Weak lensing correlations in open and flat universes

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ABSTRACT
Correlations between the magnification or polarization of background sources, induced by gravitational lensing due to their large-scale structure, and the positions of foreground galaxies, are investigated. We find that the amplitude of those correlations is enhanced with respect to those for a single population. We analyse the dependence of the correlations with the density parameter \( \Omega \) considering a non-linear evolution of the matter-power spectrum. The contribution of linear evolution is negligible at scales below several arcmin. Detailed results regarding the dependence of the correlations on the redshift of the foreground and background populations for different cosmological models are presented. The effect reaches its maximum amplitude for foreground populations with relatively small redshifts due to the rapid increase of the non-linear matter-power spectrum at recent times.

Key words: cosmology: theory – gravitational lensing – large-scale structure of Universe.

1 INTRODUCTION
The gravitational deflection of photons can be used as a probe of the matter distribution along the line of sight to the sources. The latter may be at the last scattering surface \((z \approx 10^3)\), in the case of the cosmic microwave background (Seljak 1996; Martínez-González, Sanz & Cayón 1997; Bernardeau 1997), or may be high-z objects such as QSOs or galaxies (Blandford et al. 1991; Kaiser 1992; Kaiser & Squires 1993; Bartelmann 1995; Villumsen 1995b; Villumsen 1996; Bernardeau, van Waerbeke & Mellier 1996; Kaiser 1997; Jain & Seljak 1996). Information about the matter fluctuations can be obtained on different scales ranging from galaxy haloes to the large-scale structure of the Universe.

Many of the theoretical studies on this subject have dealt with the polarization or ellipticity produced on background galaxies by the large-scale structure of the Universe, and there are currently several ongoing observational projects trying to detect and quantify this effect. Nevertheless, measuring shear amplitudes such as ones predicted by the above-mentioned calculations is very difficult from a technical point of view (although see Mould et al. 1994; Villumsen 1995a), and it is not totally clear whether such precision will be routinely achievable in the near future.

However, there is another observable phenomenon produced by gravitational lensing of background sources by foreground mass distributions which may have been already detected: QSO–galaxy associations due to the magnification bias effect (Canizares 1981). The surface density of a sample of flux-limited background sources behind a lens which magnifies them by a factor \( \mu \) is changed in the form \( N' (> S) \propto \mu^{-1}N(> S) \), where \( N(> S) \) is the unperturbed background source density. If \( N(> S) \propto S^{-\alpha} \) [or \( N(< m) \propto 10^{\alpha m} \)], the change in the density can be characterized by the factor \( q = N'/N = \mu^{-1} \). Thus, depending on the slope \( \alpha \), there may be an excess of background sources \((\alpha > 1)\), a depletion \((\alpha < 1)\), or the density may remain unchanged \((\alpha = 1)\). If we properly choose the background source population, so that it has a slope \( \alpha \) considerably different from 1, there may be a correlation (or anticorrelation) between the position of the matter overdensities acting as lenses and the background sources. Now, these matter perturbations will be traced, up to a bias factor, by galaxies, and thus there will be a correlation between these foreground galaxies (or any other tracers of dark matter) and the background sources.

There are several reported associations between foreground galaxies and high-redshift background active galactic nuclei (AGNs) (see Schneider, Ehlers & Falco 1992, Narayan & Bartelmann 1996 and Wu 1996 for reviews), but only a few of these studies extend to relatively large scales. Bartelmann & Schneider (1994) found a strong association between galaxies selected from the IRAS Faint Source Catalogue and high-z AGNs from the 1-Jy catalogue. In Benítez & Martínez-González (1995) it was found that the auto-
matic plate measurement (APM) galaxies tracing large-scale structures were correlated with 1-Jy QSOs. Another sample of radio-loud QSOs, extracted from the Parkes survey catalogue, has also been found to be correlated with COSMOS galaxies (Benitez & Martínez-González 1997), with a correlation scale of several arcmin. Other studies considering the correlation between galaxy clusters and high-z QSOs (Seitz & Schneider 1995; Wu & Han 1995) have also found positive results.

In this paper, we shall study the effects of weak gravitational lensing by foreground matter fluctuations on a population of background sources at high-z. We consider different values of $\Omega$ and model the fluctuations assuming a cold dark matter (CDM) universe with a power spectrum whose evolution in time follows a standard Ansatz (Hamilton et al. 1991; Peacock & Dodds 1996; linear and non-linear contributions are considered). We assume that these matter perturbations are traced, up to a global bias parameter $b$, by galaxies. More specifically, we shall explore the behaviour of $C_{\mu\nu}$, i.e. the large-scale correlation between the ellipticity of background galaxies and the position of foreground ones, which apparently has not been considered in the literature. We shall also consider in detail other correlations such as $C_{\phi\phi}$, i.e. magnification–foreground galaxies and $C_{\mu\mu}$, i.e. magnification–magnification galaxies; in particular, we will study the dependence of the correlations on $\Omega$. $C_{\mu\mu}$ can be indirectly estimated through the galaxy–galaxy correlation function (Villumsen 1995b). However, measuring $C_{\mu\mu}$ offers several advantages over $C_{\phi\phi}$ from the observational point of view. In the first place, $C_{\mu\mu}$ has an amplitude several times higher than $C_{\phi\phi}$. In any case, if the foreground and background galaxy populations are properly selected so that there is no redshift overlap between them (e.g. high-z QSOs and bright galaxies), one does not have to bother about intrinsic correlations; any measured effect should be caused by gravitational lensing.

Section 2 develops the formalism dealing with weak gravitational lensing for a flat and open cosmological model, the concepts of magnification and polarization (or ellipticity) and the different correlation. In Section 3 the main theoretical results are given, as well as comments on different observational perspectives. Finally, in Section 4 we present the conclusions of the paper.

2 FORMALISM

2.1 Geodesics in a perturbed Friedmann universe

We will consider the propagation of photons from a source at redshift $z$ to the observer ($z = 0$), the universe being a perturbed Friedmann model with vanishing pressure. For scalar perturbations, the metric in the conformal Newtonian gauge is given in terms of the scalefactor $a(t)$ and a single potential $\phi(x, z)$ that satisfies the Poisson equation, as follows (Martínez-González et al. 1997):

$$ds^2 = a^2(t) [-(1 + 2\phi) dt^2 + (1 - 2\phi) \gamma^{-2} \delta_\gamma dx^i dx^j],$$

where

$$\gamma = 1 + \frac{k}{4} z^2.$$  

We take units such that $c = 8\pi G = a_0 = 2H_0 = 1$ and $k = (4\Omega)^{-1} = 0, -1, +1$ denote the flat, open and closed Friedmann background universe.

Assuming a perturbation scheme ('weak lensing'), the null geodesic equation for the previous metric can be integrated in the form $x = \lambda n + \epsilon$, where $n$ is the direction of observation and $\lambda$ is the distance to the photon in the background metric, i.e.

$$\lambda = \tau_0 - \tau \quad (k = 0),$$

$$\lambda = (1 - \Omega)^{-1} \tanh[(1 - \Omega)(\tau_0 - \tau)] \quad (k = -1).$$  

The perturbation $\epsilon$ can be decomposed in a term parallel to the direction of observation $n$ and a term, $\epsilon_x$, orthogonal to such a direction. The last term is given by

$$\epsilon_x = 2 \int_0^\lambda d\epsilon' W(\lambda, \epsilon') \nabla_\lambda \phi(\epsilon', x = \lambda n),$$  

where $W(\lambda, \epsilon')$ is a window function

$$a(\lambda) = \frac{(1 - \lambda)^2}{1 + k\lambda^2/4}, \quad W(\lambda, \epsilon') = \frac{1 + k\lambda^2/4}{1 + k\lambda^2/4}.$$  

For photons that are propagated from a source at redshift $z$ (distance $\lambda$) to the observer ($z_0 = 0$ or $\lambda_0 = 0$), the lensing vector $\beta$ is defined in the usual way:

$$\beta = \frac{n - x - x_0}{|x - x_0|}.$$

Thus we find

$$\beta = \frac{1}{\lambda} \epsilon_x(\lambda).$$  

Once we have obtained the expression for the trajectory of the photon in the conformal Newtonian gauge, it is easy to calculate everything in the conformal synchronous-comoving gauge (Martínez-González et al. 1997). The lensing vector in such a gauge (that is the appropriate one from the point of view of observations) is given by the expression (5) plus some additional terms that can be interpreted as Doppler contributions at the source and observer and an acceleration term at the observer. The last two terms can be estimated from the Doppler velocity with respect to the cosmic microwave background and from our local infall towards the Virgo cluster (or Great Attractor). These extra contributions are very small, so the lensing vector $\beta$ in the synchronous-comoving gauge is approximately given by $\beta$, as defined by equations (3) and (5).

2.2 Magnification and polarization

Let us assume a population of background sources (e.g. quasars or galaxies), placed at different distances $\lambda$ with a distribution $R_b(\lambda)[1, dR_b(\lambda)] = 1$. Then, we can define the integrated lensing vector $\beta(n) = \int_0^\lambda dR_b(\lambda) \beta(\lambda, n)$ and taking into account equations (3)–(5) we obtain

$$\beta(n) = DS, \quad D' = (\delta^i - n_i n^i) \frac{\partial}{\partial n^i},$$

$$S(n) = \int_0^n d\tau T_b(\tau) \phi(\lambda, x = \lambda n),$$

$\Omega = 0$
If we take the derivative along the plane orthogonal to the direction of observation \( \mathbf{n} \), we get the convergence tensor
\[
\varepsilon_{ij} = D_{\mathbf{n}j} = D_{\mathbf{n}i} \delta_{ij}
\]
which can be decomposed in the form
\[
(8)
\]
For weak lensing, the convergence scalar \( K(\mathbf{n}) \) is related to the magnification by
\[
f_L(\mathbf{n}) = 1 + 2K(\mathbf{n}).
\]
Moreover, the polarization component in the plane orthogonal to \( \mathbf{n} \) can be defined in the standard way
\[
P_1 = P_{12} - P_{21} = 2P_{12}
\]
and the complex polarization is
\[
P = P_1 + iP_2.
\]
So, we can define the scalar polarization
\[
p_2 = pp^* = 2P_{ij}p_{ij}.
\]

### 2.3 Background–foreground correlations

Let us consider a second population of foreground sources (e.g. galaxies) placed at different distances \( \lambda \) with a distribution \( R_f(\lambda) \). If \( \delta(\lambda, x) \) represents the density fluctuation [that satisfies the Poisson equation \( (V^2 + 3k)c\lambda = \rho_0 \) ], we shall assume that there is a constant bias factor \( b \) relating the number fluctuation, \( \langle n \rangle / N \), and such an overdensity. Then we can define the integrated overdensity
\[
\delta(\mathbf{n}) = \int_0^1 \delta R_f(\lambda) \lambda d\lambda.
\]
Taking into account equations (3)–(7), we obtain
\[
\langle \beta(n) \rangle = D_x \langle S(n) \delta(n) \rangle,
\]
\[
\langle S(n) \delta(n') \rangle = 2 \int_0^1 d\lambda T_x(\lambda) \int_0^1 d\lambda' R_x(\lambda') C_{\delta\delta}(\lambda, \lambda'; x),
\]
where
\[
C_{\delta\delta}(\lambda, \lambda'; x) = \langle \phi(\lambda, x) \delta(\lambda', x') \rangle.
\]
Using the Limber approximation (see Appendix), i.e. only a small region \( r \) is contributing with \( \lambda \approx \lambda \), the previous equation can be approximated by
\[
\langle S(n) \delta(n') \rangle \approx 4 \int_0^1 d\lambda T_x(\lambda) R_x(\lambda) \int_0^1 d\lambda' R_x(\lambda') C_{\delta\delta}(\lambda, \lambda'; x).
\]
where the correlation now appears at a single time and \( s \) is given by \( s = 1 - (1 - \Omega)^{1/2} \). Introducing the power spectrum \( P(\lambda, k) \) of the matter density fluctuations defined by
\[
\delta_0^2(\lambda) = P(\lambda, k) \delta^2(k - k')
\]
and the relation \( P_\delta(\lambda, k) = -6\Omega \delta_0^2(\lambda) a^{-1}(\lambda) \), obtained via the Poisson equation for small scales, the last expression (11) is
\[
\langle S(n) \delta(n') \rangle \approx \frac{6\Omega}{\pi} \int_0^1 d\lambda T_x(\lambda) R_x(\lambda) \int_0^1 d\lambda' R_x(\lambda') C_{\delta\delta}(\lambda, \lambda'; x).
\]
where
\[
C_{\delta\delta}(\lambda, \lambda'; x) = \int_0^1 d\lambda' \left( \frac{\lambda}{1 - \lambda} \right)^2
\]
Finally, taking another \( D_x \) derivative on the equation (9), we get the following correlations
\[
C_{\mu\mu}(\theta) \equiv 2 \langle \kappa(n) \delta(n') \rangle \propto A_0,
\]
\[
C_{\mu\delta}(\theta) \equiv \langle \mu(n) \delta(n') \rangle \propto A_2,
\]
where \( J_1 \) is the Bessel function of the first kind. These are the basic formulae to be applied when two different populations are correlated. In particular, the background–foreground correlation function is given by Bartelmann (1995).
\[
\xi_{\mu\delta}(\theta) \propto b(\chi - 1) C_{\mu\delta},
\]
where \( \chi \) is the slope of the background source number counts.

If the background population is concentrated at a certain redshift \( \chi_b \), corresponding to a distance \( \lambda_b \), having a Dirac delta distribution \( R_b(\lambda) = \delta(\lambda - \chi_b) \), then the window \( T_b \) is given by
\[
T_b(\lambda) = 0, \lambda \geq \lambda_b.
\]
where
\[
T_b(\lambda) = \left( \frac{1}{\lambda - \chi_b} \right) \left( \frac{1 - (1 - \Omega) \lambda_b}{1 - (1 - \Omega) \lambda} \right), \lambda \leq \lambda_b.
\]
If the foreground population has also a Dirac delta distribution form \( R_f(\lambda) = \delta(\lambda - \lambda_f) \), then the window \( T_f \) in equation (16) is
\[
T_f(\lambda) = T_f(\lambda_b) = 0, \lambda \geq \lambda_b.
\]
where \( \xi_{\mu\delta}(\theta) \propto b(\chi - 1) C_{\mu\delta} \).

### 2.4 Background autocorrelations

Let us consider a population of background sources (e.g. galaxies) placed at different distances \( \lambda \) with a distribution \( R_b(\lambda) \). We are now interested in the gravitational lensing properties induced by the population itself. The magnification–magnification \( C_{\mu\mu} \), polarization–polarization \( C_{pp} \), and magnification–overdensity \( C_{\mu\delta} \) correlations are defined by
\[
C_{\mu\mu}(\theta) \equiv 4 \langle \kappa(n) \delta(n') \rangle \propto D_D D'_D \langle S(n) S(n') \rangle,
\]
\[
C_{pp}(\theta) \equiv 2 \langle p(n) p(n') \rangle \propto \langle p(n) p(n') \rangle \propto D_D D'_D \langle S(n) S(n') \rangle - \frac{1}{2} C_{\mu\delta}(\theta),
\]
\[
C_{\mu\delta}(\theta) \equiv 2 \langle \kappa(n) \delta(n') \rangle \propto D_D \langle S(n) \delta(n') \rangle.
\]
After a straightforward calculation, one obtains

\[ \langle S(n)S(n') \rangle = \frac{72 \Omega}{\pi} \int_{0}^{1} d\lambda \frac{T_{A}^{2}(\lambda)}{(1 - \lambda)^{4}} \]

\[ \times \int_{0}^{1} dk k^{-3} P(\lambda, k) J_{0}(ks \theta), \]  

(23)

where the Limber approximation has been assumed and \( T_{b} \)

is given by equation (7). Now, taking into account the definitions

for the correlations, one gets (see also Kaiser 1992, 1997; Jain 

& Seljak 1996)

\[ C_{\mu \nu}(\theta) = \frac{72 \Omega}{\pi} \int_{0}^{1} d\lambda \frac{T_{A}^{2}(\lambda)}{(1 - \lambda)^{4}} \]

\[ \times \int_{0}^{\infty} dk k P(\lambda, k) J_{0}(ks \theta), \]  

(24)

\[ C_{\mu}(\theta) = C_{\mu \mu}(\theta), \]  

(25)

the last expression being in agreement with Villumsen (1995b). For a population with a Dirac delta distribution \( R_{A} = \delta(\lambda - \lambda_{0}) \), the window \( T_{b}(\lambda) \) is given by equation (18).

Moreover, \( C_{\mu}(\theta) \) is given by equation (14) but now \( R_{A} \)

must be substituted by \( R_{b} \). Trivially, for a strongly peaked \( z \)

distribution such a correlation is very small (see the depend­

ence on \( \lambda_{b} - \lambda_{f} \) in equation 19).

It is interesting to note that the observed correlation function \( w(\theta) \)

associated to a population is given by the following contributions

(Villumsen 1995b)

\[ \frac{\delta N}{N} (n) = b \delta(n) + (\alpha - 1) \mu(n), \]  

(26)

\[ w(\theta) = \left( \frac{\delta N}{N} (n) \frac{\delta N}{N} (n') \right) \]

\[ = b^{2} C_{\delta \delta} + 2b (\alpha - 1) C_{\delta \mu} + (\alpha - 1)^{2} C_{\mu \mu}, \]  

(27)

where \( C_{\delta \delta} \) is the matter correlation function. So gravita­

tional lensing modifies the intrinsic correlation approxi­

mately by the magnification term.

3 RESULTS

With the formalism presented in the previous section we have calculated the correlations \( C_{\delta \delta}, C_{\delta \mu} \) and \( C_{\mu \mu} \). We assume a CDM model with a primordial Harrison–Zeldo­

vich spectrum, a Hubble parameter \( h = 0.5 \) (\( H = 100h \) km 

s\(^{-1}\) Mpc\(^{-1}\)) and flat as well as open universe models. For the power spectrum we have used the fit given by equation (G3) of Bardeen et al. (1986) which is normalized to the cluster abundance: \( \sigma_{8} = 0.6 \Omega_{m}^{-0.58} \) \( F(\Omega) = 0.34 + 0.28 \Omega - 0.13 \Omega^{2} \),

following Viana & Liddle (1996) (see also White, Efstathiou 

& Frenk 1993; Eke, Cole & Frenk 1996; and see Kaiser 1997 for a discussion on alternative normalizations). For the non-linear evolution of the power spectrum we use the recently improved fitting formula given by Peacock & Dodds (1996). That formula is based on the Hamilton et al. (1991) scaling procedure to describe the transition between linear and non-linear regimes. It accounts for the correction

introduced by Jain, Mo & White (1995) for spectra with \( n \leq -1 \) and applies to flat as well as open universes.

For the redshift distributions of the background and foreground sources we consider a Dirac delta distribution peaked at \( \lambda_{b} \) and \( \lambda_{f} \) respectively, \( R_{b}(\lambda) = \delta(\lambda - \lambda_{b}) \) and \( R_{f}(\lambda) = \delta(\lambda - \lambda_{f}) \). These simple distributions are very useful since they reduce the calculations and the results differ only slightly when compared to other more realistic distributions (see below). \( C_{\delta \delta} \) and \( C_{\mu \mu} \) are computed using equations (14), (15), (19). \( C_{\mu\mu} = C_{\mu\mu} \) is obtained from equation (25) where the function \( T_{b} \) is given by equation (18).

3.1 Background magnification–foreground matter cross-correlations

The cross-correlation \( C_{\delta \mu}(\theta) \) for a population of background sources peaked at \( z_{b} = 1 \) and another of foreground lenses at \( z_{f} = 0.2 \) is given in Fig. 1. The effect is maximum at zero lag and rapidly decreases for scales above a few arcmin. The amplitude is always above a few per cent for scales \( \leq 1 \) arcmin, and at these scales the linear contribution is negligible compared to the non-linear one for all the \( \Omega \) values. Notice that \( C_{\mu \mu} \) increases when \( \Omega \) decreases, whereas considering only the linear contribution the situation is reversed.

From the observational point of view it is useful to calculate the average cross-correlation (or equivalently the mean relative excess of background sources around foreground lenses) within a given radius \( b \). \( C_{\delta \mu}(\theta) \). The variation of \( C_{\delta \mu}(1 \) arcmin) with \( z_{f} \) for different values of \( z_{b} \) and a 1 arcmin radius is shown in Fig. 2. A maximum amplitude of a few per cent is obtained at a \( z_{f} \) in the range 0.1–0.25 for all background populations and all cosmological models. (At this point it is important to recall that, to compare with observations of background–foreground object correlation \( C_{\delta \mu}(\theta) \), the bias factor \( b \) of the foreground population and the slope \( \alpha \) of the background population enter into the calculation following equation 16.) It is interesting to note that there are already large galaxy samples available, like the APM or COSMOS catalogues, which peak at a redshift

Figure 1. \( C_{\delta \mu}(\theta) \) for a foreground population peaked at \( z_{f} = 0.2 \) and a background one at \( z_{b} = 1 \) and for three values of \( \Omega \): 1 (solid), 0.3 (dotted) and 0.1 (dashed). The three bottom lines represent the linear contribution for the same \( \Omega \) values.
within that range (for the APM catalogue \(\langle z \rangle = 0.16\) for a magnitude limit \(B_J = 20.5\), see Efstathiou 1995). Moreover, the use of realistic redshift distributions to represent the foreground and background populations (with a bell-like shape similar to that of the APM one) changes the results in only \(\approx 10\) per cent for the relevant angular scales compared to the Dirac delta distribution used here. Those catalogues, which except for a bias factor are assumed to follow the large-scale matter distribution, are therefore very suitable to cross-correlate with a background source population. This has already been done by Benítez & Martínez-González (1995, 1997) for the 1-Ly and Parkes samples of radio-loud QSOs as background populations, finding clear evidence of positive cross-correlations. In Fig. 3, we show \(C_{\mu}(\theta, z_b)\) for \(0.1 \leq \Omega \leq 1\) and for \(z_r = 0.15\) and \(z_r = 1\), the mean redshift values appropriate for the galaxy and radio QSO samples considered above. The dependence of \(C_{\mu}\) with \(\Omega\) is more relevant at small angular scales. In Fig. 4, we also represent the average cross-correlation \(\bar{C}_{\mu}(1\text{ arcmin})\) as a function of \(\Omega\). A rough comparison between its expected amplitude and the measured value for the COSMOS–Parkes samples, as given in Fig. 4 of Benítez & Martínez-González (1997), shows agreement for realistic values of \(\Omega\) and the bias parameter. A detailed comparison of the theoretical calculations with the observational results will be given elsewhere. Dolag & Bartelmann (1997) have recently presented calculations of the QSO-galaxy correlation function for QSO and galaxy populations produced by gravitational lensing due to the large-scale structure, following a similar theoretical scheme.

### 3.2 Background polarization–foreground matter cross-correlations

The cross-correlation between the polarization of a background source population peaked at \(z_b\) and the matter density fluctuations peaked at \(z_r\), \(C_{\mu}(\theta)\), is given in Fig. 5 for \(z_r = 1\) and \(z_r = 0.3\). The maximum value is in the angular range 0.4–1 arcmin, and this scale is smaller for low \(\Omega\) models. As in the case of \(C_{\mu}\), the linear contribution is negligible; however, it peaks at a much larger angle of \(\sim 10\) arcmin as a consequence of the much larger scales which contribute to the linear level. Including the non-linear evolution, a correlation of \(\sim 1\) per cent can be expected at angular scales \(\leq 1\) arcmin for realistic models of structure formation. The use of realistic redshift distributions to represent the foreground and background populations (with a bell-like shape typical of magnitude limited samples) changes the results in only \(\approx 10\) per cent of the sample for the relevant angular scales compared to the Dirac delta distribution used here.

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**Figure 2.** \(\bar{C}_{\mu}(1')\) as a function of the foreground population redshift \(z_f\) and for background redshifts \(z_b = 0.5, 1, 2, 5\). (a) \(\Omega = 1\); (b) \(\Omega = 0.3\); (c) \(\Omega = 0.1\).

**Figure 3.** \(C_{\mu}(\theta, \Omega)\) for a foreground population peaked at \(z_f = 0.15\) and a background one at \(z_b = 1\).

**Figure 4.** \(\bar{C}_{\mu}(1')\) as a function of \(\Omega\) for the same values of \(z_f\) and \(z_b\) as in Fig. (3).
3.3 Magnification and polarization autocorrelations

In this subsection we concentrate on a single population of background sources peaked at a given $z_b$ and calculate $C_{pp}(\theta)$. This is done in Fig. 7 for $z_b=1$. The maximum effect is at zero lag and its amplitude is relatively small, < 1 per cent. The non-linear contribution clearly dominates over the linear one but in this case the amplitude grows with $\Omega$, contrary to the cross-correlations $C_{pl}(\theta)$ and $C_{lp}(\theta)$. Our results are in agreement with Jain & Seljak (1996), and with Kaiser (1997) for one-degree scales (see those papers also for a more detailed analysis of $C_{pp}$, where the dominant contribution is the linear one. Considering also the slope of the population of sources following equation (26), $C_{pp}$ could be estimated from the observed autocorrelation of faint galaxies. Nevertheless, measuring $C_{pp}$ should be more feasible from the practical point of view, as we do not have to disentangle the contribution caused by lensing from the intrinsic correlations; it is usually assumed (and strongly hoped) that the intrinsic ellipticities of background galaxies are not correlated.

4 CONCLUSIONS

We have obtained the expressions for the correlations between the magnification or polarization of background sources and the foreground matter distribution as function of the non-linear evolution of the power spectrum. These formulae are valid for flat and open universes.

For the cross-correlation of the background magnification and foreground matter distribution, $C_{pl}(\theta)$, the maximum is at zero lag and the amplitude remains above a few per cent for scales $\lesssim 1$ arcmin. $C_{pl}(\theta)$ increases significantly when $\Omega$ decreases. The linear contribution is negligible compared to the non-linear one for the relevant scales below a few arcmin. Varying the redshift of the foreground population, $z_f$, a maximum amplitude of a few per cent for the integrated correlation $C_{pl}(1\text{ arcmin})$ is obtained at $0.1 \lesssim z_f \lesssim 0.25$ for all background populations and all cosmological models.

Figure 5. $C_{pl}(\theta)$ for a foreground population peaked at $z_f=0.3$ and a background one at $z_b=1$ and for three values of $\Omega$: 1 (solid), 0.3 (dotted) and 0.1 (dashed). The three bottom lines represent the linear contribution for the same $\Omega$ values.

Figure 6. $C_{pl}(\theta)$ as a function of the foreground population redshift $z_f$ and for background redshifts $z_b=0.5, 1, 2, 5$. (a) $\Omega=1$; (b) $\Omega=0.3$; (c) $\Omega=0.1$.

Figure 7. $C_{pp}(\theta)$ [or equivalently $C_{pp}(\theta)$] for a population peaked at $z_b=1$ and for three values of $\Omega$: 1 (solid), 0.3 (dotted) and 0.1 (dashed). The three bottom lines represent the linear contribution for the same $\Omega$ values.
The cross-correlation of the background polarization and foreground matter distribution \( C_{\rho}(\theta) \), presents a maximum of the order of 1 per cent at a non-null angle, typically in the range \( 0.4-1 \) arcmin. Fixing the redshifts \( z_f \) and \( z_b \) of the two populations, the angular scale of the maximum decreases with \( \Omega \). \( C_{\rho}(\theta) \) increases significantly when \( \Omega \) decreases. The linear contribution is negligible compared to the non-linear one for the relevant scale below a few arcmin. Varying the redshift of the foreground population, \( \frac{\partial C_{\rho}}{\partial \Omega} \) presents a maximum amplitude of \( \sim 1 \) per cent for the integrated correlation \( C_{\rho}(1 \text{ arcmin}) \) is obtained within a relatively wide range \( 0.2 \leq z_b \leq 0.5 \). This amplitude grows appreciably with \( z_b \), being a factor of \( \approx 2 \) different between \( z_b = 0.5 \) and 2.

Finally, the correlation of the magnifications for a single population, \( C_{\mu}(\theta) \), has a relatively small maximum amplitude \( \lesssim 1 \) per cent in all cases. The amplitude grows with \( \Omega \), contrary to the cross-correlations \( C_{\rho}(\theta) \).

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APPENDIX

Some equations (e.g. equation 11) have been obtained assuming the Limber approximation, i.e. one assumes that the maximum scale of appreciable correlation is small compared to the typical distances to the foreground and background objects in the samples. So, if we consider a double integral with the same form as equation (10), the contribution to the integral is appreciable only when the points are nearly at the same time \( \lambda \approx \lambda' \) and their separation is small, \( |x - x'|_0 \ll \lambda \). When \( \theta \ll 1 \), the spatial separation of two points is

\[
r^2 = s^2 + s'{}^2 - 2s s' \cos \theta \approx (s - s')^2 + ss' \theta^2; \tag{28}
\]

Defining the new variable \( t = \lambda' - \lambda \), the separation can be rewritten as \( r \approx |t^2 + \theta^2|^{1/2} [1 - (1 - \Omega) \lambda^2]^{-1} \).

Therefore, the double integral can be approximated by

\[
I \sim \int_0^1 \int_0^1 d\lambda' R_s(\lambda') T_s(\lambda') \int_0^{1-x} d\lambda C[\lambda'; t(r)] \tag{29}
\]

where \( C[\lambda'; t(r)] \equiv C[\lambda'; t(r)] \) and \( T_s(\lambda + t) \approx T_s(\lambda) \), the latter being a smooth function. The previous integral for the variable \( t \) can be approximated by \( 2 \int_0^1 \) if one assumes that the foreground sources are placed at distances far away from the observer and Hubble distance (case of practical interest). Finally, changing the variable \( r \) by \( r \) one gets equation (11). We remark that equation (11) is also valid when a Dirac distribution is assumed to represent the foreground redshift distribution. The Limber approximation in Fourier space has also been considered by Kaiser (1992) in the context of weak gravitational lensing.

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\text{REFERENCES}
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