would seem that an initial tilt would be followed by some rather complex motion.

One may well ask how can a seal work; and the answer probably lies in the fact that rotor waviness and run out as well as stator waviness are of the same order as the tilt and act to preserve a stable film between rotor and stator. Whereas one may sometimes observe some strange behavior as to stator motions, most of the time one sees an essentially synchronous (but not necessarily in phase) motion of the stator as the wavy, slightly misaligned rotor turns. Conduction tests reassure one that the liquid film is indeed supporting the face loads.

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The author shows proper caution in invoking the half-Sommerfeld film pressure condition. My own confidence in the use of this condition is strengthened greatly by the work of Stanghan-Batch and Iny.[3] I might also note that they offer strong arguments for the role of 2-lobe (2nd mode) waviness in assuring proper seal action.

Now that I have stated certain reservations, mainly to warn the reader that there is more to the problem, let me note that the selected problem has been dealt with in a masterful way, and will represent a pivotal study as we seek to discover when the stabilizing effects of waviness are overridden and the transverse moments take over to disrupt proper stator motion.

## R. Metcalfe[4]

In his three papers on misalignment of flat-faced radial end face seals, the author has derived useful relationships for the associated forces and moments. It is particularly commendable that limits of accuracy of the closed form solutions have been identified.

The author's equation for hydrodynamic transverse moment $M_z$ for the full fluid film condition is of greatest interest to this reviewer. It has been related to measured values of the diametral tilt moment required just to cause touch of conical-face seals [18]. Using the author's nomenclature, measured and calculated results are correlated in the table below.

| Test number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Sealed pressure, $\Delta p$ (MPa) | 2.76 | 2.76 | 2.76 | 2.76 | 2.76 | 0.535 | 2.74 | 6.75 |
| Balance ratio | 0.76 | 0.84 | 0.89 | 0.90 | 0.96 | 0.645 | 0.672 | 0.707 |
| Rotational speed, $n$ (rad/s) | 0 | 50 | 105 | 157 | 208 | 157 | 157 | 157 |
| Calculated average clearance, $C$ ($\mu$m) | 4.4 | 1.75 | 0.92 | 0.73 | 0.05 | 24 | 13 | 8.7 |
| Measured applied tilt moment just to cause touch (N.m) | 2.5 | 20 | 31 | 35 | 49 | 0 | 0 | 0 |
| Calculated hydrostatic restoring moment, $M_x$ (N.m) | 3.2 | 8.3 | 7.0 | 6.6 | 2.5 | −3.2 | −11.8 | −14.2 |
| Calculated hydrodynamic Transverse moment at touch (i.e. $\epsilon = 1$), $M_z$ (N.m) | 0 | 250 | 1900 | 4520 | 128000 | 4.2 | 14.2 | 31.8 |

Seal details: $r_i = 19.6$ mm, $r_0 = 36.4$ mm, $\mu = 6.8 \times 10^{-4}$ Pa.s, nominal amount of conical inward convergence between the seal faces, $D = 4.86\ \mu$m.

The method of obtaining the measured results has been described [19]. Calculated values for average clearance, $C$, between the seal faces and hydrostatic restoring moment, $M_x$, are derived from existing performance curves for full film lubrication of seals with inward leakage and convergence [20]. Hydrodynamic transverse moment, $M_z$, for the tilted condition where faces just touch has been calculated from the author's equation for flat faces, using tilt parameter $\epsilon = 1$.

If both rings of the test seals were in steady state (no wobble or whirl) at the moment of touch, it should be expected that the vector sum of the three moments should be zero for each test. Remembering that $M_x$ and $M_z$ are 90 deg apart, it is clear from the table that there is no such equilibrium. The author's predictions of wobbling appear justified—it happens! The conclusion from the tabulated results must be that, for the test arrangement and conditions, wobbling of the stationary seal ring is only slightly constrained. The author has derived the "fully constrained" relationship for the hydrodynamic transverse moment, but its practical magnitude will normally be much less than obtained by using shaft speed $\omega$ in the equation. In fact, if angular speed of wobble, $\omega_s$, could be measured for the test seal stators, it is believed that hydrodynamic transverse moment $M_z$ would be proportional to $\omega - 2\omega_s$, not to $\omega$ alone. This moment could then be used in dynamic analysis to assess damping properties of elastomers and inertia effects.

Could the author please comment, particularly on the relevance of his flat-face seal relationships to the test results for conical-face seals. How should a high pressure seal be designed to derive greatest benefit from hydrodynamics, or must we always rely on hydrostatics for stability?

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## Author's Closure

The author wishes to thank the discussers for their comments and interest in the paper.

Dr. Burton and Dr. Lebeck comments on seal flatness are correct. It is true that seal faces are generally not flat and that both waviness and coning can be combined with tilt. It is also true, however, that face seals are generally lapped dead flat to begin with. Hence, one may ask what are the mechanisms that cause out of flatness of a seal that is originally flat. One possible answer is that seal wobble generated by the transverse moment wears the seal faces into a wavy form. In studying the behavior of flat misaligned seal we may prevent this sort of wear and perhaps eliminate unnecessary waviness. The common thought that waviness is essential for proper operation of face seals is not correct since angular misalignment alone produces separating force too. If wobble starts squeeze effects are also helping in supporting the face load.

There are some evidences of wobble in face seals. The experiment described by Dr. Metcalf in his discussion is one but wobble was also observed and measured elsewhere [21, 22].

Partial contact in a misaligned flat seal is represented by a tilt parameter $\epsilon = 1$. As can be seen from the results the hydrodynamic transverse moment $M_z$ exists over the whole range of $\epsilon$. Hence, wobble can occur with partial contact as well as with no contact. If by partial contact asperity contact is meant than the answer depends on the nature of the mean film thickness distribution. If it is similar to that of a misaligned seal then the same type of instability is expected.

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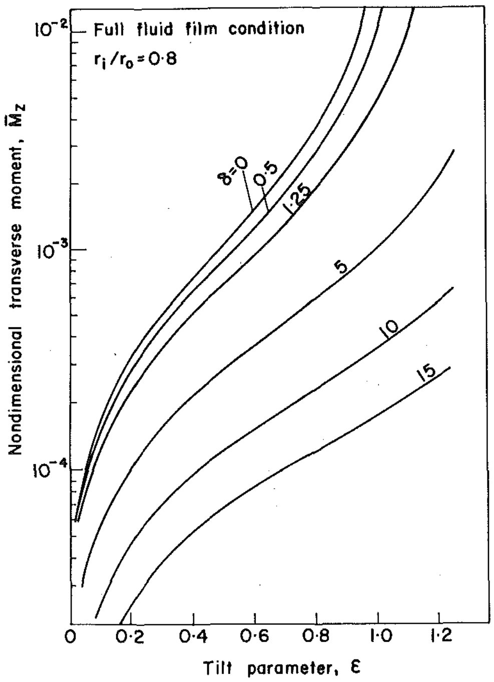


**Fig. 6 Effect of coning on the nondimensional transverse moment**

The tabulated results of Dr. Metcalf are indeed of great interest. It is true that the transverse moment is proportional to $\omega - 2\omega_s$ where $\omega_s$ is the precession of the wobbling seal. This can partly explain the enormous high values of $M_z$ in the table. However, the main reason is the neglect of face coning in Dr. Metcalf's calculations. The effect of coning is to reduce the hydrodynamic transverse moment. This is demonstrated in Fig. 6 taken from [23]. The parameter $\delta$ is related to the amount of conical convergence, $D$, by the equation

$$\delta = Dr_0/(r_0 - r_i)C$$

As can be seen from Fig. 6 an increase in $\delta$ from $\delta = 0$ to $\delta = 15$ at $\epsilon = 1$ reduces $M_z$ by two orders of magnitude. Values of the coning parameter $\delta$ calculated for the various tests of Dr. Metcalf are listed below:

| Test No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| Coning Parameter, $\delta$ | 28.5 | 72 | 137 | 173 | 2500 | 5.2 | 9.7 | 14.5 |

It is clear that these are very large values especially for test No. 5. Hence, the actual values of $M_z$ would be much less than these given in the table.

The answer to the question of how should a high pressure seal, or any seal, be designed in order to be stable may be given only after a thorough dynamic analysis is performed. It is the author's belief that too many degrees of freedom of the flexibly mounted ring is a possible source of dynamic instability. Until a better concept of face seal design is found it seems that reducing the hydrodynamic transverse moment to a minimum will result in a more stable seal.

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