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Representation of Infiltration in Adaptive Rainfall – Runoff Models

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The reliability of the extended Time Compression Approximation (TCA), commonly adopted in watershed models in order to represent the infiltration associated with erratic rainfalls, is investigated. This approximation is considered as a component of an adaptive real-time flood forecasting model. The forecasted flows are compared with those obtained replacing the extended TCA with the Complex Storm Point Infiltration Model (CSPIM) recently proposed by Smith *et al.* (1993). The discharge forecasted through the infiltration component based on the numerical solution of Richards' equation is used as a bench mark. The models were applied to situations representative of real areas in Central Italy. The CSPIM based watershed model was found to provide excellent results. The TCA based model, in spite of the adaptive component, yielded poor results for various rainfall patterns. However, it seems to be a reasonable approximation when a uniform rainfall spatial distribution is involved.

Introduction

Rainfall-runoff conceptual models should give primary importance to the representation of the infiltration process. In fact, infiltration provides a considerable contribution to losses in a wide variety of soils characterizing most natural basins. Many approximate representations have been proposed for a continuous infiltration process within a given storm with rainfall rate, r, sufficiently larger than natural saturated hydraulic conductivity, K_s. These representations usually rely on the assumption of a constant initial soil water content. Among them the physically based approach proposed by Mein and Larson (1973), then reformulated by Chu (1978) for variable rainfall patterns, and that by Smith and Parlange (1978) have been more extensively used. They have been applied in watershed modeling both considering a simple point formulation with spatially "equivalent" soil hydraulic properties (Corradini and Melone 1986) and coupling the point formulation with an explicit representation of the spatial variability of these properties (Bresler and Dagan 1983, Dagan and Bresler 1983, Smith and Hebbert 1983). The aforementioned approaches have been widely tested and may be successfully used for many field situations, particularly in fine textured soils. However, there are situations which require the computation of the infiltration associated with intermittent rainfall. For storms producing immediate ponding after a rainfall hiatus this problem is commonly solved through the classical Time Compression Approximation, TCA (Reeves and Miller 1975, Sivapalan and Milly 1989). For the other storms, producing a pre-ponding period after a rainfall hiatus, a procedure which really is an extension of the TCA is frequently adopted. Specifically, the extended TCA is based on the cumulative depth approximation shown by Smith (1982) and on the continuity of the infiltration rate, v_0, under ponding conditions (see Péschke and Kutílek 1982, Corradini *et al.* 1987). It may be considered reliable only for a very limited number of storms with relatively low rainfall rates. Smith *et al.* (1993) have recently proposed a much more adequate point infiltration model for complex storms (designated henceforth as CSPIM) which allows an accurate representation of infiltration in fine textured soils characterized by a small hysteresis loop. The model is physically based and represents, on a continuous basis, the processes of infiltration during a first storm leading the surface to natural saturation, soil water redistribution during a moderately long period with $r<K_s$, and then re-infiltration associated with a successive storm with $r>K_s$. It has been found that within a short period at the beginning of the re-infiltration the CSPIM provided a substantial improvement in comparison with the previously available methods. However, despite the fact that the CSPIM is inherently a very efficient model, its usefulness as a component of adaptive watershed modeling must be proved. In fact it requires more experimental data on hydraulic soil properties and produces a relative increase in model complexity, furthermore the adaptive estimate of model parameters could in principle reduce its utility.

The main objective of this paper is to address the last issue using a simplified framework of the on-line flood forecasting model earlier proposed by Corradini and Melone (1986) and then utilized by Corradini (1991). This model is used in three different versions obtained incorporating as infiltration component the extended TCA, the CSPIM and the numerical solution of Richards' equation. The last model version is used as a yardstick and is denoted hereafter as the reference model.

Definition of the Infiltration Problem

Given a time-varying rainfall rate, r, we consider the problem of one-dimensional infiltration into a homogeneous soil. Starting at a time $t = 0$ when the vertical distribution of soil water is uniform, rainfall is assumed to produce natural saturation at the soil surface at a time $t = t_{p1}$, and then to keep surface saturation up to a time $t = t_h$. Afterwards the rainfall rate is suddenly dropped to zero and, with $r = 0$, during an interval from t_h to t_n evaporation is disregarded. Then rainfall starts again, surface natural saturation is re-established at a time $t = t_{p2}$, and saturation kept until storm end. The Richards' equation describing the water flow is

$$\frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z}\left[K(\theta)\frac{d\psi}{d\theta}\frac{\partial \theta}{\partial z}\right] - \frac{dK}{d\theta}\frac{\partial \theta}{\partial z} \qquad (1)$$

where z is a vertical coordinate directed downward; θ is the volumetric soil water content; $K(\theta)$ is the hydraulic conductivity; $K(\theta)d\psi/d\theta = D(\theta)$ is the soil water diffusivity, with ψ representing the soil water matric potential. The initial condition is

$$\theta = \theta_i \qquad z \geq 0 \qquad t = 0 \qquad (2)$$

The boundary condition imposed in the pre-ponding stages concerns the surface flux, v_0; it is

$$-D(\theta)\frac{\partial \theta}{\partial z} + K(\theta) = v_0(t) = r(t) \qquad z = 0 \qquad \begin{cases} 0 < t \leq t_{p1} \\ t_n < t \leq t_{p2} \end{cases} \qquad (3)$$

while in the post-ponding stages involving natural saturation water content, θ_s, at the surface, where water accumulation is neglected, it is replaced by

$$\theta = \theta_s \qquad z = 0 \qquad \begin{cases} t_{p1} < t \leq t_h \\ t > t_{p2} \end{cases} \qquad (4)$$

Further, in the no-rainfall period Eq. (3) becomes

$$-D(\theta)\frac{\partial \theta}{\partial z} + K(\theta) = 0 \qquad z = 0 \qquad t_h < t \leq t_n \qquad (5)$$

The rainfall infiltration in the post-ponding stages is formally given by Darcy's law through the water content profile determined as a solution of the posed problem.

The hydraulic soil properties may conveniently be described by the relations (Smith *et al.* 1993)

$$\psi(\theta) = \psi_b\left[\left(\frac{\theta - \theta_r}{\theta_s - \theta_r}\right)^{-c/\lambda} - 1\right]^{1/c} + d \qquad (6)$$

$$K(\theta) \equiv K_s \left(\frac{\theta - \theta_r}{\theta_s - \theta_r} \right)^{b+a/\lambda} \tag{7}$$

where ψ_b is the air entry pressure; $\theta_r < \theta_i$ is the residual soil water content; c, λ and d are empirical coefficients making it possible to obtain a very flexible $\psi(\theta)$ curve which fits most experimental data well; K_s is the natural saturation hydraulic conductivity; and b and a are coefficients assumed, according to Burdine's method, equal to 3 and 2, respectively. Note that Eq. (6) is an extended form of both the Brooks and Corey (1964) and the van Genuchten (1980) equations.

Water flow problems of the kind considered above have to be frequently investigated in rainfall-runoff modeling for medium and large basins, where different storms separated by many hours may contribute to the same flood event generating multiple peaks.

Some Remarks on the CSPIM

The physically based model proposed by Smith *et al.* (1993) is addressed to fine textured soils characterized by D increasing rapidly near θ_s. Its main feature is the assumption of a compound wetting profile in an interval of the re-infiltration period ranging from t_n to a time t_m. The latter denotes the time when a primary wetting profile produced by the first storm and a secondary one associated with the successive storm merge into a single front moving deeper into the soil. A detailed description of this approach can be found in the cited reference; we provide in Appendix, for the sake of completeness, a short account.

On-Line Flood Forecasting Model

We consider a basin with measurements of point rainfall at various locations and of discharge at the outlet, Q_E, all available in real time. Representative space invariant soil hydraulic properties are adopted. The basin is divided into n zones, assumed as homogeneous, by isochrones of travel time from the outlet. Within each zone the mean areal rainfall rate is used for r. Infiltration is considered to determine the major loss. The effective rainfall in each zone is used to compute the discharge, Q, following in the classical Clark translation-routing procedure which involves as parameters the time of concentration, T_c, and the storage coefficient, T, of a fictitious linear reservoir placed at the basin outlet (Linsley *et al.* 1982). Through this linear approach Q may be expressed by

$$Q(t) = B + \sum_{1}^{n}{}_m \int_0^t [r_m(\tau) - v_{0,m}(\tau)] h_m(t-\tau) d\tau \tag{8}$$

where B is the base flow assumed as time invariant; m is a subscript designating a given zone; and h_m denotes the instantaneous unit hydrograph of the m-th zone given by

$$
\begin{aligned}
h_m(t-\tau) &= \frac{A_m}{T} \exp\left[-\frac{t-\tau-\eta_m}{T}\right] && t-\tau > \eta_m \\
h_m(t-\tau) &= 0 && 0 \le (t-\tau) \le \eta_m
\end{aligned}
\qquad (9)
$$

with A_m and η_m area and average travel time, respectively, for the m-th zone.

Flow forecasts are performed at the current time, t_b, for a lead time L and then repeated at successive time steps. The forecasted flow $Q_F(t_b + L)$ is given formally by Eq. (8) with $t = t_b + L$ and $r_m - v_{0,m} = 0$ beyond t_b.

When infiltration is represented by the CSPIM or by the solution of Richards' equation the estimate of Q_F requires the knowledge of the hydraulic soil properties, Eqs. (6) and (7), of the parameters T_c and T, determined in advance through calibration events, and of the initial soil water content. The last quantity is considered to be known in the reference model while when the CSPIM is involved it is adapted on-line using the objective function $M(\theta_i)$. This function is formally expressed by the sum of the squares of the errors in discharge obtained for the current time and for the previous time origin of forecast, $t_b - \Delta t$. It is subject to a constraint on the trend of the computed discharge. Specifically, we have

$$
\begin{aligned}
M(\theta_i;\psi(\theta),K(\theta),T_c,T) &= \sum_{0}^{1}{}_j [Q_E(t_b-j\Delta t)-Q(t_b-j\Delta t)]^2 = \\
&\sum_{0}^{1}{}_j \left[Q_E(t_b-j\Delta t)-[B+\sum_{1}^{n}{}_m \int_{0}^{t_b-j\Delta t} [(r_m(\tau)-v_{0,m}(\tau)]h_m(t_b-j\Delta t-\tau)d\tau\right]^2 \to \min
\end{aligned}
\qquad (10)
$$

$$
\frac{Q_E(t_b)-Q_E(t_b-\Delta t)}{Q(t_b)-Q(t_b-\Delta t)} > 0 \qquad (11)
$$

However, if no solutions to Eqs. (10) and (11) exist, then the solution satisfying only Eq. (10) is used. Because the selected objective function generally results in $Q(t_b) \neq Q_E(t_b)$, a runoff scaling factor correcting the residual error is derived. This quantity is then used together with the optimal θ_i in estimating Q_F. We remark in reality θ_i should be a constant for a given event, but, because of the difficulty in its practical determination, in the CSPIM based model it is used as a fitting parameter. In a given event, a narrow range of θ_i values would mean therefore that the model is well-suited.

For the extended TCA based model the estimate of Q_F is made according to Corradini (1991). In particular the same scheme of the CSPIM based model is used, but with the "sorptivity" incorporated as a fitting parameter adapted on-line without the constraint on its maximum value.

Experimental Data and Model Results

A synthetic basin with most characteristics similar to those known for a real basin in Central Italy (Tiber River basin at S. Lucia) was selected. The basin area was 934 km^2, with T_c = 11h and T = 8h (Corradini 1991). Variations of the last two parameters within 20% were also considered. Base flow was set equal to zero.

The hydraulic soil properties and the parameters of the CSPIM were typical of clay loam and referred to Soil B of Smith *et al.* (1993). With ψ in mm and K in mmh^{-1}, according to Eqs. (6) and (7) we have

$$\psi(\theta) = -800\left[\left(\frac{\theta-0.1225}{0.21}\right)^{-25}-1\right]^{0.2} + 100$$

and

$$K(\theta) = 0.4\left(\frac{\theta-0.1225}{0.21}\right)^{13}$$

moreover, the CSPIM parameters defined in Appendix were β = 0.85, p = 2 and α = 0.8.

Synthetic rainfall patterns representative of the above-mentioned area were chosen. Rainfall was distributed over three sub-areas, each having spatially uniform rainfall rate, and in three time intervals each with constant rainfall rate. The rainfall distributions are summarized in Table 1.

Flow forecasts were performed for a maximum lead time of 6h and updated at 1-h intervals. The discharges forecasted by the application of the reference and the CSPIM based models were very similar, with the $Q_F(t_b + L)$ curves for L = 6h nearly coincident for all the events analysed. This result may be ascribed to the fact

Table 1 – Synthetic rainfall events over the selected basin. Successive rainfall periods of duration t_u, in h, and constant rainfall rate r, in mmh^{-1}, are indicated for three basin sub-areas.

Event serial number	Upstream region (272 km^2)						Central region (363 km^2)							Downstream region (299 km^2)						
	t_u	r	t_u	r	t_u	r	t_u	r	t_u	r	t_u	r	t_u	r	t_u	r	t_u	r	t_u	r
1	4	0	4	7	5	0	5	8	6	0	2	10	5	8	4	0	2	10	2	0
2	10	0	4	7	5	0	5	8	12	0	2	10	5	8	10	0	2	10	2	0
3	20	0	4	7	5	0	5	8	22	0	2	10	5	8	20	0	2	10	2	0
4	6	8	4	0	6	7	16	0					5	7	0	6	7	3	0	
5	6	8	10	0	6	7	22	0					13	0	6	7	3	0		
6	6	8	20	0	6	7	32	0					23	0	6	7	3	0		
7	6	7	4	0	4	7	6	7	4	0	4	7	6	7	4	0	4	7		
8	6	7	10	0	4	7	6	7	10	0	4	7	6	7	10	0	4	7		
9	6	7	20	0	4	7	6	7	20	0	4	7	6	7	20	0	4	7		

Table 2 – Percent peak errors, $\epsilon_{p\theta}$ and $\epsilon_{p\theta_i}$, for the TCA based model in the adaptive and non-adaptive version, respectively.

Event serial number	Rainfal spatial distribution	$\epsilon_{p\theta}$	$\epsilon_{p\theta_i}$
1	variable	+9.7	+9.7
2	variable	+16.8	+16.8
3	variable	+27.6	+27.6
4	variable	−3.9	+2.2
5	variable	−9.2	+3.9
6	variable	−17.5	+5.6
7	uniform	+4.5	+6.7
8	uniform	+7.1	+13.3
9	uniform	+2.4	+21.6

that despite the CSPIM with the correct θ_i produced typical errors on the post-hiatus infiltration within ± 10% (Smith *et al.* 1993), these errors were substantially corrected by the adaptive procedure.

The extended TCA component, which neglects redistribution, provides underestimates of infiltration in the initial part of the post-hiatus period when therefore, in the absence of the adaptiveness, the forecasted flows should be overestimated. However, the adaptive component, which tends to counterbalance empirically the error at the time of forecast, may modify substantially the errors in flow prediction. A summary of the errors in the forecasted peak flow is given in Table 2. As can be seen the adaptive approach may also be worse than the non-adaptive one. A deep analysis of the results for a few sample events allows us to clarify the origin of these errors. Figs. 1-3 show the behaviour of the direct runoff predicted by the TCA based model for events involving no-rainfall periods of 20h. The curves designated as "observed" were derived by the reference model but with the additional knowledge of r_m beyond t_b; they may therefore be considered representative of the actual hydrographs. It is apparent as the adaptive component may lead to opposite prediction errors. In event 3 of Fig. 1 the flow forecasts $Q_F(t_b + 6\text{h})$ performed by the TCA based model earlier than the main rising limb of the "observed" hydrograph (29h$\leqslant t_b \leqslant$32h) were greatly overestimated because:

a) the variability in space of rainfall was such that a single storm occurred in the zone nearest to the outlet, where therefore the infiltration computed by the extended TCA and CSPIM were coincident, and two storms occurred in the remaining part of the basin, where the two infiltration components in principle produced rather different results;
b) at a given time of forecast t_b with the "observed" discharge caused mainly by the

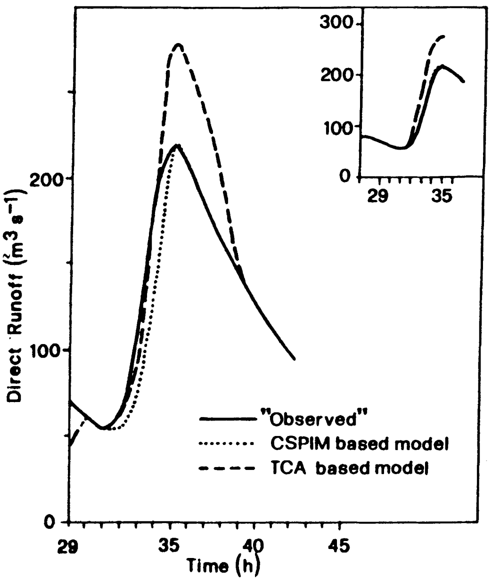

Fig. 1. Comparison of "observed" and forecast direct runoff hydrographs for event 3 of Table 1 and for a 6-h lead time. Forecasts performed 6h earlier than the "observed" peak for lead times up to 6h are also shown in the upper part.

effective rainfall occurred in the sub-area closer to the outlet, the adaptive estimate of the parameters for the TCA based model was not affected by errors due to the computation of infiltration by the extended TCA;

c) at the time t_b + 6h, the effective rainfall coming from farther sub-areas and overestimated by the TCA based model reached the outlet but its contribution to Q_F was not counterbalanced by the adaptive estimate previously carried out at t_b.

Instead, the opposite behaviour of $Q_F(t_b + 6\text{h})$ in the period 28h$\leq t_b \leq$33h of Fig. 2, associated with event 6, can be ascribed to the fact that:

a) the discharge "observed" at $t = t_b$ and produced by the effective rainfall coming from the nearest sub-area was erroneously computed by the TCA based model because of the redistribution period between the two successive storms;

b) the adaptive correction of the model parameters counterbalanced the error reducing properly the effective rainfall in the nearest sub-area, but in the meanwhile the effective rainfall was incorrectly reduced in the remaining basin area;

c) at the time t_b + 6h, the effective rainfall from farther sub-areas reached the outlet and Q_F was therefore underestimated.

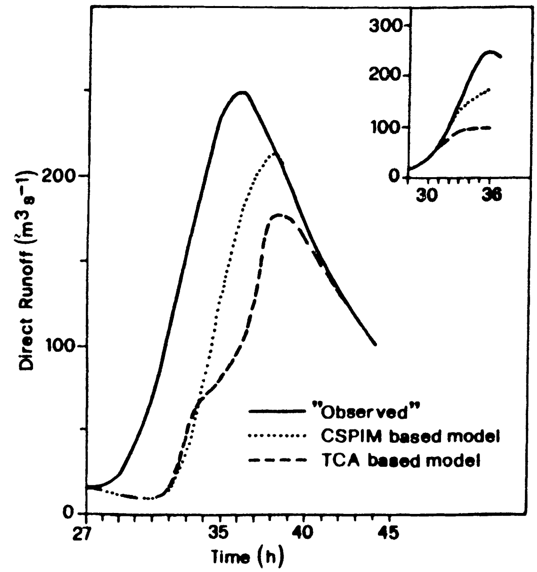

Fig. 2. As in Fig. 1 except for event 6 of Table 1.

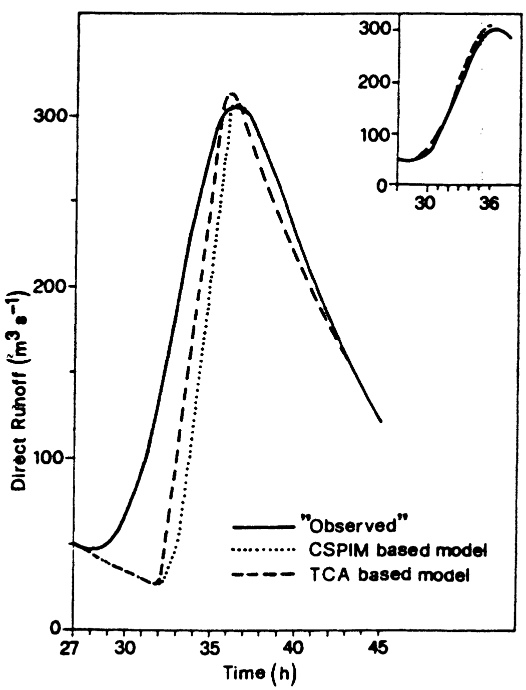

Fig. 3. As in Fig. 1 except for event 9 of Table 1.

Table 3 – As in Table 2 except for the events with uniform rainfall spatial distribution and for different values of the parameters T_c and T of the translation-routing procedure.

Event serial number	$T_c \equiv 11.0h$	$T \equiv 8.0h$	$T_c \equiv 8.8h$	$T \equiv 6.4h$	$T_c \equiv 13.2h$	$T \equiv 9.6h$
	$\epsilon_{p\theta}$	$\epsilon_{p\theta_i}$	$\epsilon_{p\theta}$	$\epsilon_{p\theta_i}$	$\epsilon_{p\theta}$	$\epsilon_{p\theta_i}$
7	+4.5	+6.7	+2.5	+6.1	+3.2	+6.6
8	+7.1	+13.3	+0.8	+11.9	+3.8	+13.1
9	+2.4	+21.6	+10.7	+18.6	+1.1	+20.0

For rainfall distribution uniform in space, the forecast errors from the TCA based model were quite limited. In the representative event of Fig. 3, for $26h<t_b\leqslant30h$ the relative role of the effective rainfall coming from different sub-areas was again different at a given t_b and at t_b + 6h. However, being the error in computing the infiltration by the extended TCA of the same type in the entire basin, the adaptive correction of the model parameter worked adequately and the quantity $Q_F(t_b$ + 6h) was much more appropriate than in the events with rainfall variable in space. In addition, the results for uniform rainfall were found to have substantially a light sensitivity to variations in the characteristics of the transformation effective rainfall-direct runoff (see Table 3).

In order to extend our analysis to the hypothetical situation of models involving a correct rainfall prediction, we repeated the previous computations for r_m known beyond t_b. The forecasts from the CSPIM based model were clearly coincident with those obtained through Richards' equation. Those from the TCA based model experienced significant variations, but their relative differences remained practically unchanged.

Lastly, the model results revealed that decreasing the redistribution time interval between two successive storms the TCA based model errors became smaller, with minor values for $t_n - t_h$ = 4h. This was expected because for short $t_n - t_h$ the extended TCA component, independently of adaptiveness, was inherently more appropriate.

Conclusions

1) On the basis of synthetic data, it appears that the CSPIM proposed by Smith *et al.* (1993) is a point infiltration formulation adequate as a component of on-line flood forecasting models. Its use within a simple adaptive scheme provided excellent results. We stress that this approach remains to be tested on actual data.

2) The CSPIM can not be usefully replaced by the extended TCA. In fact, in practical situations involving rainfall variability in space and moderately long periods of soil water redistribution the infiltration would be inadequately represented. The adaptive component of the watershed model does not correct for the errors due to the extend TCA scheme even if lead times limited to a few hours are considered.

3) The infiltration described in terms of the extended TCA as a component of on-line flood forecasting models seems to be reasonable in the limits of short redistribution periods or for a nearly uniform rainfall spatial distribution.

The above conclusions may be someway linked with the specific simplifications involved in the representation of the other physical processes and with the selected adaptive procedure. Nevertheless, the real possibility of making marked errors in flow forecasts through the extended TCA supports in any case the adoption of the CSPIM approximation.

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Appendix

During the first storm up to t_{p1}, the infiltration rate v_0 is expressed by

$$v_0 = r \qquad 0 < t \leq t_{p1} \tag{A1}$$

while in the post-ponding period it is represented by the soil infiltration capacity f in the form

$$v_0 = f = K_s\left[1 + \frac{\alpha}{\exp[(I-K_i t)\alpha/\{(\theta_s-\theta_i)G(\theta_i,\theta_s)\}]-1}\right] \qquad t_{p1} < t \leq t_h \tag{A2}$$

where α is a parameter linked with the variation of $K(\theta)$ near θ_s (Parlange *et al.* 1982); K_i stands for $K(\theta_i)$; I is the cumulative infiltration depth; $G(\theta_i, \theta_s)$ is expressed through a generalized definition of sorptivity, $S(\theta_a, \theta_b)$, obtained by an extension of the sorptivity as used by Smith and Parlange (1978). We have

$$G(\theta_a,\theta_b) \equiv \frac{S^2(\theta_a,\theta_b)}{2K_s(\theta_b-\theta_a)} = \frac{1}{K_s}\int_{\theta_a}^{\theta_b} D\,d\theta \tag{A3}$$

which in Eq. (A2) is referred to the specific case with $\theta_a = \theta_i$ and $\theta_b = \theta_s$. The evolution in time of f is obtained combining Eq. (A2) with the following relation for I (t)

$$K_s(t-t_{p1})(1-\alpha) \equiv I-I_{p1}-(\theta_s-\theta_i)\left(\frac{K_s}{K_d}\right)G \times \tag{A4}$$

$$\times \ln\left[\frac{\exp[\alpha(I-K_i t)/\{(\theta_s-\theta_i)G\}]-1+\alpha K_s/K_d}{\exp[\alpha(I_{p1}-K_i t_{p1})/\{(\theta_s-\theta_i)G\}]-1+\alpha K_s/K_d}\right]$$

where $K_d = K_s - K_i$, G stands for $G(\theta_i, \theta_s)$ and I_{p1} is the value of I at the time t_{p1}. This latter corresponds to the time when Eq. (A2) with $f = r$ is first satisfied.

We represent the time evolution of the water content profile during redistribution by an equivalent rectangular shaped function of constant θ, denoted by θ_0, from the surface to the location of the equivalent front but scaled by a factor β. The value of θ_0 after a redistribution period $t - t_h$ may be computed by

$$-\frac{d\theta_0}{(\theta_0-\theta_i)^2 G(\theta_i,\theta_0)\beta p K_s + (I_h-K_i t)(\theta_0-\theta_i)(K(\theta_0)-K_i)} = \frac{dt}{(I_h-K_i t)^2} \tag{A5}$$

with $I_h = I(t_h)$ and p is a factor equal to 2.0. Eq. (A5) may be easily solved numerically for $\theta_0(t_n)$ which is then used, together with the cumulative water content in the profile $I(t_n) - K_i t_n$, as an initial condition when rainfall starts again.

In the post-hiatus period a secondary wetting profile is considered to advance alongside the earlier profile, until, at $t = t_m$, they have approximately the same depth and merge to form a single wetting front. The original model may refer to any post-hiatus rainfall value provided $r>K_s$, however, for simplicity we may limit our analysis to significant rainfall rates verifying the condition (Smith *et al.* 1993)

$$r > K_s\left[1 + \frac{\alpha}{\exp[\alpha\{I(t_n)-K_i t_n\}/\{(\theta_n-\theta_i)G(\theta_i,\theta_s)\}]-1}\right] \quad \text{(A6)}$$

Infiltration for $t_n<t\leq t_m$ is described by forms of (A2) and (A4) with cumulative infiltration now accumulated after t_n, with θ_n in all the functions earlier dependent on θ_i, and with t_{p2} and I_{p2} instead of t_{p1} and I_{p1}, respectively. Further, a correction of these relations for $\beta \neq 1$ is applied. Then for $t>t_m$, infiltration is still represented by forms of Eqs. (A2)-(A4) obtained from their original version modified by incorporating the actual cumulative infiltration in the wetting front, I', instead of $I - K_i t$ and by changing t with $t - (t_m - t_a)$ in Eq. (A4). The last correction, where t_a is computed as the value of t which satisfies Eq. (A4) with $I'(t_m)$ instead of $I - K_i t$, results from continuity of f at $t = t_m$.

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