Fig. 1 Coexistence of lean phase flow and packed bed flow showing decreasing pressure profile down the hole (Leung and Jones, reference [1])

in the downhole without taking into account the gas-solid slip and the voidage requires more qualifications before the analysis can be applied constructively.

In his earlier analysis, Kojabashian [4] took into account the gas-solid slip and defined the flow regimes based on $\frac{\partial u_s}{\partial P} w_s w_g$. In their recent analysis, Leung and Jones [1] classified the flow regimes based on $\frac{\partial u_g}{\partial P} w_s$ and presented quantitative flow regime diagram for an alumina catalyst. A similar approach may be used to analyze the present problem without having to resort to the no-slip assumption.

An Experimental Study of the Flow-Induced Motions of a Flexible Cylinder in Axial Flow

J. B. MILES. The authors have made some worthwhile measurements and observations related to an interesting and complex fluid-structure interaction phenomenon. As the authors are no doubt aware, much remains to be done before a thorough understanding of the problem is obtained. In the interest of suggesting avenues for future study, and also to caution readers about extrapolating these results to other configurations, the following comments are offered:

1. A quantitation description of the important properties of the subject cylinder is lacking. As an example, what are the values of $E$ and $I$, suggested as being important by equation (2). Would the reported results likely be different if the Tygon tubing had thicker or thinner walls? What about the effect of other than circular cross-section; of pressurizing the tubing?

2. The phenomenon of "static divergence" was given considerable attention by the authors, but I was unable to decipher their meaning of static divergence. In support of the importance of being quantitative (or at least identifying) important tubing properties, as per Comment #1, the authors cite a prior study in which static divergence is not observed on a polypropylene cylinder with a rough surface.

3. Care was taken that the Tygon cylinder was neutrally buoyant, and this probably eliminated from immediate consideration the effect of relative density of structure and fluid. However, is this not in fact an important consideration?

4. The availability of multiple channels for the optical displacement measurements would be a significant advance. This would allow for simultaneous determination of hori-
ontal as well as vertical displacements, recognizing that horizontal displacements are equally as significant as vertical displacements. Also, two-point space-time correlations should enable the investigation of such phenomenon as travelling waves.

5. In the interest of generalizing the measured spectrum, why not appropriately normalize them, by flow velocity squared as an example. Similar consideration might be given to the frequency variable on these spectrums.

6. My interpretation of equation (5) and the associated text is that the local tube displacement as a function of time, \( D(t) \), can be uniquely constructed from the power spectrum of this displacement. This seems impossible in view of the random nature of this displacement.

7. The last paragraph of the Conclusions highlights my concerns about applying the reported observations to other systems. Certainly the statement about vibration sensitivity in the 1-10 Hz range must be restricted to systems "similar" to the one tested. Moreover, as yet we do not even know what constitutes similarity, so the restriction might actually be to identical systems.

P. A. PFUND\(^3\). The authors have presented some interesting results on the flow induced motions of a flexible cylinder in axial flow. They seem to have arrived at a reasonable understanding of the behavior of the long, flexible cylinder. I would hope that they could comment on some of the features of that behavior.

The authors were careful to ensure that the downstream termination did not affect the results. Do the authors have any plans to study the effects of downstream terminations that might be expected to affect the results (e.g., a sphere of larger diameter than the cylinder)? Such terminations might represent towed sensors.

The authors' drag coefficients agree more closely with values representative of a cylinder in an infinite medium than in an annular flow. Is there an explanation for this? The drag coefficients increase by approximately 30 percent between Reynolds numbers of 45,000 and 55,000. The authors suspect that this is due to the flow induced motions, but none of their motion data reflects noticeable differences between Reynolds numbers of 45,000 and 55,000. Can the authors provide any additional insight?

A. D. MADDARUS\(^4\). The authors describe an interesting study of a flow induced vibration problem with some unusual characteristics. This writer would like to know the possibility of an application occurring in which the cable system has a resonance in the 3-10 Hz frequency range, or, for that matter, a strong resonance occurring at higher frequencies. In such a case it would be useful to be able to predict the amplitude of the motion.

The latter objective could be accomplished following the method suggested by Liepmann\(^5\) who demonstrated that, when force is a random function, the relationship between force, displacement, and system impedance is still valid for the power spectral densities (PSD). Thus

\[ g(\omega) = f(\omega)/z(\omega)^2 \]

using the latter author's notation for \( g \) and \( f \) (the PSD of motion and force, respectively) with \( z \) as the system impedance and \( \omega \) the frequency.

It is clear that once the PSD of motion is determined by experiment, the above equation gives the PSD of force. Then, for a system having the same flow geometry but a different mechanical impedance, the PSD, as well as the mean square value of displacement may be found; the latter is expressed as

\[ \langle y^2 \rangle = \int_0^{\infty} g(\omega) \ d\omega \]

For obvious reasons, parameterization of \( f(\omega) \) with respect to flow velocity is a useful thing to do in such an approach.

The writer recognizes that the current problem may be more difficult to handle using the above approach than those considered by Liepmann, but also believes that the potential for success may be sufficient.

Authors' Closure

The authors wish to express their gratitude for the important comments made by the Discussers. Concerning the points made by Dr. Pfund, we do not plan further studies of downstream terminations at this time, though we agree they would be useful. The minimum in the drag coefficient versus Reynolds number data (Fig. 7) occurs at a Reynolds number in the 40,000 to 45,000 range. The onset of static divergence near the downstream end of the tube is in the 40,000 range. This correspondence prompted the authors' suggestion that the higher values of drag coefficient at larger Reynolds number might be associated with flow-induced deformations of the cylinder.

The suggestion of Dr. Maddaus that Liepmann's concepts can be extended to complex fluid-structure interactions such as studied by the authors is an intriguing one. It should be pursued. A potential limitation should be recognized at the outset, however. The flow-induced forces should depend on the magnitude of the motions executed by the flexible member. Hence, conclusions drawn from a study of a very flexible member, such as reported by the authors, might be of limited utility or even misleading if applied to a much more rigid cylinder whose flow-induced motions were only a small fraction of a cylinder diameter.

Professor Miles correctly emphasizes the need for additional work to provide a comprehensive understanding of the flow-induced motions studied and to allow generalization of the results to other configurations. Also, equation (5) should read

\[ D(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)e^{j\omega t} \ d\omega, \]

where \( F(\omega) \) is the Fourier series of the sampled value of the displacement. \( D(t) \) cannot be calculated from a knowledge of the bandwidth and power spectrum only. Finally, the values of \( E \) and \( J \) for use in equation (2) were obtained from the measurements of force versus elongation (Fig. 2) and the measured diameter and wall thickness of the Tygon cylinder.

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