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Application of an Adaptive Forecast Algorithm to the River Västerdalälven

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During the last two decades, advances in electronic engineering, hydrological modelling and systems theory have given considerable benefits to the hydrological forecast developments. Today several powerful adaptive techniques are available, which can improve the reliability of hydrological forecasting. One of these techniques is the self-tuning predictor based on an ARMA type model using direct parameter estimation by recursive least square algorithm. The self-tuning predictor has been tested on the River Västerdalälven in Sweden.

Introduction

Developed forecasting models may be deterministic, stochastic or coupled structural stochastic. It has become obvious in practice that there is a general need for stochastic models, because of their relatively simple computational algorithms, easy model parameter estimation, and small computer memory demand.

The widely used regressional approaches have the disadvantage of not being able to account for the changes in the water regime. This fact may cause significant forecast errors. Attempts to reduce such errors led to a very simple adaptive technique, *i.e.* the preceding time step error is added to the forecast to be issued. Today there are more powerful techniques of adaptive estimation. For cases, in which the structure and parameters of the systems are subject to changes and adaptation, Kalman gave a widely used algorithm (1958), which is nowadays commonly referred to as Kalman – filter. An adaptive on-line prediction algorithm called self-tuning predictor has been developed by Wittenmark (1974). Hydrological applications of the self-tuning predictor are reported by Ganendra (1976) and Ambrus (1980). Ambrus' work served as a model for the studies on River Västerdalälven described in this paper.

Self-Tuning Predictor

The self-tuning predictor is an autoregressive moving average (ARMA) type model.

The moving average model expresses a sequence of events, *e.g.* water level or discharge, in terms of deviations at time $t,\varepsilon(t)$, from the mean, y of the process or sequence of events, $y(t)$. The deviation from the mean of the process is expressed as

$$\varepsilon(t) = e(t) - \Theta_1 e(t-1) - \Theta_2 e(t-2) - \ldots - \Theta_q e(t-q) \qquad (1)$$

in which the Θ_i are the weighted parameters and $e(t-i)$ are random elements (white noise). Eq. (1) represents a moving average process of order g.

The autoregressive process expresses the deviation from the mean of the process, as a finite weighted sum of previous deviations plus a random variable $e(t)$. Thus

$$\varepsilon(t) = \Phi_1 \varepsilon(t-1) + \ldots + \Phi_2 \varepsilon(t-2) + \ldots + \Phi_p \varepsilon(t-p) + e(t) \qquad (2)$$

is an autoregressive process of order p.

Solving many hydrological problems, it may be necessary to include both autoregressive and moving average terms to obtain as parsimonious model. Box and Jenkins (1975) described such as model by expressing the deviation of a variable from its mean as a finite weighted sum of previous deviations plus a finite weighted sum of random variates plus a random element. Thus

$$\varepsilon(t) - \Phi_1 \varepsilon(t-1) - \ldots - \Phi_p \varepsilon(t-p) = e(t) - \Theta_1 e(t-1) - \ldots$$
$$\ldots - \Theta_q e(t-q) \qquad (3)$$

is an autoregressive moving average [ARMA (p,q)] model of autoregressive order p and moving average order q.

The advantage of the self-tuning predictor is that the identification of the original ARMA model is avoided and the parameters of the predictor function are estimated directly. Detailed theoretical description of the self-tuning predictor is given by Wittenmark (1974). This paper gives only the basic recursive formulas of model updating and forecasting after Ambrus (1980).

Prediction

Predicted value of the process, $y(t)$, at time $(t+k)$ is $y(t+k|t)$, where $k(>0)$ denotes the lead time of the forecasts. The condition being that measurements, $y(t)$ are available up to time t. Prediction using the previously estimated parameters and measured variables. Thus

$$\hat{y}(t+k\mid t) = -\alpha(q^{-1})\hat{y}(t+k-1\mid t-1) + \beta(q^{-1})u(t) + \gamma(q^{-1})\mu(t) \quad (4)$$

where the operator polynomials are

$$\alpha(q^{-1}) = \alpha_1 + \alpha_2 q^{-1} + \ldots + \alpha_p q^{1-p}$$

$$\beta(q^{-1}) = \beta_1 + \beta_2 q^{-1} + \ldots + \beta_r q^{1-r}$$

$$\gamma(q^{-1}) = \gamma_1 + \gamma_2 q^{-1} + \ldots + \gamma_s q^{1-s}$$

and q^{-1} is the backward operator, $u(t)$ is time series of the observations at the upstream station; $\mu(t)$ is the forecast error series, p,r and s are the orders of the operator polynomials to be chosen during the adaptation procedure of the model.

The terms in Eq. (4) can be also written in vectorial form

$$\underline{x}^T(t) = [\hat{y}(t+k-1\mid t-1),\ldots,\hat{y}(t+k-p\mid t-p),u(t),\ldots,u(t-r+1),$$
$$\ldots,\mu(t),\ldots,\mu(t-s+1)]$$

which is the vector of variables and

$$\underline{a}^T = [\alpha_1,\alpha_2,\ldots,\alpha_p,\beta_1,\beta_2,\ldots,\beta_q,\gamma_1,\gamma_2,\ldots,\gamma_s]$$

which is vector of the parameters (T is for transpose). Thus Eq. (4) becomes

$$\hat{y}(t+k\mid t) = \underline{x}^T(t)\,\hat{\underline{a}}(t)$$

where $\hat{\underline{a}}(t)$ is the estimate of the parameter vector, $\underline{a}^T$, at the time step t. The estimation is done via recursive least squares method (LS). After receiving the latest measurement data, the weighting matrix and parameter vector are updated recursively at every time step.

Recursive Estimation of the Weighting Matrix $\underline{\underline{P}}$

The matrix $\underline{\underline{P}}$ is the parameter estimation error covariance matrix divided by the variance of the white noise defined as

$$\underline{\underline{P}} = \frac{1}{\delta_\mu^2} E\{(\underline{a}-\hat{\underline{a}})(\underline{a}-\hat{\underline{a}})^T\}$$

It can be easily proved (Kendal and Stuart 1977) that $\underline{\underline{P}}$ is linearly proportional to the prediction error. The weighting matrix of the LS parameter estimation method is actually a convergence matrix if the LS method is considered as a gradient algorithm. The updating of $\underline{\underline{P}}$ is done according to Young (1974), *i.e.*

$$\underline{\underline{P}}(t+1) = \underline{\underline{P}}(t) - P(t)\underline{x}(t)[1 - \underline{x}^T(t)\underline{\underline{P}}(t)\underline{x}(t)]^{-1}\underline{x}^T(t)\underline{\underline{P}}(t) \quad (5)$$

Note that the term in the brackets is a scalar, therefore no matrix inversion is required.

Updating of the Parameter Vector

The parameter updating is based on the forecast error

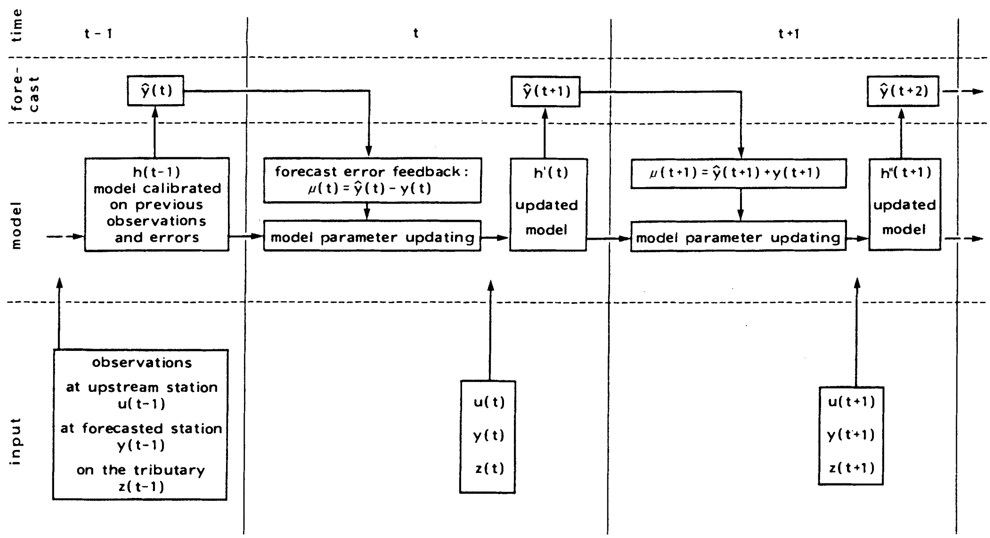

Fig. 1. Scheme of the adaptive forecasting by self-tuning predictor.

$$\underline{\hat{a}}(t+1) = \underline{\hat{a}}(t) - \underline{\underline{P}}(t)\underline{x}(t)[1 + \underline{x}^T(t)\underline{\underline{P}}(t)\underline{x}(t)]^{-1}\,\mu(t) \tag{6}$$

The next forecast is computed by expression Eq. (4) in which the parameters are updated. These recursive steps are repeated during the forecast computations (Fig. 1).

Initial values for the calculation can be chosen arbitrarily. To obtain a fast convergence, $\underline{a}(0) = 0$ and $\underline{\underline{P}}(0) = 10^3 \times \underline{\underline{I}}$ are chosen as suggested by Szöllösi-Nagy (1987). $\underline{\underline{I}}$ is the identity matrix.

The first forecast is made for the m^{th} element of the $y(t)$ series where $m = \max(p,r,s) + 1$. The first p elements and $\underline{x}(0)$ vector are those of $y(t)$ preceding $y(m)$. The next r elements of $\underline{x}(0)$ are taken from the $u(t)$ series in the same way. The last s elements of $\underline{x}(0)$, which are forecast errors preceding the m^{th} time interval, can obviously be chosen as zeros. Given the initial values, the first forecast can be obtained and all the other steps follow recursively. The model, thus step-by-step approaches a stability condition.

Application of the Model

In recent years high floods and water stages have been observed on the River Västerdalälven. Therefore, this river basin (Fig. 2) was chosen in an attempt to apply the model.

The most important step in applying the model, is to choose the orders of the polynomials (dimensions of the model). There is no general rule for dimensioning. Therefore, the computations have to be performed with various autoregressive (observations at the forecasted station), crossregressive (observations at the up-

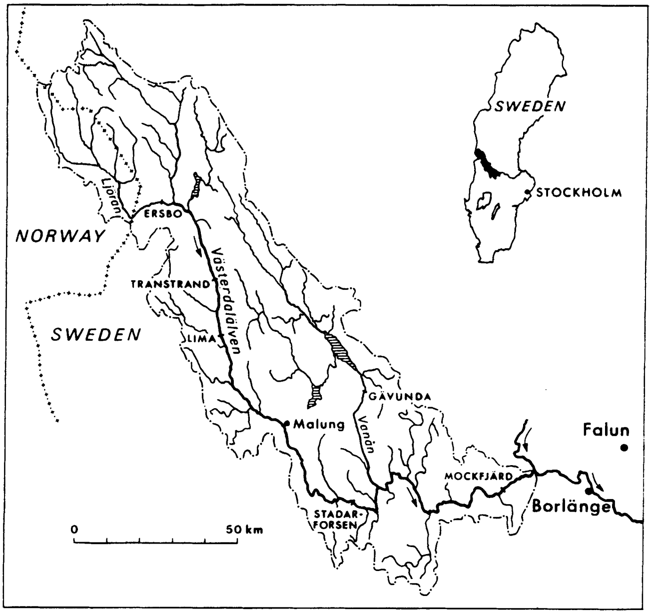

Fig. 2.
The drainage basin of River Västerdalälven.

stream station), auxiliary (observations on tributary, if any) and forecast error parameters. The forecasts were evaluated by the correlation index

$$R = 1 - \left(\frac{\delta_\mu}{\delta_y} \right)^2 \qquad (7)$$

where δ_μ is the standard deviation of the prediction error and δ_y is the standard deviation of the measured time series y itself.

Daily flow data from gauges at Lima, Stadarforsen, Mockfjärd located on the main river and Ersbo and Gävunda on the tributaries were used (see some characteristics of the stations in Table 1). Discharge series for 15 years were handled by the data management part of STATGRAF program package, providing a relatively flexible data handling and statistical analysis.

Before starting the computations, the aims of forecast were defined: Reliable flow forecasts should be issued during any phase of the water regime for the station Mockfjärd with the longest possible lead time, as well as downstream between the closest stations.

It was first assumed that physically longest lead time for the station Mockfjärd could be obtained from the data collected at Ersbo. Forecasts with three, two and one day lead time were computed. Fig. 3 shows that the forecasts were of the highest accuracy when one day lead time was used, even though the flood peak travel time is about 3 days. The discharge at Ersbo slightly influences the water regime at Mockfjärd, so the model became autoregressive. In the regression equations (Table 2), the discharges at Mockfjärd and the forecast errors had the highest

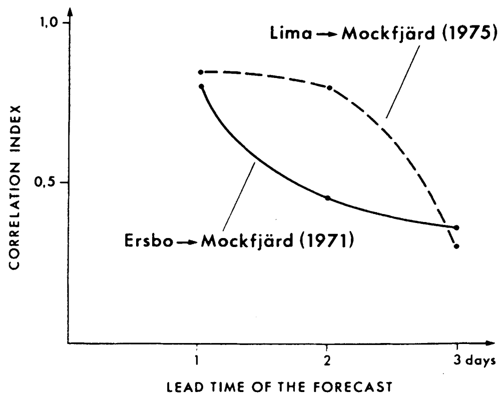

Fig. 3.
Accuracy of the forecasts as function of the lead time - upstream station: Ersbo and Lima, forecasted station: Mockfjärd.

weights. Thus, prediction of river discharges at Mockfjärd was not possible using hydrological data from Ersbo as input.

The next observation station downstream from Ersbo was Lima. The three day ahead forecast possibility between Lima and Mockfjärd was tested, but the model gave late forecasts. Again, the model became autoregressive and followed the phenomena with a definite lag (Fig. 4). The discharges observed at Lima appeared to have a greater effect on the water regime in the case of two-day ahead forecast. The correlation index became as high as 0.81 in 1975. Auxiliary station Gävunda was found to be important for the reach, because it characterized the ungauged flow between stations Lima and Mockfjärd.

The second goal, as stated before, was that forecasts should be issued for the downstream gauges of the closest stations. The computations show that one day ahead predictions between them are reliable (Figs. 5-7).

The initial model dimension was chosen as (3-3-0-1), *i.e.* the model included three autoregressive, three crossregressive, no auxiliary and one error parameter. The number of the parameters were kept constant throughout the computation,

Table 1 – Characteristics of the discharge stations in the Västerdalälven.

	River	Elevation m	Basin km^2	Mean flow m^3/s	Max flow m^3/s	Mean max m^3/s	Distance to Mockfjärd
Ersbo	Ljöran	393	1,101	25	401	200	196
Transtrand	Dalälven	346	2,644	49	582	315	175
Lima	Dalälven	341	3,266	57	647	346	150
Stadarforsen	Dalälven	250	4,506	71	722	411	65
Gävunda	Vanån	265[a)]	2,067	25[a)]	217[a)]	105	32+60[b)]
Mockfjärd	Dalälven		8,493	114	1,468	646	0

a) valid for Landobyn just upstream of Gävunda where the basin is 2,045 km^2
b) 32 km travel distance in Vanån.

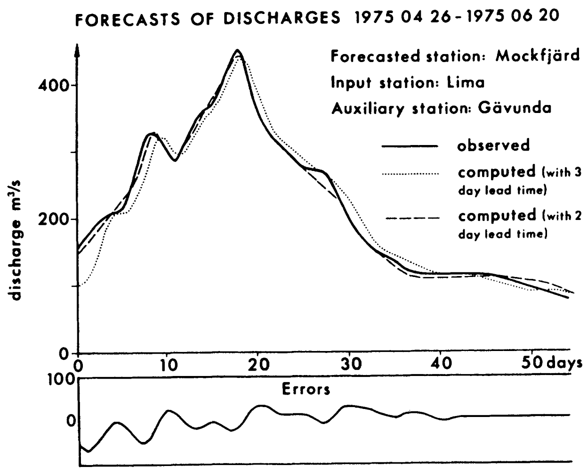

Fig. 4. Forecast simulation by self-tuning predictor assuming different lead time intervals.

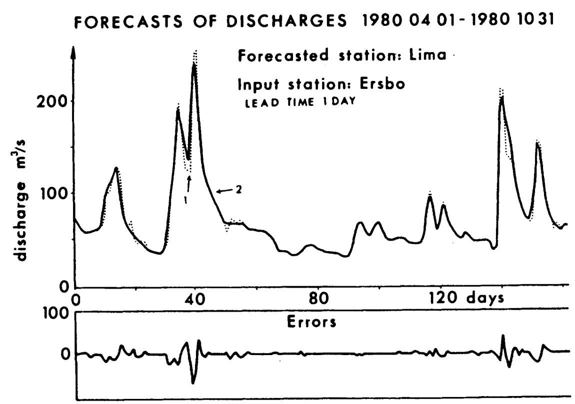

Fig. 5. Forecast simulation by self-tuning predictor. (1-computed, 2-observed hydrograph).

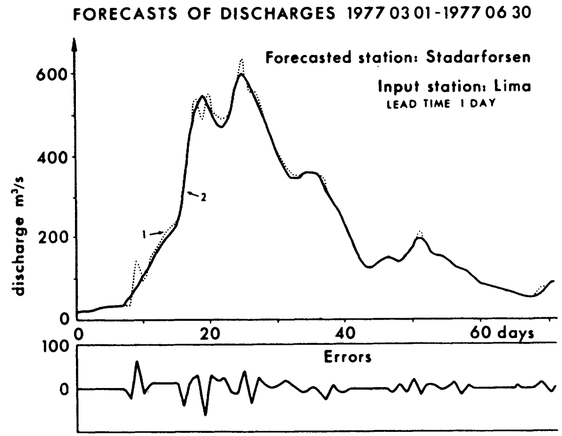

Fig. 6. Forecast simulation by self-tuning predictor. (1-computed, 2-observed hydrograph).

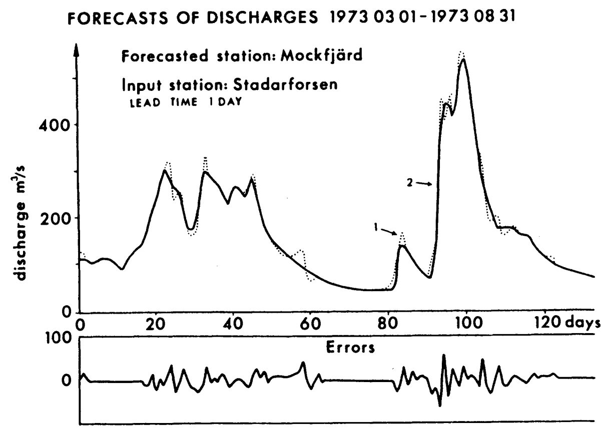

Fig. 7. Forecast simulation by self-tuning predictor. (1-computed, 2-observed hydrograph).

Table 2 – Self-tuning dimensions and parameter values for the reach between the stations Ersbo and Mockfjärd (1972 and 1975). Changes of parameter values are shown for the cases of highest correlation index of the given year.

Lead time (day)	Correlation index	Number of parameters in the regressional equation Autoregressive (Mockfjärd)	Crossregressive (Ersbo)	Error
3	0.34	3	3	1
2	0.46	3	3	1
1	0.57	3	3	1
1	0.52	1	1	1
1	0.62	3	1	1
1	0.74	3	1	3
1	0.78	3	3	3
Parameter values at				
100th step		1.37 −0.58 0.04	−1.44 0.53 1.44	1.32 −0.61 0.06
200th step		1.84 −1.22 0.36	0.63 −0.32 0.31	1.90 −1.06 0.37
300th step		1.83 −1.18 0.35	0.62 −0.28 −0.35	1.87 −1.03 0.36
1	0.66	3	1	3
1	0.76	3	3	3
Parameter values at				
100th step		0.90 −0.15 0.20	6.25 −4.23 −1.22	1.00 −0.10 0.26
200th step		1.65 −0.80 0.14	0.34 0.04 −0.37	1.71 −0.73 0.16
300th step		1.63 −0.78 0.14	0.31 0.13 −0.44	1.62 −0.71 0.17

Table 3 – Chosen model dimensions for the one-day ahead forecast between neighbouring stations.

Forecasting from station	to station Lima	Stadarforsen	Mockfjärd	Correlation index	for period
Ersbo	3-3-0-1*			0.78	1980.04-01-10.31
Lima		1-3-0-1*		0.91	1980.04.01-10.31
Stadarforsen			3-3-0-1*	0.86	1981.01.01.-12.31

* number of autoregressive-crossregressive-auxiliary-error components

except for the reach between stations Lima and Stadarforsen, where the model dimensions were changed to (1-3-0-1). The zero dimension means that no auxiliary station (tributary) was included. The dimensions presented in Table 3 were chosen after a series of computations.

It seems that the self-tuning predictor effectively uses the information of the inputs, since for efficient forecasts the error series shows only slight autocorrelation at lag one (Fig. 8).

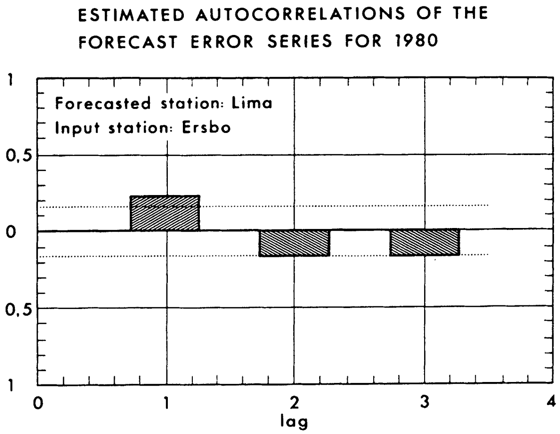

Fig. 8.
Analysis of the forecast error series. (The lead-time of the forecast is one day).

Conclusion

A self-tuning predictor based upon an ARMA type model has been adopted for the river flow forecasting in the Dalälven Basin. The advantage of this model is that its parameters do not require prior estimation. After receiving the latest measurement data, the parameter vector and the weighting matrix, which is also the covariance matrix of the parameter estimation error, are updated recursively at every discrete time interval and forecasts are made.

Parameters are set to arbitrary values at the beginning of a model run. The parameters converge fast and obtain the optimal values after 40 to 50 steps. During a model run, changes of the water regime are reflected by continued parameter updating.

There is no general rule for the selecting of model dimensions. The optimal model dimensions for every different reach should be chosen by trial-and-error. Using the flow data from the last 15 years, numerous computations have been made. It turned out that for a considerable number of cases the best forecasts were given by three autoregressive, three crossregressive and one error parameter.

The addition of an auxiliary station only improved the results if the flow at that station made a significant contribution to the downstream flow of the main river flow. In other cases the auxiliary station only increased the number of model parameters, but the additional information did not compensate the instability caused by the increased parameter number (Ambrus 1980). It is also important to note that during a model run several steps are needed to obtain the optimal parameter values. If the tributary's discharges have a relatively low rate in the main river flow, but its water regime has sudden changes and/or the changes are not synchronized with the main flow, the updating procedure is not able to adopt them into optimal parameter values.

The results show that reliable forecasts can be issued for the gauge at Mockfjärd two days ahead using Lima as a input station. Lead time between the closest stations is one day. The computation error feed back provided a possibility for automatic account of the lateral flow between the upstream and forecasted station. If the data were available for the tributaries and they contributed significant volume to the main flow, there was a possibility to include them into the model inputs.

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